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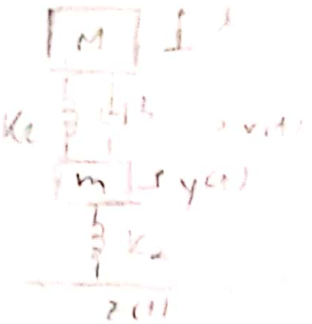
2.

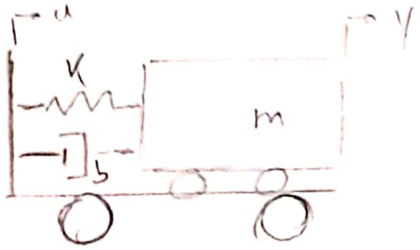


$$M\ddot{x} = -k_1(x-y) - b(\dot{x}-\dot{y})$$

$$m\ddot{y} = k_1(x-y) + b(\dot{x}-\dot{y}) - k_2(y-z)$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{M} & -\frac{b}{M} & \frac{k_1}{M} & \frac{b}{M} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m} & -\frac{b}{m} & \frac{-k_1-k_2}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_2}{m} \end{bmatrix} z(t)$$

3.



$$M\ddot{x} - K(y-x) - b(\dot{y}-\dot{x}) = u(t)$$

$$m\ddot{y} + K(y-x) + b(\dot{y}-\dot{x}) = 0$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M} & -\frac{b}{M} & \frac{K}{M} & \frac{b}{M} \\ 0 & 0 & 0 & 1 \\ \frac{K}{m} & \frac{b}{m} & -\frac{K}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix} + u(t)$$

4.

$q = [x_1\ x_2\ x_G\ \theta\ \dot{x}_1\ \dot{x}_2\ \dot{x}_G\ \dot{\theta}]^T$

$\dot{x} = [\dot{x}_1\ \dot{x}_2\ \dot{x}_G\ \dot{\theta}\ \ddot{x}_1\ \ddot{x}_2\ \ddot{x}_G\ \ddot{\theta}]^T$

$\dot{x} = Ax + Bu$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{(k_1+k_2)}{m_1} & 0 & \frac{k_1}{m_1} & \frac{k_1 l}{m_1} & -\frac{k_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 l}{m_1} \\ 0 & -\frac{(k_1+k)}{m_1} & \frac{k_2}{m_2} & -\frac{k_2 l}{m_2} & 0 & -\frac{b_2}{m_2} & \frac{b_2}{m_2} & -\frac{b_2 l}{m_2} \\ \frac{k_1}{M} & \frac{k_2}{M} & -\frac{(k_1+k_2)}{M} & \frac{(k_2-k_1) l}{M} & \frac{b_1}{m} & \frac{b_2}{m_2} & -\frac{(b_2+b_1)}{M} & \frac{b_2 l - b_1 l}{m} \\ \frac{k_1 l}{J} & -\frac{k_2 l}{J} & \frac{(k_2-k_1) l}{J} & -\frac{l^2(k_1+k_2)}{J} & \frac{b_1 l}{J} & \frac{b_2 l}{J} & -\frac{b_2 l}{J} & \frac{l b_1 b_2}{J} \end{bmatrix}$$

5-

$(M+m)\ddot{x} + ml\ddot{\theta} = u$

$J\ddot{\theta} + ml\ddot{x} - mgl\theta = 0$

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m^2 l^2}{J(M+m) - m^2 l^2} & 0 & 0 \\ 0 & \frac{gml(M+m)}{J(M+m) - m^2 l^2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M+m - \frac{m^2 l^2}{J}} \\ \frac{-gml}{J(M+m) - m^2 l^2} \end{bmatrix} u$$

6-

$m\ddot{x} = mg - \frac{KI^2}{x^2}$

$L\dot{I} + RI = V$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2KI_0^2}{mx_0^3} & 0 & -\frac{2KI_0}{mx_0^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} V$$