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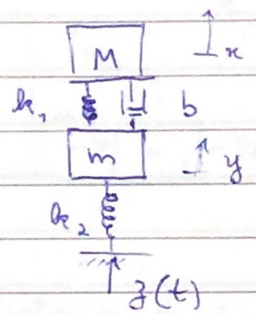
- Exercícios dos dias 1º/10 e 06/10 -

2-



$$M\ddot{x} + k_1(x-y) + b(\dot{x}-\dot{y}) = 0$$

$$\Rightarrow \ddot{x} = \frac{-k_1}{M}\cdot x + \frac{k_1}{M}y - \frac{b}{M}\dot{x} + \frac{b}{M}\dot{y}$$

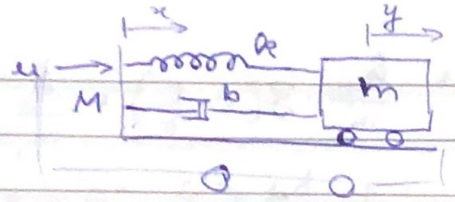
$$m\ddot{y} - k_1(x-y) + k_2(y-z) - b(\dot{x}-\dot{y}) = 0$$

$$\Rightarrow \ddot{y} = \frac{k_1}{m}\cdot x - \frac{(k_1+k_2)}{m}y + \frac{k_2}{m}\cdot z + \frac{b}{m}\dot{x} - \frac{b}{m}\dot{y}$$

$X = [x \quad y \quad \dot{x} \quad \dot{y}]$ | $Y = [x \quad \dot{y}]$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{k_1}{M} & \frac{k_1}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k_1}{m} & \frac{-(k_1+k_2)}{m} & \frac{b}{m} & -\frac{b}{m} \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_2}{m} \end{bmatrix}}_{B} \cdot z$$

$$\begin{bmatrix} x \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

3 - 

$$m\ddot{y} - k(x-y) - b(\dot{x}-\dot{y}) = 0$$

$$\Rightarrow \quad \ddot{y} = \frac{k}{m}x - \frac{k}{m}y + \frac{b}{m}\dot{x} - \frac{b}{m}\dot{y}$$

$$M\ddot{x} + k(x-y) + b(\dot{x}-\dot{y}) - u = 0$$

$$\Rightarrow \quad \ddot{x} = -\frac{k}{M}x + \frac{k}{M}y - \frac{b}{M}\dot{x} + \frac{b}{M}\dot{y} + \frac{u}{M}$$

$$X = [x \ y \ \dot{x} \ \dot{y}]$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{M} & \frac{k}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k}{m} & -\frac{k}{m} & \frac{b}{M} & -\frac{b}{M} \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ 0 \end{bmatrix}}_{B} \cdot u$$

$$C = [0 \ 1 \ 0 \ 0]$$

4 - $$m_1\ddot{x}_1 + (k_p + k_1)x_1 + b_1\dot{x}_1 - k_p \cdot z(t) - k_1(x_G + b\theta) +$$
$$-b_1(\dot{x}_G + b\dot{\theta}) = 0$$

$$\Rightarrow \quad \ddot{x}_1 = \frac{-(k_p + k_1)x_1}{m_1} - \frac{b_1}{m_1}\dot{x}_1 + \frac{k_p}{m} \cdot z(t) + \frac{k_1}{m}(x_G + b\theta) +$$
$$+ \frac{b_1}{m}(\dot{x}_G + b\dot{\theta})$$

$$m_2\ddot{x}_2 + (k_p + k_1)x_2 + b_2\dot{x}_2 - k_p \cdot z\left(t - \frac{l}{v}\right) - k_2(x_G - a\theta) - b_1(\dot{x}_G - a\dot{\theta}) =$$
$$= 0$$
$$\Rightarrow$$

$$\Rightarrow \ddot{x}_2 = -\frac{(k_p + k_2)}{m_2} x_2 - \frac{b_2}{m_2} \cdot \dot{x}_2 + \frac{k_p}{m_2} \cdot g\left(t - \frac{l}{v}\right) + \frac{k_2}{m_2}(x_G - a\theta) + \frac{b_2}{m_2}(\dot{x}_G - a\dot{\theta})$$

