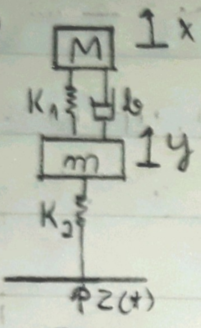
Exercícios da Aula dos dias 01 e 06/10/2020

(2)



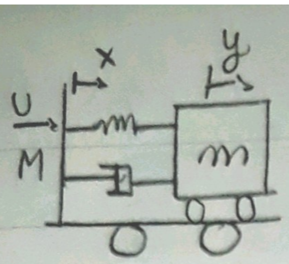
- $x > y$ e $\dot{x} > \dot{y}$
- M: $M\ddot{x} = -K_1(x-y) - b(\dot{x}-\dot{y}) \Rightarrow \ddot{x} = -\frac{K_1}{M}x + \frac{K_1}{M}y - \frac{b}{M}\dot{x} + \frac{b}{M}\dot{y}$
- m: $m\ddot{y} = K_1(x-y) + b(\dot{x}-\dot{y}) - K_2(y-z) \Rightarrow$

$$\Rightarrow \ddot{y} = \frac{K_1}{m}x - \frac{(K_1+K_2)}{m}y + \frac{b}{m}\dot{x} - \frac{b}{m}\dot{y} + \frac{K_2}{m}z$$

- $X = [x\ y\ \dot{x}\ \dot{y}]$; $y = [x\ \dot{y}]$; $U = z$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_1}{M} & \frac{K_1}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{K_1}{m} & -\frac{(K_1+K_2)}{m} & \frac{b}{m} & -\frac{b}{m} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_2}{m} \end{bmatrix} \cdot U$$

$$\begin{bmatrix} x \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

③ 

$\cdot\ m\ddot y = k(x-y) + b(\dot x - \dot y) \Rightarrow \ddot y = \frac{k}{m}x - \frac{k}{m}y + \frac{b}{m}\dot x - \frac{b}{m}\dot y$

$\cdot\ M\ddot x = U - k(x-y) - b(\dot x - \dot y) \Rightarrow \ddot x = \frac{U}{M} - \frac{k}{M}x + \frac{k}{M}y - \frac{b}{M}\dot x + \frac{b}{M}\dot y$

$\cdot\ x = [x\ \ y\ \ \dot x\ \ \dot y]$

$$\cdot\ \begin{vmatrix} \dot x \\ \dot y \\ \ddot x \\ \ddot y \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{M} & \frac{k}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k}{M} & -\frac{k}{M} & \frac{b}{M} & -\frac{b}{M} \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ \dot x \\ \dot y \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ \frac{1}{M} \\ 0 \end{vmatrix} U$$

$\cdot$ Com $y = y$

$C = [0\ 1\ 0\ 0]$ $D = [0]$

④ $\cdot\ m_1\ddot x_1 + (k_p + k_1)x_1 + b_1\dot x_1 = k_p z(t) + k_1(x_g + b\theta) + b_1(\dot x_g + b\dot\theta) \Rightarrow$

$\Rightarrow \ddot x_1 = -\frac{(k_p+k_1)}{m_1}x_1 - \frac{b_1}{m_1}\dot x_1 + \frac{k_p}{m_1}\cdot z(t) + \frac{k_1 x_g}{m_1} + \frac{1}{m_1}k_1 b\theta + b_1\dot x_g\frac{1}{m_1} + b_1 b\dot\theta\frac{1}{m_1}$

$\cdot\ m_2\ddot x_2 + (k_p + k_2)x_2 + b_2\dot x_2 = k_p\cdot z(t - \frac{l}{v}) + k_2(x_g - a\theta) + b_2(\dot x_g - a\dot\theta) \Rightarrow$

$\Rightarrow \ddot x_2 = -\frac{(k_p+k_2)}{m_2}x_2 - \frac{b_2}{m_2}\dot x_2 + \frac{k_p}{m_2}z(t - \frac{l}{v}) + \frac{k_2}{m_2}x_g - \frac{k_2}{m_2}a\theta + \frac{b_2}{m_2}\dot x_g - \frac{b_2}{m_2}a\dot\theta$

$\cdot\ M\ddot x_g + k_1(x_g + b\theta) + b_1(\dot x_g + b\dot\theta) + k_2(x_g - a\theta) + b_2(\dot x_g + a\dot\theta) = k_1x_1 + b_1\dot x_1 + k_2x_2 + b_2\dot x_2 \Rightarrow$

