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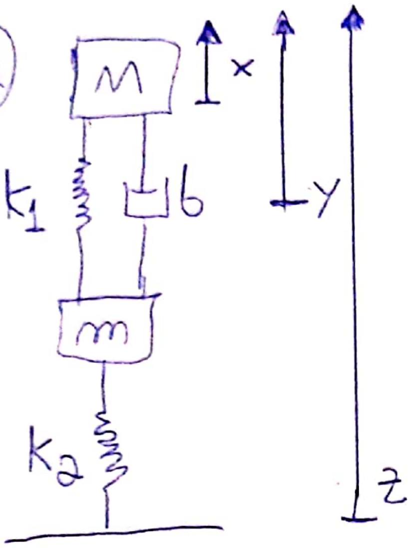
Modelagem de sistemas Dinâmicos - PME 3380

②



$$\begin{cases} M\ddot{x} = -k_1(x-y) - b(\dot{x}-\dot{y}) \\ m\ddot{y} = k_1(x-y) + b(\dot{x}-\dot{y}) - k_2(y-z) \end{cases}$$

$$\begin{cases} \ddot{x} = -\frac{k_1}{M}(x-y) - \frac{b}{M}(\dot{x}-\dot{y}) \\ \ddot{y} = \frac{k_1}{m}(x-y) + \frac{b}{m}(\dot{x}-\dot{y}) - \frac{k_2}{m}(y-z) \end{cases}$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix}}_{\dot{u}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/M & k_1/M & -b/M & b/M \\ \frac{k_1}{m} & \frac{-(k_1+k_2)}{m} & \frac{b}{m} & \frac{-b}{m} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}}_{u} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ k_2/m \end{bmatrix}}_{B} \cdot \underbrace{U}_{U}$$

$$\dot{u} = A\,u + B \cdot U$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

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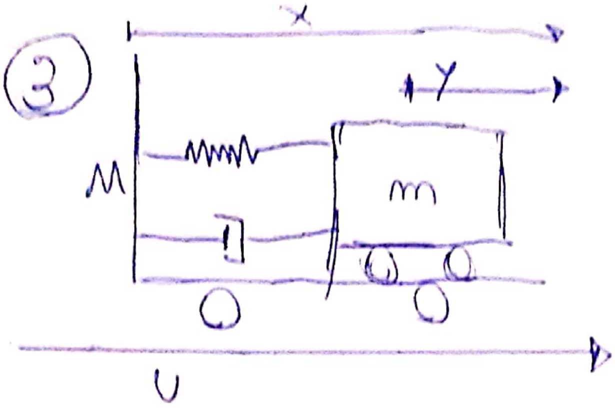

$$\begin{cases} m\ddot{y} = k(x-y) + b(\dot{x}-\dot{y}) \\ M\ddot{x} = U - k(x-y) - b(\dot{x}-\dot{y}) \end{cases}$$

$$\begin{cases} \ddot{y} = \frac{k}{m}(x-y) + \frac{b}{m}(\dot{x}-\dot{y}) \\ \ddot{x} = \frac{U}{M} - \frac{k}{M}(x-y) - \frac{b}{M}(\dot{x}-\dot{y}) \end{cases}$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix}}_{\dot{w}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{M} & \frac{k}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k}{M} & -\frac{k}{M} & \frac{b}{M} & -\frac{b}{M} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}}_{w} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ 0 \end{bmatrix}}_{B} \underbrace{U}_{U}$$

$$C = [0 \ 1 \ 0 \ 0]$$

④

$$\begin{cases} m_1\ddot{x}_1 + (k_p + k_1)x_1 + b_1\dot{x}_1 = k_p \cdot z(t) + k_1(x_G + b\theta) + b_1(\dot{x}_G + b\dot{\theta}) \\ m_2\ddot{x}_2 + (k_p + k_2)x_2 + b_2\dot{x}_2 = k_p z\left(\frac{vt-l}{v}\right) + k_2(x_G - a\theta) + b_1(\dot{x}_G - a\dot{\theta}) \end{cases}$$

