Modelagem

Ex 01 e 06/10 Espaço de estados

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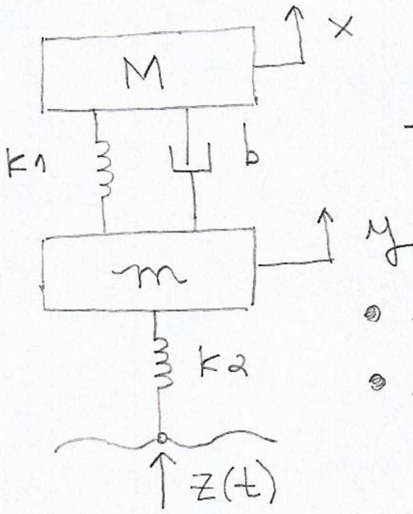
2) Modelagem 1/4 de carro



Equações sistema:

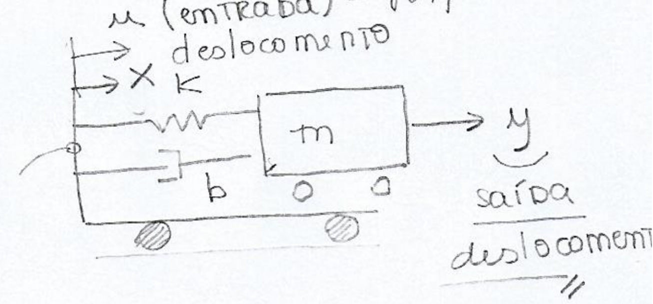
$$\begin{cases} M\ddot{x} + K_1(x-y) + b(\dot{x}-\dot{y}) = 0 \\ m\ddot{y} + K_1(y-x) + b(\dot{y}-\dot{x}) + yK_2 = zK_2 \end{cases}$$

- saídas: x; $\dot{y}$
- entrada: z

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} ; \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} ; \quad \begin{matrix} x_1 = x \\ x_2 = \dot{x} \\ x_3 = y \\ x_4 = \dot{y} \end{matrix}$$

$D = 0$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -K_1/M & -b/M & K_1/M & b/M \\ 0 & 0 & 0 & 1 \\ K_1/m & b/m & -\frac{(K_1+K_2)}{m} & -b/m \end{bmatrix}}_{A} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ K_2 \end{bmatrix}}_{B} z ; \quad y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$



3) Carreta

$$(M+m)\ddot{x} + (x-y)k + b(\dot{x}-\dot{y}) = u$$

$$m\ddot{y} + (y-x)k + b(\dot{y}-\dot{x}) = 0$$

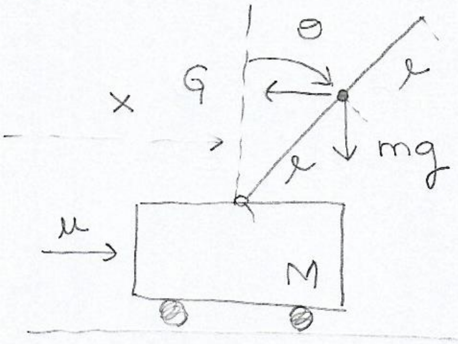
3.1) $M = 0$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m & -b/m & k/m & b/m \\ 0 & 0 & 0 & 1 \\ k/m & b/m & -k/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u ; \quad y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$

5.4) M N ...

$$\begin{bmatrix}\dot x\\ \ddot x\\ \dot y\\ \ddot y\end{bmatrix} = \begin{bmatrix}0&1&0&0\\ -\frac{k}{(M+m)} & -\frac{b}{(M+m)} & \frac{k}{(M+m)} & \frac{b}{(M+m)}\\ 0&0&0&1\\ k/m & b/m & -k/m & -b/m\end{bmatrix}\begin{bmatrix}x\\ \dot x\\ y\\ \dot y\end{bmatrix} + \begin{bmatrix}0\\1\\0\\0\end{bmatrix} u \; ; \; y = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot x\\ y\\ \dot y\end{bmatrix}$$

5)



Δ Por Lagrange

$$T = \frac{1}{2}(M+m)\dot x^2 + ml\dot x\dot\theta\cos\theta + \frac{1}{2}(J+ml^2)\dot\theta^2$$

$$V = mgl(\cos\theta - 1)$$

Δ Equações

$$\begin{cases}(I+ml^2)\ddot\theta + ml\ddot x\cos\theta - mgl\,\mathrm{sen}\,\theta = G\\ (M+m)\ddot x + ml\ddot\theta\cos\theta - \underbrace{\boxed{ml\dot\theta^2\sin\theta}}_{f} = u\end{cases} \quad \bar{N}\ \underline{\underline{lin}}$$

Δ linearização, $\theta \sim 0$
pequenos deslocamentos;
$\cos\theta \cong 1$; $\mathrm{sen}\,\theta \cong \theta$

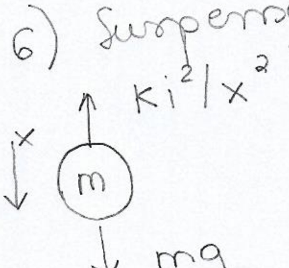
E.E:

$$\underbrace{\begin{bmatrix}\dot\theta\\ \ddot\theta\\ \dot x\\ \ddot x\end{bmatrix}}_{[\dot y]} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{+mgl}{(I+ml^2)} & 0 & 0 & \frac{-ml}{(I+ml^2)}\\ 0 & 0 & 0 & 1\\ 0 & \frac{-ml}{(M+m)} & 0 & 0\end{bmatrix}[y] + \begin{bmatrix}0&0\\1&0\\0&0\\0&1\end{bmatrix}\begin{bmatrix}G\\ u\end{bmatrix}$$