$\Rightarrow \ddot x_g = -\frac{k_1}{M}x_g - \frac{k_1 b}{M}\theta - \frac{b_1}{M}\dot x_g - \frac{b_1 b}{M}\dot\theta - \frac{k_2}{M}x_g + \frac{k_2}{M}a\theta - \frac{b_2}{M}\dot x_g + \frac{b_2}{M}a\dot\theta$
$+ \frac{k_1x_1}{M} + \frac{k_2}{M}x_2 + \frac{b_1}{M}\dot x_1 + \frac{b_2}{M}\dot x_2$

$\cdot\ J\ddot\theta + k_1 b(x_g + b\theta) + b_1 b(\dot x_g + b\dot\theta) - k_2 a(x_g - a\theta) - b_1 a(\dot x_g - a\dot\theta) = \Rightarrow$
$= k_1x_1 b + b_1\dot x_1 b - k_2x_2 a - b_2\dot x_2 a$

$\Rightarrow \ddot\theta = \frac{1}{J}(k_2 a - k_1 b)x_g + \frac{1}{J}(b_2 a - b_1 b)\dot x_g - \frac{1}{J}(k_2 a^2 + k_1 b^2)\theta - \frac{1}{J}(b_2 a^2 + b_1 b^2)\dot\theta$
$+ \frac{1}{J}k_1x_1 b + \frac{1}{J}b_1\dot x_1 b - \frac{k_2}{J}x_2 a - \frac{b_2}{J}\dot x_2 a$

$\cdot$ Definindo: $x = [x_1\ \ x_2\ \ x_g\ \ \theta\ \ \dot x_1\ \ \dot x_2\ \ \dot x_g\ \ \dot\theta]^T$; $y = [x_g\ \ \theta]^T$

$U = [z(t)\ \ \ z(t - l/v)]^T$

$\cdot$ Sistema do tipo: $\dot x = Ax + Bu$, $y = Cx + Du$

$\cdot$ Onde:

$$B = \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_p}{m_1} & 0 \\ 0 & \frac{k_p}{m_2} \\ 0 & 0 \\ 0 & 0 \end{vmatrix} \qquad C = \begin{vmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{vmatrix}, \quad D = 0(2\times 2)$$

$$A = \begin{vmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{(k_p+k_1)}{m_1} & 0 & \frac{k_1}{m_1} & \frac{k_1 b}{m_1} & -\frac{b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 b}{m_1} \\ 0 & -\frac{(k_p+k_2)}{m_2} & \frac{k_2}{m_2} & -\frac{k_2 a}{m_2} & 0 & -\frac{b_2}{m_2} & \frac{b_2}{m_2} & -\frac{b_2 a}{m_2} \\ \frac{k_1}{m} & \frac{k_2}{M} & -\frac{(k_1+k_2)}{M} & \frac{(k_2 a - k_1 b)}{M} & \frac{b_1}{M} & \frac{b_2}{M} & -\frac{(b_1+b_2)}{M} & \frac{(b_2 a - b_1 b)}{M} \\ \frac{k_1 b}{J} & -\frac{k_2 a}{J} & \frac{(k_2 a - k_1 b)}{J} & -\frac{(k_2 a^2 + k_1 b^2)}{J} & \frac{b_1 b}{J} & -\frac{b_2 a}{J} & \frac{(b_2 a - b_1 b)}{J} & -\frac{(b_2 a^2 - b_1 b^2)}{J} \end{vmatrix}$$

5) $\begin{cases} \cdot (M+m)\ddot{x} + ml\ddot{\theta} = U \quad (I) \\ \cdot J\ddot{\theta} = lmg\theta - ml\ddot{x} \quad (II) \end{cases}$

· Substituindo (II) em (I), isolando $\ddot{\theta}$:

$$(M+m)\ddot{x} + ml\left(\frac{lmg\theta}{J} - \frac{ml}{J}\ddot{x}\right) = U \Rightarrow (M+m)\ddot{x} + \frac{m^2 l^2 g}{J}\theta - \frac{m^2 l^2}{J}\ddot{x} = U \Rightarrow$$

$$\Rightarrow \left(M+m-\frac{m^2l^2}{J}\right)\ddot{x} + \frac{m^2 l^2}{J}g\theta = U \Rightarrow \ddot{x} = \left(\frac{-m^2l^2}{MJ+mJ-m^2l^2}\right)\theta + \frac{U}{M+m-\frac{m^2l^2}{J}}$$

