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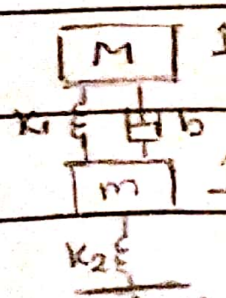
Exercícios das Aulas 01, 06 / 10

2)



$$\begin{cases} M\ddot{x} + b(\dot{x}-\dot{y}) + K_1(x-y) = 0 \\ m\ddot{y} + b(\dot{y}-\dot{x}) + K_1(y-x) + K_2(y-z) = 0 \end{cases}$$

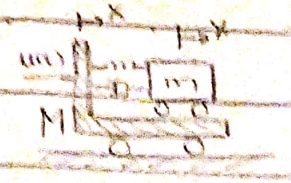
$$u = [x \;\; y \;\; \dot{x} \;\; \dot{y}]^T$$

Espaço de Estados:

$$\begin{cases} \dot{u} = Au + Bz \\ r = Cu + Dz \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\dot{x}} \\ \dot{\dot{y}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_1/M & K_1/M & -b/M & b/M \\ K_1/m & -\frac{(K_1+K_2)}{m} & b/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_2/m \end{bmatrix} z$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} z$$

3) 

$$\begin{cases} M\ddot{x} + b(\dot{x}-\dot{y}) + k(x-y) = u(t) \\ m\ddot{y} + b(\dot{y}-\dot{x}) + k(y-x) = 0 \end{cases}$$

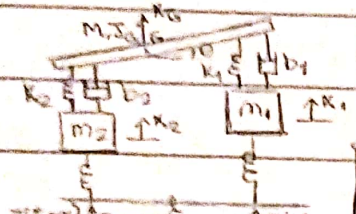
3.2) $X = [x, y, \dot{x}, \dot{y}]^T \rightarrow \dot{X} = AX + Bu$ | $Y = [\dot{x}\ \dot{y}]^T \rightarrow Y = CX + Du$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/M & k/M & -b/M & b/M \\ k/m & -k/m & b/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} u(t) \qquad \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(t)$$

3.1) Para $m \gg M$: $m\ddot{y} = u(t)$

$X = [y\ \dot{y}]^T \rightarrow \dot{X} = AX + Bu$ | $Y = \dot{y} \rightarrow Y = CX + Du$

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u(t) \qquad \dot{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + 0 \cdot u(t)$$

4) 

$$\begin{cases} m_1\ddot{x}_1 - b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) - k_1(x_G - x_1 + l\theta) + k_{p1}(x_1 - z(t)) = 0 \\ m_2\ddot{x}_2 - b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) - k_2(x_G - x_2 - l\theta) + k_{p2}(x_2 - z(t-\alpha)) = 0 \\ M\ddot{x}_G + b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) + k_1(x_G - x_1 + l\theta) + k_2(x_G - x_2 - l\theta) = 0 \\ J_G\ddot{\theta} + b_1 l(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2 l(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) + k_1 l(x_G - x_1 + l\theta) - k_2 l(x_G - x_2 - l\theta) = 0 \end{cases}$$

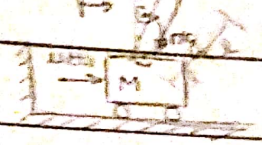
$X = [x_1\ x_2\ x_G\ \theta\ \dot{x}_1\ \dot{x}_2\ \dot{x}_G\ \dot{\theta}]^T$ $\rightarrow \begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases}$

$U = [z(t)\ \ z(t-\alpha)]^T$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{(k_{p1}+k_1)}{m_1} & 0 & k_1/m_1 & k_1 l/m_1 & -b_1/m_1 & 0 & b_1/m_1 & b_1 l/m_1 \\ 0 & -\frac{(k_{p2}+k_2)}{m_2} & k_2/m_2 & -k_2 l/m_2 & 0 & -b_2/m_2 & b_2/m_2 & -b_2 l/m_2 \\ k_1/M & k_2/M & -\frac{(k_1+k_2)}{M} & \frac{l(k_2-k_1)}{M} & b_1/M & b_2/M & -\frac{(b_1+b_2)}{M} & l(b_2-b_1)/M \\ k_1 a/J_G & -k_2 l_2/J_G & \frac{l(k_2-k_1)}{J_G} & -\frac{l^2(k_2+k_1)}{J_G} & b_1 l_1/J_G & -b_2 l_2/J_G & \frac{l(b_2-b_1)}{J_G} & -l^2(b_2+b_1)/J_G \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_{p1}/m_1 & 0 \\ 0 & k_{p2}/m_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5)



Equações linearizadas: $\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u(t) \\ J\ddot{\theta} + ml\ddot{x} - mgl\theta = 0 \end{cases}$

$X = [x\ \theta\ \dot{x}\ \dot{\theta}]^T$

$Y = [x\ \theta]^T$

$\begin{cases} \dot{X} = AX + Bu \\ Y = CX \end{cases}$

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m^2l^2}{J(M+m)-m^2l^2} & 0 & 0 \\ 0 & \frac{gml(M+m)}{J(M+m)-m^2l^2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{J}{(M+m-\frac{m^2l^2}{J})} \\ \frac{-Ml}{J(m+M)-m^2l^2} \end{bmatrix} u(t)$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

6) Equações: $\begin{cases} m\ddot{x} + \frac{KI^2}{x^2} - mg = 0 \\ L\dot{I} + RI = V \end{cases}$

$X = [x, \dot{x}, I]^T$, $U = V \rightarrow \dot{X} = AX + BU$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2KI_0^2}{mx_0^3} & 0 & \frac{-2KI_0}{mx_0^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} V$$

8) TMQM: $\boxed{J_x\ddot{\theta}_x + 2b\dot{\theta}_x + Kl\theta_x = J\omega\theta_z}$

$x(t) = [\theta_x\ \dot{\theta}_x]^T$

$u(t) = \dot{\theta}_z(t)$

$\begin{cases} \dot{x} = Ax + Bu \\ Y = Cx \end{cases}$

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{Kl}{J_x} & -\frac{2b}{J_x} \end{bmatrix} \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} + \begin{bmatrix} 0 \\ -J/J_x \end{bmatrix} \dot{\theta}_z$$

$$Y = [1\ 0] \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix}$$