$$M\ddot{x}_G - k_1 x_1 - b_1 \dot{x}_1 - k_2 x_2 - b_2 \dot{x}_2 + k_1(x_G + b\theta) + b_1(\dot{x}_G + b\dot{\theta}) + k_2(x_G - a\theta) + b_2(\dot{x}_G - a\dot{\theta}) = 0$$

$$\Rightarrow \ddot{x}_G = \frac{k_1}{M} x_1 + \frac{b_1}{M} \dot{x}_1 + \frac{k_2}{M} x_2 + \frac{b_2}{M} \dot{x}_2 - \frac{k_1}{M}(x_G + b\theta) - \frac{b_1}{M}(\dot{x}_G + b\dot{\theta}) - \frac{k_2}{M}(x_G - a\theta) - \frac{b_2}{M}(\dot{x}_G - a\dot{\theta})$$

$$J\ddot{\theta} - k_1 x_1 b - b_1 \dot{x}_1 b + k_2 x_2 a + b_2 \dot{x}_2 a + k_1 b(x_G + b\theta) + b_1 b(\dot{x}_G + b\dot{\theta}) - k_2 a(x_G - a\theta) - b_2 a(\dot{x}_G - a\dot{\theta}) = 0$$

$$\Rightarrow \ddot{\theta} = \frac{k_1}{J} x_1 b + \frac{b_1}{J} \dot{x}_1 b - \frac{k_2}{J} x_2 a - \frac{b_2}{J} \dot{x}_2 a + \frac{1}{J}(k_2 a - k_1 b) x_G + \frac{1}{J}(b_2 a - b_1 b)\dot{x}_G - \frac{1}{J}(k_2 a^2 + k_1 b^2)\theta - \frac{1}{J}(b_2 a^2 + b_1 b^2)\dot{\theta}$$

$X = [x_1 \; x_2 \; x_G \; \theta \; \dot{x}_1 \; \dot{x}_2 \; \dot{x}_G \; \dot{\theta}]^T$ | $Y = [x_G \; \theta]^T$

$U = [g(t) \; g(t - \frac{l}{v})]^T$

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_G \\ \ddot{\theta} \end{bmatrix}
=
\underbrace{\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{-(k_p+k_1)}{m_1} & 0 & \frac{k_1}{m_1} & \frac{k_1 b}{m_1} & -\frac{b_1}{m_1} & 0 & \frac{b_1}{m_2} & \frac{b_1 b}{m_1} \\
0 & \frac{-(k_p+k_2)}{m_2} & \frac{k_2}{m_2} & -\frac{k_2 a}{m_2} & 0 & -\frac{b_2}{m_2} & \frac{b_2}{m_2} & -\frac{b_2 a}{m_2} \\
\frac{k_1}{M} & \frac{k_2}{M} & \frac{-(k_1+k_2)}{M} & \frac{k_2 a - k_1 b}{M} & \frac{b_1}{M} & \frac{b_2}{M} & \frac{-(b_1+b_2)}{M} & \frac{(b_2 a - b_1 b)}{M} \\
\frac{k_1 b}{J} & -\frac{k_2 a}{J} & \frac{k_2 a - k_1 b}{J} & \frac{-(k_2 a^2 + k_1 b^2)}{J} & \frac{b_1 b}{J} & -\frac{b_2 a}{J} & \frac{b_2 a - b_1 b}{J} & \frac{-(b_2 a^2 - b_1 b)}{J}
\end{bmatrix}}_{A}
$$

$$
\cdot \begin{bmatrix} x_1 \\ x_2 \\ x_G \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \end{bmatrix}
+
\underbrace{\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\frac{k_p}{m_1} & 0 \\
0 & \frac{k_p}{m_2} \\
0 & 0 \\
0 & 0
\end{bmatrix}}_{B} \cdot u
$$

$$\begin{bmatrix} x_G \\ \theta \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_G \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

5- $\begin{cases} (M+m)\ddot{x} + m\ell\ddot{\theta} = u & \text{(I)} \\ J\ddot{\theta} - m\ell g\theta + m\ell\ddot{x} = 0 & \text{(II)} \end{cases} \Rightarrow \ddot{\theta} = \frac{m\ell g}{J}\theta - \frac{m\ell}{J}\ddot{x}$

Substituindo $\ddot{\theta}$: $(M+m)\ddot{x} + m\ell\left(\frac{m\ell g}{J}\theta - \frac{m\ell}{J}\ddot{x}\right) = u$