· Substituindo (II) em (I), isolando $\ddot{x}$:

$$(M+m)\left(\frac{lmg\theta}{lm} - \frac{J\ddot{\theta}}{ml}\right) + ml\ddot{\theta} = U \Rightarrow (M+m)g\theta - \left((M+m)\frac{J}{ml} - ml\right)\ddot{\theta} = U \Rightarrow$$

$$\Rightarrow \left(\frac{-ml}{M+m} + \frac{J}{ml}\right)\ddot{\theta} = g\theta - \frac{U}{M+m} \Rightarrow \ddot{\theta} = \left(\frac{gml(M+m)}{J(M+m)-m^2l^2}\right)\theta - \left(\frac{gml}{J(M+m)-m^2l^2}\right)U$$

· Definido: $x = [x\ \theta\ \dot{x}\ \dot{\theta}]^T$; $y = [x\ \theta]$; $U = U$

· Sistema do tipo: $\dot{x} = Ax + BU$; $y = Cx$

$$A = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \left(\frac{-m^2l^2}{MJ+mJ-m^2l^2}\right) & 0 & 0 \\ 0 & \left(\frac{gml(M+m)}{J(M+m)-m^2l^2}\right) & 0 & 0 \end{vmatrix}, \quad B = \begin{vmatrix} 0 \\ 0 \\ \left(\frac{1}{M+m-\frac{m^2l^2}{J}}\right) \\ \left(\frac{}{J(M+m)-m^2l^2}\right) \end{vmatrix}; \quad C = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$D = 0\ (2\times1)$

6) $\begin{cases} \cdot m\ddot{x} = mg - \frac{Ki^2}{x^2} \\ \cdot L\dot{i} + Ri = V \end{cases}$ · Definido: $x = [x, \dot{x}, i]^T$; $\dot{x} = [\dot{x}, \ddot{x}, \dot{i}]^T$

$y = [x]$; $U = V$

E.E.: $\begin{matrix} f_1 = \\ f_2 = \\ f_3 = \end{matrix} \begin{vmatrix} \dot{x} = \dot{x} \\ \ddot{x} = g - \frac{K}{m}\cdot\frac{i^2}{x^2} \\ \dot{i} = \frac{V}{L} - \frac{R}{L}\cdot i \end{vmatrix}$ $\quad g = y = x$

· Linear: sistema da forma: $\dot{x} = Ax + BU$; $y = Cx$

$$A = \begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \dot{x}} & \frac{\partial f_1}{\partial i} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} & \frac{\partial f_2}{\partial i} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial \dot{x}} & \frac{\partial f_3}{\partial i} \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ \frac{-2Ki_0^2}{mx_0^3} & 0 & \frac{-2Ki_0}{mx_0^2} \\ 0 & 0 & -\frac{R}{L} \end{vmatrix}; \quad B = \begin{vmatrix} 0 \\ 0 \\ \frac{1}{L} \end{vmatrix}$$

$C = [1\ 0\ 0]$; $D = 0\ (1\times1)$

7) a) → Motor.

· Circuito da armadura: $L\dot{i} + Ri = V_a - e_b \cong V_a - K_b\Omega_1$

· Parte mecânica: $J_m\dot{\Omega}_1 = T_i - B_m\Omega_1 - 2|\theta_1-\theta_2|(\theta_1-\theta_2) \Rightarrow$

$\Rightarrow J_m\dot{\Omega}_1 = Ki - B_m\Omega_1 - 2|\theta_1-\theta_2|(\theta_1-\theta_2)$

· Carro: $m\ddot{x}_1 = 2|\theta_1-\theta_2|\cdot(\theta_1-\theta_2)\frac{1}{R} - 2\dot{x}_1^3 - Kx_1$

· Vínculo cinemático: $x_1 = \theta_2 R$, $\dot{x}_1 = \Omega_2\cdot R \Rightarrow$ sistema de 5ª ordem

· Vetor de estados: $x = [i\ \theta_1\ x_1\ \dot{\theta}_1\ \dot{x}_1]^T$

b) termos ñ lineares

· $f_1 = 2|\theta_1-\theta_2|\cdot(\theta_1-\theta_2) = 2\left|\theta_1 - \frac{x_1}{R}\right|\left(\theta_1 - \frac{x_1}{R}\right)$

