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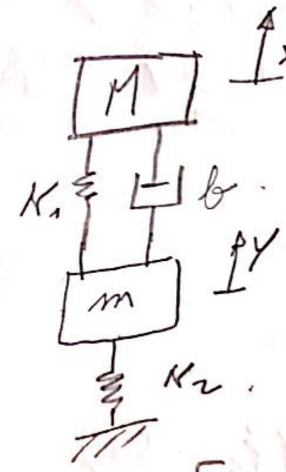
(2)

$: x > y;\ \dot{x} > \dot{y};$



$\boxed{M}$: $M\ddot{x} = -K_1(x-y) - b(\dot{x}-\dot{y}) = 0$

$$\ddot{x} = -\frac{K_1 x}{M} + \frac{K_1}{M}y - \frac{b}{M}\dot{x} + \frac{b}{M}\dot{y}$$

$\boxed{m}$: $m\ddot{y} = K_1(x-y) + b(\dot{x}-\dot{y}) - K_2(y-z)$

$$\ddot{y} = \frac{K_1 x}{m} - \frac{(K_1-K_2)}{m}y + \frac{b}{m}\dot{x} - \frac{b}{m}y + z\frac{K_2}{m}$$

$\rightarrow x = [x\ y\ \dot{x}\ \dot{y}]$; $y = [x\ \dot{y}]$, $u = z$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_1}{M} & \frac{K_1}{M} & \frac{-b}{M} & \frac{b}{M} \\ \frac{K_1}{m} & -\frac{(K_1+K_2)}{m} & \frac{b}{m} & \frac{-b}{m} \end{bmatrix}}_{A} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_2}{m} \end{bmatrix}}_{B} \cdot u$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

(3)

$$M\ddot{X} = u - K(x-y) - b(\dot{x}-\dot{y})$$

$$\hookrightarrow \ddot{X} = \frac{u}{M} - \frac{Kx}{M} + \frac{Ky}{M} - \frac{b\dot{x}}{M} + \frac{b\dot{y}}{M}$$

$$m\ddot{y} = K(x-y) + b(\dot{x}-\dot{y})$$

$$\hookrightarrow \ddot{y} = \frac{Kx}{m} - \frac{Ky}{m} + \frac{b\dot{x}}{m} - \frac{b}{m}\dot{y}$$

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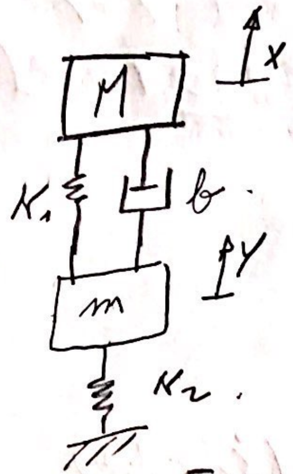
②

$;x > y;\ \dot{x} > \dot{y};$



$\boxed{M}: \quad M\ddot{x} = -K_1(x-y) - b(\dot{x}-\dot{y}) = 0$

$$\ddot{x} = -\frac{K_1}{M}x + \frac{K_1}{M}y - \frac{b}{M}\dot{x} + \frac{b}{M}\dot{y}$$

$\boxed{m}: \quad m\ddot{y} = K_1(x-y) + b(\dot{x}-\dot{y}) - K_2(y-z)$

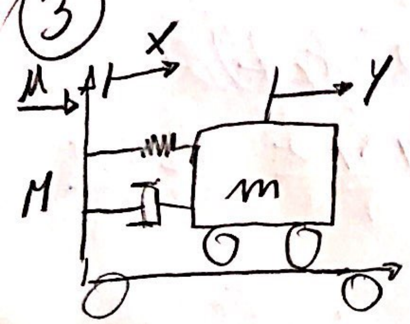
$$\ddot{y} = \frac{K_1 x}{m} - \frac{(K_1-K_2)}{m}y + \frac{b}{m}\dot{x} - \frac{b}{m}y + z\frac{K_2}{m}$$

$\rightarrow x = [x\ \ y\ \ \dot{x}\ \ \dot{y}]\ ;\ y = [x\ \ \dot{y}]\ ,\ u = z$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \ddot{x}\\ \ddot{y}\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{-K_1}{M} & \frac{K_1}{M} & \frac{-b}{M} & \frac{b}{M}\\ \frac{K_1}{m} & \frac{-(K_1+K_2)}{m} & \frac{b}{m} & \frac{-b}{m}\end{bmatrix}}_{A} \cdot \begin{bmatrix}x\\ y\\ \dot{x}\\ \dot{y}\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\\ 0\\ \frac{K_2}{m}\end{bmatrix}}_{B} \cdot u$$

$$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix}x\\ y\\ \dot{x}\\ \dot{y}\end{bmatrix}$$