$\Rightarrow \left(M + m - \frac{m^2\ell^2}{J}\right)\ddot{x} + \frac{m^2\ell^2 g}{J}\theta = u$

$\Rightarrow \ddot{x} = -\left(\frac{m^2\ell^2}{MJ + mJ - m^2\ell^2}\right)\theta + \frac{u}{M + m - \frac{m^2\ell^2}{J}}$

$\ddot{\theta} = \frac{m\ell g}{J}\theta - \frac{m\ell}{J}\ddot{x}$

$\Rightarrow \frac{m\ell}{J}\ddot{x} = \frac{m\ell g}{J}\theta - \ddot{\theta} \Rightarrow \ddot{x} = g\theta - \frac{J}{m\ell}\ddot{\theta}$

Substituindo $\ddot{x}$: $(M+m)\left(g\theta - \frac{J}{m\ell}\ddot{\theta}\right) + m\ell\ddot{\theta} = u \Rightarrow$

$$\Rightarrow \left(\frac{-ml}{M+m} + \frac{J}{ml}\right)\ddot{\theta} = g\theta - \frac{u}{M+m}$$

$$\Rightarrow \ddot{\theta} = \left(\frac{gml\;(M+m)}{J(M+m)-m^2l^2}\right)\theta - \left(\frac{gml}{J(M+m)-m^2l^2}\right)u$$

$X = [x\ \theta\ \dot{x}\ \dot{\theta}]$ | $Y = [x\ \theta]$

$$\begin{bmatrix} \dot{x} \\ \theta \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m^2l^2}{MJ+mJ-m^2l^2} & 0 & 0 \\ 0 & \frac{-gml(M+m)}{J(M+m)-m^2l^2} & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} \\ \\ \frac{1}{M+m-\frac{m^2l^2}{J}} \\ \frac{-gml}{J(M+m)-m^2l^2} \end{bmatrix}}_{B} u$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

6-
$$\begin{cases} m\ddot{x} - mg + \frac{k i^2}{x^2} = 0 \\ L\dot{i} + Ri = v \end{cases}$$

$X = [x\ \dot{x}\ i]^T$ | $Y = [x]$

$f_1 = \dot{x} = \dot{x}$ | $f_2 = \ddot{x} = g - \frac{k i^2}{m}$ | $f_3 = \dot{i} = \frac{u}{L} - \frac{Ri}{L}$ | $g_1 = y = x$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \dot{x}} & \frac{\partial f_1}{\partial i} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} & \frac{\partial f_2}{\partial i} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial \dot{x}} & \frac{\partial f_3}{\partial i} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \cdot v$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{i} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -\frac{2ki_0^2}{mx_0^3} & 0 & -\frac{2ki_0}{mx_0^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}}_{A} \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}}_{B} \cdot v$$

$$X = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \cdot \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}$$

7- a) Motor: $L \cdot \dot{i} + R \cdot i = V_a - k_b \Omega$

$$J_m \cdot \dot{\Omega}_1 = T_1 - B_m \cdot \Omega_1 - 2\,|\theta_1 - \theta_2|\,(\theta_1 - \theta_2)$$
$$\Rightarrow J_m \cdot \dot{\Omega}_1 = k \cdot i - B_m \Omega_1 - 2\,|\theta_1 - \theta_2|\,(\theta_1 - \theta_2)$$

Carro: $m\ddot{x}_1 + 2\dot{x}_1^3 + kx_1 - \frac{2}{R}\,|\theta_1 - \theta_2|\,(\theta_1 - \theta_2) = 0$

Vínculo cinemático: $x_1 = \theta_2 R \Rightarrow \dot{x}_1 = \Omega_2 R$

Vetor de estados: $X = [i \;\; \theta_1 \;\; x_1 \;\; \dot{\theta}_1 \;\; \dot{x}_1]^T \rightarrow$ Sistema de 5ª Ordem e.) $f_1 = 2|\theta_1 - \theta_2|(\theta_1 - \theta_2) = 2\left|\theta_1 - \frac{x_1}{R}\right|\left(\theta_1 - \frac{x_1}{R}\right)$

$$f_1 \cong f(0) + \left.\frac{\partial f}{\partial \theta_1}\right|_{eq}(\theta_1 - \theta_{1eq}) + \left.\frac{\partial f}{\partial \theta_2}\right|_{eq}(\theta_2 - \theta_{2eq})$$

$$= 2\left|\theta_{1eq} - \frac{x_{1eq}}{R}\right|\left(\theta_{1eq} - \frac{x_{1eq}}{R}\right) + 4\left|\theta_{1eq} - \frac{x_{1eq}}{R}\right|(\theta_1 - \theta_{1eq}) +$$

$$- 4\left|\frac{x_{1eq}}{R} - \theta_{1eq}\right|(\theta_2 - \theta_{2eq})$$

definindo:

$\delta_0 = \left(\theta_{1eq} - \frac{x_{1eq}}{R}\right)$ | $\theta_i = (\theta_i - \theta_{ieq})$

$$f = 4\,\delta_0(\theta_1 - \theta_2) + \delta_0^2$$

$f_2 = 2\dot{x}_1^3$; $f_2 \cong f_{eq} + \left.\frac{\partial f}{\partial \dot{x}_1}\right|_{eq}(\dot{x}_1 - \dot{x}_{1eq})$

$$= F_{0eq} + 6\,\dot{x}_{1eq}^2(\dot{x}_1 - \dot{x}_{1eq})$$

definindo: $\dot{x}_{1eq} = (\dot{x}_1 - \dot{x}_{1eq})$

$$f = 6\cdot\dot{x}_{1eq}^2\cdot\dot{x}_1 + F_{0eq}$$

$$\begin{cases} \frac{V_a}{L} - \frac{k_b}{L}\cdot\dot{\theta}_1 - \frac{R\cdot i}{L} = \dot{i} \\ \frac{k}{J_m}i - \frac{R_m}{J_m}\dot{\theta}_1 - \frac{4\delta_0}{J_m}\left(\theta_1 - \frac{x_1}{R}\right) - \frac{\delta_0^2}{J_m} = \ddot{\theta}_1 \\ \frac{4\delta_0}{mR}\left(\theta_1 - \frac{x_1}{R}\right) + \frac{\delta_0^2}{mR} - 6\dot{x}_{1eq}^2\cdot\dot{x}_1 + F_{0eq} - kx_1 = \ddot{x}_1 \end{cases}$$

c) $X = [\, i \quad \theta_1 \quad x_1 \quad \dot{\theta}_1 \quad \dot{x}_1 \,]^T$

Cada estado representa a pertubação em torno do valor no ponto de equilíbrio

$$\begin{bmatrix} \dot{i} \\ \dot{\theta}_1 \\ \dot{x}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & 0 & 0 & -\frac{k_b}{L} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & -\frac{4S_0}{J_m} & \frac{4S_0}{J_m R} & -\frac{B_m}{J_m} & 0 \\ 0 & \frac{4S_0}{mR} & -\frac{4S_0}{mR^2} - k & 0 & -6\dot{x}_{1eq}^2 \end{bmatrix}}_{A} \begin{bmatrix} i \\ \theta_1 \\ x_1 \\ \dot{\theta}_1 \\ \dot{x}_1 \end{bmatrix} +$$

$$+ \overbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{B} \cdot e \qquad Y = \underbrace{[\, 0 \; 0 \; 1 \; 0 \; 0 \,]}_{C} \begin{bmatrix} i \\ \theta_1 \\ x_1 \\ \dot{\theta}_1 \\ \dot{x}_1 \end{bmatrix} = x_1$$

8 - a) $H_x = J_x \cdot \omega_x$

$$\dot{\omega}_x + H_x \left(\frac{1}{J_d} - \frac{1}{J_x} \right) \omega_z = \frac{\tau_x}{J_d}$$

$$\dot{\omega}_z + H_x \left(\frac{1}{J_d} - \frac{1}{J_x} \right) \omega_x = \frac{\tau_z}{J_d} = 0$$

$$\tau_x = -2kL \cdot \theta_x - B \cdot \omega_x$$

Considerando $J_{xy} = J_z = J_d$:

$$\dot{\omega}_x + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \cdot \omega_x = -\frac{2kL}{J_d}\,\theta_x - \frac{B\,\omega_x}{J_d}$$

$$X = [\,\theta_x \quad \dot{\theta}_x\,]$$

b) Espaço de estados

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{2kL}{J_d} & -\frac{B}{J_d} \end{bmatrix}}_{A} \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} + \begin{bmatrix} 0 \\ -H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \end{bmatrix} \cdot \omega_z$$