$f_1 \cong f(0) + \left.\frac{\partial f}{\partial \theta_1}\right|_{eq}(\theta_1-\theta_{10}) + \left.\frac{\partial f}{\partial \theta_2}\right|_{eq}(\theta_2-\theta_{20})$

$f_1 \cong 2\left|\theta_{10} - \frac{x_{10}}{R}\right|\left(\theta_{10} - \frac{x_{10}}{R}\right) + 4\left|\theta_{10} - \frac{x_{10}}{R}\right|(\theta_1-\theta_{10}) - 4\left|\frac{x_{10}}{R} - \theta_{10}\right|(\theta_2-\theta_{20})$

· Definindo: $\left(\theta_{10} - \frac{x_{10}}{R}\right) = \delta_0$;

$(\theta_1 - \theta_{10}) = \theta_1$; $\Rightarrow f = 4\delta_0(\theta_1-\theta_2) + \delta_0^2$

$(\theta_2 - \theta_{20}) = \theta_2$.

· $f_2 = 2\dot{x}_1^3$; $f_2 \cong f_{eq} + \left.\frac{\partial f}{\partial \dot{x}_1}\right|_{eq}(\dot{x}_1 - \dot{x}_{10}) = F_{20} + 6\dot{x}_{10}^2\cdot(\dot{x}_1 - \dot{x}_{10})$

· Definindo: $(\dot{x}_1 - \dot{x}_{10}) = \dot{x}_{1a} \Rightarrow f = 6\dot{x}_{10}^2\cdot\dot{x}_1 + F_{20}$

· Reescrevendo as equações linearizadas:

$\dot{i} = \frac{V_a}{L} - \frac{K_b}{L}\dot{\theta}_1 - \frac{Ri}{L}$; $\ddot{\theta}_1 = \frac{K}{J_m}\cdot i - \frac{B_m}{J_m}\dot{\theta}_1 - \frac{4\delta_0}{J_m}\left(\theta_1 - \frac{x_1}{R}\right) - \frac{\delta_0^2}{J_m}$

$\ddot{x}_1 = \frac{4\delta_0}{mR}\left(\theta_1 - \frac{x_1}{R}\right) + \frac{\delta_0^2}{mR} - 6\dot{x}_{10}^2\cdot\dot{x}_1 + F_{20} - Kx_1$

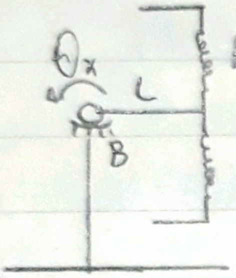
c) Redefinindo $x = [i\ \theta_1\ x_1\ \dot{\theta}_1\ \dot{x}_1]^T$

· Sistema do tipo: $\dot{x} = Ax + Bu$, $y = Cx$, onde: $B = \begin{vmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$

$C = [0\ 0\ 1\ 0\ 0]$

$$A = \begin{vmatrix} -\frac{R}{L} & 0 & 0 & -\frac{K_b}{L} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{K}{J_m} & \frac{-4\delta_0}{J_m} & \frac{4\delta_0}{J_m R} & -\frac{B_m}{J_m} & 0 \\ 0 & \frac{4\delta_0}{mR} & \left(\frac{-4\delta_0}{mR^2} - K\right) & 0 & -6\dot{x}_{10}^2 \end{vmatrix}$$

8)



$$H_x = J_x W_x$$

$$\dot{w}_x + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) w_z = \frac{\tau_x}{J_d}$$

$$\dot{w}_z + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) w_x = \cancel{\frac{\tau_z}{J_d}} \to 0$$

• $\tau_x = -\theta_x L \cdot 2K - B w_x$; também: $J_y = J_z = J_d$

• Reescrevendo: $\dot{w}_x + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) w_z = -\frac{2LK}{J_d}\theta_x - \frac{B w_x}{J_d}$

• Definindo: $x = [\theta_x \ \dot{\theta}_x]$ e $\dot{\theta}_x = w_x$, $w_z = U$

• $\ddot{\theta}_x = \frac{-2LK}{J_d}\theta_x - \frac{B}{J_d}\dot{\theta}_x - H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \cdot U$; $\dot{\theta}_x = \dot{\theta}_x$

b) no EE: $\begin{vmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ \frac{-2LK}{J_d} & \frac{-B}{J_d} \end{vmatrix} \begin{vmatrix} \theta_x \\ \dot{\theta}_x \end{vmatrix} + \begin{vmatrix} 0 \\ -H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \end{vmatrix} w_z$