③



$$M\ddot{x} = u - K(x-y) - b(\dot{x}-\dot{y})$$

$$\hookrightarrow \ddot{x} = \frac{u}{M} - \frac{Kx}{M} + \frac{Ky}{M} - \frac{b\dot{x}}{M} + \frac{b\dot{y}}{M}$$

$$m\ddot{y} = K(x-y) + b(\dot{x}-\dot{y})$$

$$\hookrightarrow \ddot{y} = \frac{Kx}{m} - \frac{Ky}{m} + \frac{b\dot{x}}{m} - \frac{b}{m}\dot{y}$$

$x = [x\ y\ \dot{x}\ \dot{y}]$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \ddot{x}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -K/M & K/M & -b/M & b/M\\ K/M & -K/M & b/M & -b/M\end{bmatrix} \cdot \begin{bmatrix}x\\ y\\ \dot{x}\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1/M\\ 0\end{bmatrix} u \quad \text{com } Y = y \quad C = [0\,1\,0\,0]$$

(4) Equações obtidas anteriormente

- $m_1 \ddot{x}_1 + (K_p + K_1)x_1 + b_1 \cdot \dot{x}_1 = K_p \cdot z(t) + K_1(x_G + b\theta) + b_1(\dot{x}_G + b\dot{\theta})$

$$\Rightarrow \ddot{x}_1 = -\frac{(K_p+K_1)x_1}{m_1} - \frac{b_1 \dot{x}_1}{m_1} + \frac{K_p \cdot z(t)}{m_1} + \frac{K_1 \cdot x_G}{m_1} + \frac{K_1 b\theta}{m_1} + \frac{b_1 \dot{x}_G}{m_1} + \frac{b_1 b \dot{\theta}}{m_1}$$

$m_2 \ddot{x}_2 + (K_p + K_2)x_2 + b_2 \dot{x}_2 = K_p \cdot z\left(t - \frac{l}{v}\right) + K_2(x_G - a\theta) + b_2(\dot{x}_G - a\dot{\theta})$

$$\Rightarrow \ddot{x}_2 = -\frac{(K_p+K_2)}{m_2}x_2 - \frac{b_2 \dot{x}_2}{m_2} + \frac{K_p}{m_2} \cdot z\left(t-\frac{l}{v}\right) + \frac{K_2 x_G}{m_2} - \frac{K_2 a\theta}{m_2} + \frac{b_2 \dot{x}_G}{m_2} - \frac{b_2 a\dot{\theta}}{m_2}$$

$M \ddot{x}_G + K_1(x_G + b\theta) + b_1(\dot{x}_G + b\dot{\theta}) + K_2(x_G - a\theta) + b_2(\dot{x}_G - a\dot{\theta}) =$

$= K_1 x_1 + b_1 \dot{x}_1 + K_2 x_2 + b_2 \dot{x}_2 \Rightarrow \ddot{x}_G = -\frac{K_1 x_G}{M} - \frac{K_1 b\theta}{M} - \frac{b_1 \dot{x}_G}{M} - \frac{b_1 b\dot{\theta}}{M}$

$$-\frac{K_2 x_G}{M} + \frac{K_2 a\theta}{M} - \frac{b_2 \dot{x}_G}{M} + \frac{b_2 a\dot{\theta}}{M} + \frac{K_1 x_1}{M} + \frac{K_2 x_2}{M} - \frac{b_1 \dot{x}_1}{M} + \frac{b_2 \dot{x}_2}{M}$$

- $J\ddot{\theta} + K_1 b(x_G + b\theta) + b_1 b(\dot{x}_G + b\dot{\theta}) - K_2 a(x_G - a\theta) - b_2 a(\dot{x}_G - a\dot{\theta}) =$

$= K_1 x_1 b + b_1 \dot{x}_1 b - K_2 x_2 a + b_2 \dot{x}_2 a$

$$\Rightarrow \ddot{\theta} = \frac{1}{J}(K_2 a - K_1 b)x_G + \frac{1}{J}(b_2 a - b_1 b)\dot{x}_G - \frac{1}{J}(K_2 a^2 + K_1 b^2)\theta$$

$$-\frac{1}{J}(b_2 a^2 - b_1 b^2)\dot{\theta} + \frac{K_1}{J} x_1 b + \frac{b_1 \dot{x}_1 b}{J} - \frac{K_2 x_2 a}{J} - \frac{b_2 \dot{x}_2 a}{J}$$

- Ao definir: $x = [x_1\ x_2\ x_G\ \theta\ \dot{x}_1\ \dot{x}_2\ \dot{x}_G\ \dot{\theta}]^T$

$Y = [x_G\ \theta]^T$; $u = \left[z(t)\ \ z\left(t - \frac{l}{v}\right)\right]^T$

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é um sistema do tipo $\dot{x} = Ax + Bu$
e $y = Cx + Du$.