$$\begin{cases} M\ddot{x}_G + k_1(x_G + b\theta) + b_1(\dot{x}_G + b\dot{\theta}) + k_2(x_G - a\theta) + b_2(\dot{x}_G - a\dot{\theta}) = k_1x_1 + b_1\dot{x}_1 + k_2x_2 + b_2\dot{x}_2 \\ J\ddot{\theta} + k_1b(x_G + b\theta) + b_1b(\dot{x}_G + b\dot{\theta}) - k_2a(x_G - a\theta) - b_1a(\dot{x}_G - a\theta) = k_1x_1b + b_1\dot{x}_1b - k_2x_2a - b_2\dot{x}_2a \end{cases}$$

Para o sistema, temos: $x = [x_1 \ x_2 \ x_G \ \theta \ \dot{x}_1 \ \dot{x}_2 \ \dot{x}_G \ \dot{\theta}]^t$

$y = [x_G \ \theta]^t \quad U = \left[z(t) \ \ z\left(\frac{tv-l}{v}\right)\right]^t$

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$$\dot{x} = Ax + Bu \qquad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(k_p + k_1)}{m_1} & 0 & \frac{k_1}{m_1} & \frac{k_1 b}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 b}{m_1} \\ 0 & \frac{-k_p + k_2}{m_2} & \frac{k_2}{M_2} & -\frac{k_2 a}{m_2} & 0 & -\frac{b_2}{m_2} & \frac{b_2}{M_2} & \frac{-b_2 a}{M_2} \\ \frac{k_1}{M} & \frac{k_2}{M} & \frac{-(k_1 + k_2)}{M} & \frac{k_2 a - k_1 b}{M} & \frac{b_1}{M} & \frac{b_2}{M} & -\frac{(b_1 + b_2)}{M} & \frac{b_2 a - b_1 b}{M} \\ \frac{k_1 b}{J} & \frac{-k_2 a}{J} & \frac{k_2 a - k_1 b}{J} & -\frac{k_1 a^2 + k_1 b^2}{J} & \frac{b_1 b}{J} & \frac{-b_2 a}{J} & \frac{b_2 a - b_1 b}{J} & \frac{-(b_2 a^2 - b_1 b^2}{J} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ +k_p/m_1 & k_p/m_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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⑤ Equações linearizadas:

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = U \\ J\ddot{\theta} = mgl\theta - ml\ddot{x} \longrightarrow \ddot{\theta} = \dfrac{mgl\theta - ml\ddot{x}}{J} \end{cases}$$

$$(m+M)\ddot{x} + ml\left(\frac{mgl\theta - ml\ddot{x}}{J}\right) = U$$

$$\ddot{x} + \frac{ml}{(M+m)} \cdot \left(\frac{ml(g\theta - \ddot{x})}{J}\right) = \frac{U}{M+m}$$

$$\ddot{x} + \frac{(ml)^2(-\ddot{x})}{(M+m)J} + \frac{(ml)^2 g\theta}{(M+m)J} = \frac{U}{(M+m)}$$

$$\ddot{x}\left(1 - \frac{(ml)^2}{(M+m)J}\right) = \frac{U}{(M+m)} - \frac{(ml)^2 g\theta}{(M+m)J}$$

$$\ddot{x} = \theta \frac{\left(\frac{-g(ml)^2}{(M+m)J}\right)}{\frac{(M+m)J - (ml)^2}{(M+m)J}} + \frac{\frac{U}{(M+m)}}{\frac{(M+m)J - (ml)^2}{(M+m)J}} \longrightarrow \boxed{\ddot{x} = \left(\frac{-m^2l^2 g}{MJ + mJ - m^2l^2}\right)\theta + \frac{U}{MJ + mJ - m^2l^2}}$$