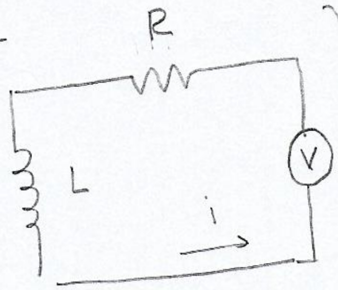
$$f(\dot\theta,\theta) = ml\bar{\dot\theta}^2\sin\bar\theta + (2ml\bar{\dot\theta}\sin\bar\theta)\,\dot\theta + ml\bar{\dot\theta}^2\cos\bar\theta\,(\theta) = 0$$

$\bar{\dot\theta} = 0$, $\bar\theta = 0$

6) Suspensão magnético esfera



$$m\ddot x = mg - \frac{ki^2}{x^2}$$



$$L\frac{di}{dt} + Ri = V$$

$$\begin{cases} m\ddot x = mg - ki^2/x^2\\ L\frac{di}{dt} + Ri = V\end{cases} \longrightarrow \text{Equações } \bar{N} \text{ lineares}$$

Linearizando:

$$\left(\frac{i^2}{x^2}\right) = \left(\frac{\bar{i}^2}{\bar{x}^2}\right) + \frac{2i}{x^2}\Big|_{\bar{x},\bar{i}}(i-\bar{i}) - 2\frac{i^2}{x^3}\Big|_{\bar{x},\bar{i}}(x-\bar{x}),$$

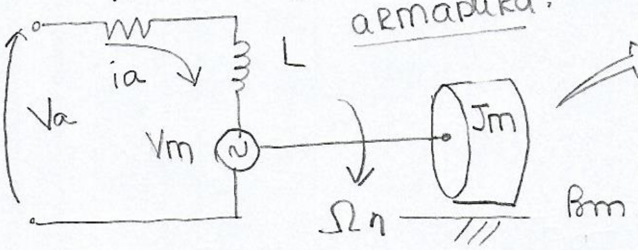
assim,

$$\ddot{x} = g - \frac{K}{m}\left(\cancel{\frac{\bar{i}^2}{\bar{x}^2}} + \frac{2\bar{i}}{\bar{x}^2}\underbrace{(i-\bar{i})}_{\delta i} - \frac{2\bar{i}^2}{\bar{x}^3}\underbrace{(x-\bar{x})}_{\delta x}\right)$$

$$\begin{bmatrix}\delta\dot{x}\\ \delta\ddot{x}\\ \delta\dot{i}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ +\frac{2K\bar{i}}{m\bar{x}^3} & 0 & -\frac{2\bar{i}}{m\bar{x}^2}\\ 0 & 0 & -R/L\end{bmatrix}\begin{bmatrix}\delta x\\ \delta\dot{x}\\ \delta i\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1/L\end{bmatrix}\delta V$$

7)

Modelo físico



Circuito armadura: $\boxed{V_m = k_m\Omega_1}$

$$L\frac{di}{dt} + Ri_a + k_m\Omega_1 = V_a$$

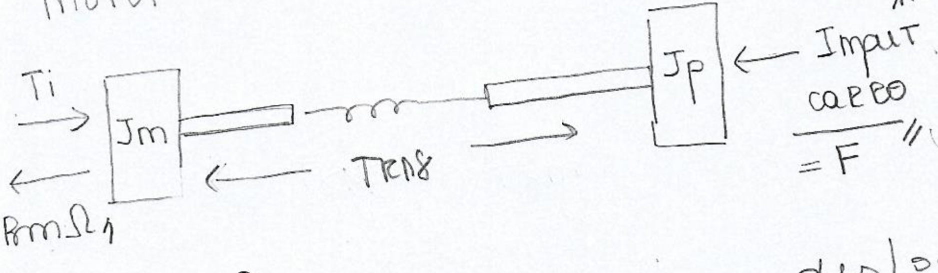
$\triangle$ $J_m\Omega_1 = -T_{NKS} - B_m\Omega_1 + T_i$

$T_i = Ki$

*

$\triangle$ $J_p\dot{\Omega}_2 - T_{KN\delta} = 0$

Circuito motor:



No carro

$M\ddot{x} - 2\dot{x}^3 + Kx = 0$; x = deslocamento cremalheira

Equações dinâmicas:

$$L\frac{di_a}{dt} + Ri_a + k_m\Omega_1 = V_a$$

$$\begin{cases} J_m\dot{\Omega}_1 = -T_{NKS} - B_m\Omega_1 + Ki_a \\ J_p\dot{\Omega}_2 = -T_{KN\delta} + fx \\ M\ddot{x} - 2\dot{x}^3 + Kx = fx \end{cases}$$

* qual a força de atrito entre a cremalheira e o pinhão ??

atrito = $f\cdot x$

$T_{nks} = 2(\theta_1 - \theta_2)\times(\theta_1 - \theta_2) \longrightarrow$ linearização .

$2\dot{x}^3$ ↓

$f(\dot{x}) \approx 2\bar{V}^3 + 6\bar{V}^2(\dot{x} - \bar{V}) = 6\bar{V}\dot{x} - 4\bar{V}^3$]

$$\begin{bmatrix} \dot{i}_a \\ \dot{\theta}_1 \\ \dot{\Omega}_1 \\ \dot{\theta}_2 \\ \dot{\Omega}_2 \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} -R/L & 0 & -Km/L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ K/Jm & -Bm/Jm & f_{nks} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & f_{nnk} & 0 & 0 & 0 & f/Jm & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -K/m & \frac{6\bar{V}^2}{m} \end{bmatrix} \begin{bmatrix} i_a \\ \theta_1 \\ \Omega_1 \\ \theta_2 \\ \Omega_2 \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ 4\bar{V}^3 \end{bmatrix}$$