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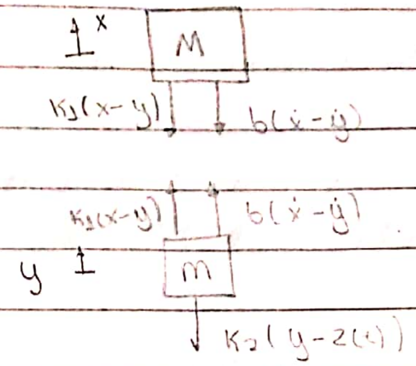
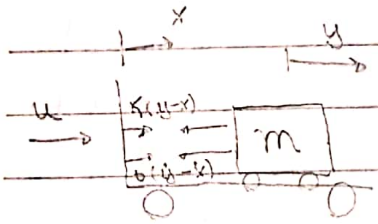
Ex 2.



$$M\ddot{x} = -K_1(x-y) - b(\dot{x}-\dot{y})$$
$$m\ddot{y} = K_1(x-y) + b(\dot{x}-\dot{y}) - K_2(y-z(t))$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{y}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -K_1/M & -b/M & K_1/M & b/M\\ 0 & 0 & 0 & 1\\ K_1/m & -b/m & \frac{-K_1-K_2}{m} & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ K_1/m\end{bmatrix} z(t)$$

Ex 3.



$$m\ddot{y} + K(y-x) + b(\dot{y}-\dot{x}) = 0$$
$$M\ddot{x} - K(y-x) - b(\dot{y}-\dot{x}) = u(t)$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{y}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -K/M & -b/M & K/M & b/M\\ 0 & 0 & 0 & 1\\ K/m & b/m & -K/m & -b/m\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0\\ 1/M\\ 0\\ 0\end{bmatrix} u(t)$$

3.1) como $m \gg M$; $m\ddot{y} = u(t)$, ficando assim com:

$$\begin{bmatrix}\dot{y}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0\\ 1/m\end{bmatrix} u(t)$$

Ex 4 para pequenos movimentos

$$m_1\ddot{x}_1 + b_1\dot{x}_1 + (K_{p1}+K_1)x_1 - K_{p1}z(t) - K_1 x_G - b_1\dot{x}_G - l_1\theta K_1 - l_1\dot{\theta} b_1 = 0$$

$$m_2\ddot{x}_2 + b_2\dot{x}_2 + (K_{p2}+K_2)x_2 - K_{p2}z(t-\alpha) - K_2 x_G - b_2\dot{x}_G + l_2\theta K_2 + l_2\dot{\theta}_2 b_2 = 0$$

$$M\ddot{x}_G + (b_1+b_2)\dot{x}_G + (K_1+K_2)x_G - K_1x_1 - b_1\dot{x}_1 - K_2x_2 - b_2\dot{x}_2 + K_1 l_1\theta$$
$$- K_2 l_2\theta + b_1 l_1\dot{\theta} - b_2 l_2\dot{\theta} = 0$$

$$J_G\ddot{\theta} + (b_1 l_1^2 + b_2 l_2^2)\dot{\theta} + (K_1 l_1^2 + K_2 l_2^2)\theta - K_1 l_1 x_1 - b_1 l_1 \dot{x}_1$$
$$+ K_2 l_2 x_2 + b_2 l_2 \dot{x}_2 + (K_1 l_1 - K_2 l_2)x_G + (b_1 l_1 - b_2 l_2)\dot{x}_G = 0$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_G \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -(K_{p1}+K_1)/m_1 & 0 & K_1/m_1 & \frac{K_1 l_1}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 l_1}{m} \\ 0 & -(K_{p2}+K_2)/m_2 & K_2/m_2 & \frac{-K_2 l_2}{m_2} & 0 & -b_2/m_2 & b_2/m_2 & -b_2 l_2/m \\ K_1/M & K_2/M & \frac{-(K_1+K_2)}{M} & \frac{K_2 l_2 - K_1 l_1}{M} & \frac{b_1}{M} & b_2/M & \frac{-(b_1+b_2)}{M} & \frac{(b_2 l_2 - b_1 l_1)}{M} \\ K_1 l_1/J_G & -K_2 l_2/J_G & \frac{K_2 l_2 - K_1 l_1}{J_G} & \frac{-(K_1 l_1^2 + K_2 l_2^2)}{J_G} & \frac{b_1 l_1}{J_G} & -\frac{b_2 l_2}{J_G} & \frac{b_2 l_2 - b_1 l_1}{J_G} & \frac{(-b_1 l_1^2 + b_2 l_2^2)}{J_G} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_G \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \end{bmatrix}$$

Ex 5

$(M+m)\ddot{x} - ml\dot{\theta}^2 \operatorname{sen}\theta - ml\ddot{\theta}\cos\theta = u$

$\frac{4ml^2\ddot{\theta}}{3} - mgl\operatorname{sen}\theta + \ddot{x}\cos\theta = 0$

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \\ \dot{\omega}_4 \end{bmatrix} = \begin{bmatrix} \omega_3 \\ \omega_4 \\ \left[\left(g - \frac{ml\omega_4^2}{M+m}\cos(\omega_2)\right)\operatorname{sen}\omega_2 - \frac{u(t)\cos\omega_2}{M+m}\right]\left(\frac{4}{3}l + \frac{ml\cos^2\omega_2}{M+m}\right)^{-1} \\ \left(u(t) + ml\omega_4^2\operatorname{sen}\omega_2 + \frac{3}{4}mg\operatorname{sen}\omega_2\cos\omega_2\right)\left(M + m + \frac{3}{4}m\cos^2\omega_2\right. \end{bmatrix}$$

Ex 6.

$$\begin{cases} m\ddot{x} - mg + \frac{KI^2}{x^2} = 0 \\ L\dot{I} + RI = V(t) \end{cases}$$

$w(t) = [x \ \dot{x} \ i]^T = [\omega_1 \ \omega_2 \ \omega_3]^T$

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} \omega_2 \\ g - \frac{K\omega_3^2}{m\omega_1^2} \\ \frac{V(t)}{L} - \frac{R}{L}\omega_3 \end{bmatrix}$$

b) $$\begin{bmatrix} \delta\dot{x} \\ \delta\ddot{x} \\ \delta\dot{I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2K\dot{x}_{eq}^2}{m x_{eq}^3} & 0 & -\frac{2K I_{eq}}{m x_{eq}^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta\dot{x} \\ \delta i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \delta V(t)$$

Sendo x_{eq}, $\dot{x}_{eq}$, I_{eq} os valores das variáveis de estado no ponto de equilíbrio do sistema

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Ex8. Aplicando o TMQM em torno do eixo x, temos:

$$J_x\ddot{\theta}_x + 2B\dot{\theta}_x + 2Kd\,\theta_x = -J_w\dot{\theta}_z$$

estado:

$$x(t) = \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix}$$

entrada:

$$u(t) = \dot{\theta}_z(t)$$

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{2Kd}{J_x} & -\frac{2B}{J_x} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta}_x \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{J_w}{J_x} \end{bmatrix} \int \dot{\theta}_z\,dt$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta}_x \end{bmatrix}$$