Da segunda equação $\ddot{x} = \dfrac{mgl\theta - J\ddot{\theta}}{ml}$.

$$(M+m)\left(\frac{mgl\theta - J\ddot{\theta}}{ml}\right) + ml\ddot{\theta} = U$$

$$-\frac{(M+m)J\ddot{\theta}}{ml} + ml\ddot{\theta} + \frac{(M+m)(mgl\theta)}{ml} = U$$

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$$\ddot{\theta}\left(ml - \frac{(M+m)J}{ml}\right) = U - \frac{(M+m)(g\theta)}{1}$$

$$\boxed{\ddot{\theta} = \frac{U\cdot(ml)}{(ml)^2-(M+m)J} - \left(\frac{(M+m)(g)(ml)}{(ml)^2-(M+m)J}\right)\theta}$$

Seja $X = [x \ \ddot{\theta} \ \dot{x} \ \dot{\theta}]^t$ $\quad y = [x \ \theta]$ $\quad U = U$

e $\dot{X} = AX + BU$, $y = CX$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{m^2l^2g}{MJ+mJ-(ml)^2} & 0 & 0 \\ 0 & -\dfrac{(M+m)g(ml)}{(ml)^2-(M+m)J} & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{M+m-(m^2l^2)/J} \\ \dfrac{-gml}{J(M+m)-(ml)^2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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6)
$$\begin{cases} m\ddot{x} = mg - \dfrac{ki^2}{x^2} \\ L\dot{i} + Ri = V \end{cases}$$

$x = [x, \dot{x}, i]^t \quad y = [x]$

$\dot{x} = [\dot{x}, \ddot{x}, \dot{i}]^t \quad u = V$

Novamente, temos $\dot{x} = Ax + Bu$ e $y = Cx$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{-2ki_0^2}{mx_0^3} & 0 & \dfrac{-2ki_0}{mx_0^2} \\ 0 & 0 & -R/L \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} \quad C = [1\ 0\ 0] \quad D = [0]$$

7)
$$\begin{cases} L\dot{i} + Ri = V - e_B = V_C - K_b\Omega_1 \\ J_m\dot{\Omega}_1 = Ki - B_m\Omega_1 - 2|\theta_1 - \theta_2|\cdot(\theta_1 - \theta_2) \\ m\ddot{x}_1 = 2|\theta_1 - \theta_2|\cdot\dfrac{(\theta_1 - \theta_2)}{R} - 2\dot{x}_1^3 - Kx_1 \\ x_1 = \theta_2 R \rightarrow \dot{x}_1 = \Omega_2 R \end{cases}$$

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⑧ $M_x = J_x \cdot \omega_x$

(i) $\dot{\omega}_x + M_x\left(\frac{J_x - J_d}{J_x J_d}\right)\omega_2 = \frac{\tau_x}{J_d}$

$\begin{cases} \dot{\theta}_x = \omega_x \\ \omega_2 = U \end{cases}$

(ii) $\dot{\omega}_2 + M_x\left(\frac{J_x - J_d}{J_x J_d}\right)\omega_x = \frac{\tau_2}{J_d}$

(iii) $\tau_x = -\theta_x L\, 2k - B\omega_x$

$\ddot{\theta}_x = -\frac{2Lk}{J_d}\theta_x - \frac{B}{J_d}\dot{\theta}_x - M_x\left(\frac{J_x - J_d}{J_x J_d}\right)U$

Substituindo (iii) em (i):

$\dot{\omega}_x + M_x\left(\frac{J_x - J_d}{J_x J_d}\right)\omega_2 = -\frac{2kLx\theta}{J_d} - \frac{B\omega_x}{J_d}$

Temos: $x = [\theta_x \;\; \dot{\theta}_x]$

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{2Lk}{J_d} & -\frac{B}{J_d} \end{bmatrix}\begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} + \begin{bmatrix} 0 \\ -M_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \end{bmatrix}\omega_2$$

M