$$A = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{(-K_p+K_1)}{m_1} & 0 & K_1/M_1 & K_1 b/m_1 & -b_1/M_1 & 0 & b_1/M_1 & b_1 b/M_1 \\ 0 & \frac{-(K_p+K_2)}{m_2} & K_2/M_2 & -K_2\partial/M_2 & 0 & -b_2/M_2 & b_2/M_2 & -\frac{b_2\partial}{M_2} \\ K_1/M & K_2/M & \frac{-(K_1+K_2)}{M} & \frac{(K_2\partial-K_1 b)}{M} & \frac{b_1}{M} & b_2/M & \frac{-(b_1+b_2)}{M} & \frac{b_2\partial-b_1 b}{M} \\ \frac{K_1 b}{J} & \frac{-K_2\partial}{J} & \frac{K_2\partial-K_2 b}{J} & \frac{-(K_2\partial^2-K_1 b^2)}{J} & \frac{b_1 b}{J} & \frac{-b_2\partial}{J} & \frac{b_2\partial-b_1 b}{J} & \frac{-(b_2\partial^2-b_1 b^2)}{J} \end{bmatrix} \\ \begin{array}{cccccccc} X_1 & X_2 & X_G & \theta & \dot{X}_1 & \dot{X}_2 & \dot{X}_G & \dot{\theta} \end{array} \end{array}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_p/M_1 & 0 \\ 0 & K_p/M_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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5

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u & \text{I} \\ J\ddot{\theta} = lmg\theta - ml\ddot{x} \end{cases}$$

$$(M+m)\ddot{x} + ml\left(\frac{lmg\theta}{J} - \frac{ml\ddot{x}}{J}\right) = (M+m)\ddot{x} +$$

$$+\frac{m^2l^2g\theta}{J} - \frac{m^2l^2\ddot{x}}{J} = u =$$

$$\Rightarrow \ddot{x} = \left(\frac{-m^2l^2}{MJ+mJ-m^2l^2}\right)\theta + \frac{u}{M+m-l^2m^2/J}$$

substituindo (2) em (1) e isolando $\ddot{x}$:

$$(M+m)\cdot\left(\frac{lmg\theta}{lm} - \frac{J\ddot{\theta}}{ml}\right) + ml\ddot{\theta} = u =$$

$$= \left(\frac{J}{ml} - \frac{ml}{M+m}\right)\ddot{\theta} = g\theta - \frac{u}{M+m}$$

$$\Rightarrow \ddot{\theta} = \left(\frac{gml(M+m)}{J(M+m)-m^2l^2}\right)\theta - \left(\frac{gml}{J(M-m)-m^2l^2}\right)u$$

$$x = [x \quad \theta \quad \dot{x} \quad \dot{\theta}]^T;$$

$$Y = [x \quad \theta]; \quad u = u$$

Ao fazer o sistema do tipo $\dot{X} = Ax + Bu$; $Y = Cx$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \left(\frac{-m^2l^2}{MJ+mJ-m^2l^2}\right) & 0 & 0 \\ 0 & \left(\frac{gml(M+m)}{J(M+m)-m^2l^2}\right) & 0 & 0 \end{bmatrix}$$

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$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{M+m-m^2l^2}{J} \\ \dfrac{-gml}{J(M+m)-m^2l^2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$D = 0 \; (2 \times 1)$

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7 a) motor:

- circuito da armadura:

$$L\dot{i} + R\cdot i = V_2 - e_B = V_2 - K_b \Omega_1$$

- parte mecânica:

$$J_m \cdot \dot{\Omega}_1 = T_i - B_m \cdot \Omega_1 - 2|\theta_1 - \theta_2|\cdot(\theta_1 - \theta_2) \Rightarrow$$

$$\Rightarrow J_m \cdot \dot{\Omega}_1 = K\cdot i - B_m \cdot \Omega_1 - 2|\theta_1 - \theta_2|\cdot(\theta_1 - \theta_2)$$

- carro:

$$m\cdot \ddot{x}_1 = 2|\theta_1 - \theta_2|\cdot(\theta_1 - \theta_2)\frac{1}{R} - 2\dot{x}_1^3 - K\cdot x_1$$

- vínculo cinemático:

$\cdot\ x_1 = \theta_2 \cdot R \qquad \cdot\ \dot{x}_1 = \Omega_2 \cdot R$

↳ sistema de quinta ordem

- vetor de estados: $\mathbf{x} = [i\ \theta_1\ x_1\ \dot{\theta}_1\ \dot{x}_1]^T$

b) termos não lineares:

$\cdot\ f_1 = 2|\theta_1 - \theta_2|\cdot(\theta_1 - \theta_2) = 2|\theta_1 - \frac{x_1}{R}|(\theta_1 - \frac{x_1}{R})$

$$f_1 = f(0) + \left.\frac{\partial f}{\partial \theta_1}\right|_{eq}(\theta_1 - \theta_{10}) + \left.\frac{\partial f}{\partial \theta_2}\right|_{eq}(\theta_2 - \theta_{20})$$

$$f_1 = 2\cdot|\theta_{10} - \frac{x_{10}}{R}|(\theta_{10} - \frac{x_{10}}{R}) + 4\cdot|\theta_{10} - \frac{x_{10}}{R}|\cdot(\theta_1 - \theta_{10}) - 4|\frac{x_{10}}{R} - \theta_{10}|(\theta_2 - \theta_{20})$$

$\cdot$ definindo: $(\theta_{10} - \frac{x_{10}}{R}) = \delta_0$

$(\theta_1 - \theta_{10}) = \theta_1$

$(\theta_2 - \theta_{20}) = \theta_2$

$\rightarrow f = 4\delta_0(\theta_1 - \theta_2) + \delta_0^2$

$\cdot\ f_2 = 2\cdot\dot{x}_1^3$

$$f_2 = f_{eq} + \left.\frac{\partial f}{\partial \dot{x}_1}\right|_{eq}(\dot{x}_1 - \dot{x}_{10})$$

$$f_2 = F_{00} + 6\dot{x}_{10}^2\cdot(\dot{x}_1 - \dot{x}_{10})$$

$\cdot$ definindo: $(\dot{x}_1 - \dot{x}_{10}) = \dot{x}_{10}$

$\rightarrow f = \dot{x}_{10}^2 \cdot \dot{x}_1 + F_{20}$

$\cdot$ reescrevendo as equações linearizadas:

$$\cdot\ \dot{i} = \frac{V_2}{L} - \frac{K_b}{L}\cdot\dot{\theta}_1 - \frac{R_i}{L}$$

$$\cdot\ \ddot{\theta}_1 = \frac{K}{J_m}\cdot i - \frac{B_m}{J_m}\cdot\dot{\theta}_1 - \frac{4\delta_0}{J_m}(\theta_1 - \frac{x_1}{R}) - \frac{\delta_0^2}{J_m}$$

$$\cdot\ \ddot{x}_1 = \frac{4\delta_0}{mR}\left(\theta_1 - \frac{x_1}{R}\right) + \frac{\delta^2}{mR} - 6\,\dot{x}_{10}^{2} \cdot \dot{x}_1 + F_{20} - kx_1$$

c) · redefinindo $x = [i\ \theta_1\ x_1\ \dot{\theta}_1\ \dot{x}_1]^T$ onde cada estado é a sua pertubação em torno de um valor do equilíbrio:

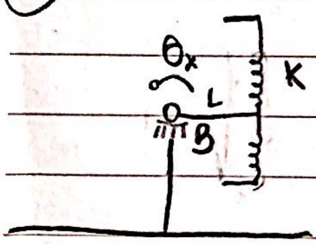
· sistema do tipo $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$ onde:

$$A = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & -\frac{k_0}{L} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{K}{J_m} & \frac{-4\delta_0}{J_m} & \frac{4\delta_0}{J_m R} & -\frac{B_m}{J_m} & 0 \\ 0 & \frac{4\delta_0}{mR} & \left(\frac{-4\delta_0}{mR^2} - k\right) & 0 & -6\dot{x}_{10}^{2} \end{bmatrix}$$

$$B = \begin{bmatrix} 1/L \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad C = [0\ 0\ 1\ 0\ 0]$$

↳ saída: x_1

8) a) vista lateral



· $H_x = J_x \cdot \omega_x$ → momento angular

· as equações de Euler podem ser escritas como:

$$\cdot\ \dot{\omega}_x + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \cdot \omega_z = \frac{\tau_x}{J_d}$$

$$\cdot\ \dot{\omega}_z + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \cdot \omega_x = \cancel{\frac{\tau_z}{J_d}} \to 0$$

· $\tau_x = -\theta_x \cdot L \cdot 2k - B\omega_x$ · Também, $J_y = J_z = J_d$

· Reescrevendo:

$$\cdot\ \dot{\omega}_x + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \cdot \omega_z = -\frac{2LK}{J_d} \cdot \theta_x - \frac{B\omega_x}{J_d}$$

e $\dot{\theta}_x = \omega_x$, $\omega_z = U$

$$> \ddot{\theta}_x = -\frac{2LK}{J_d} \cdot \theta_x - \frac{B}{J_d}\dot{\theta}_x - H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \cdot U$$

$$\dot{\theta}_x = \dot{\theta}_x$$

b) no EE:

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-2Lk}{J_d} & \frac{-B}{J_d} \end{bmatrix} \cdot \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} + \begin{bmatrix} 0 \\ -H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \end{bmatrix} \cdot \omega_2$$