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$$\begin{cases} M\ddot{x} + b\dot{x} + K_1 x - b\dot{y} - K_1 y = 0 \\ m\ddot{y} + b\dot{y} + (K_1 + K_2) y - b\dot{x} - K_2 x - K_2 z = 0 \end{cases}$$

$$X = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_1/M & K_1/M & -b/M & b/M \\ K_1/m & \frac{-(K_1+K_2)}{m} & \frac{b}{m} & -\frac{b}{m} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_2/m \end{bmatrix}$$

$$\dot{X} = AX + Bz$$

$$\eta = Cw + D\cdot z$$

$$\eta(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \Rightarrow C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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$$\begin{cases} m\ddot{y} + Ky + b\dot{y} - Kx - b\dot{x} = 0 \\ M\ddot{x} + Kx + b\dot{x} - Ky - b\dot{y} - u(t) = 0 \end{cases}$$

$$X = [x \; y \; \dot{x} \; \dot{y}]^t$$

$$\dot{X}(t) = A X(t) + B u(t)$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/m & K/m & -b/m & b/m \\ K/M & -K/M & b/M & -b/M \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M \end{bmatrix}$$

$$\eta = CX + Du$$

$$\eta(t) = \begin{bmatrix} \dot{y} \\ \dot{x} \end{bmatrix} \Rightarrow C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

4 3) Com $m >> M$, $m\ddot{y} = u(t)$

$$w(t) = [y \; \dot{y}]^t \; ; \; \dot{w} = Aw + Bu$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix} \qquad C = [1 \;\; 0]$$

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$$\begin{cases} M\ddot{x} + m\ddot{x} - ml\dot{\theta}^2 \sin\theta - ml\ddot{\theta}\cos\theta = u \\ \frac{4ml^2\ddot{\theta}}{3} - mgl\sin\theta + \ddot{x}\cos\theta = 0 \end{cases}$$

espaço de estados: $w(t) = [x \; \theta \; \dot{x} \; \dot{\theta}]^t$
entrada: $u(t)$

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \end{bmatrix} = \begin{bmatrix} w_3 \\ w_4 \\ \left(g - \frac{ml w^2_4}{M+m}\cos(w_2)\right)\sin w_2 - \frac{u(t)\cos w_2}{M+m} \Big] \left(\frac{4}{3}l + \frac{ml\cos^2 w_2}{M+m}\right)^{-1} \\ (u(t) + ml w^2 \sin w_2 + \frac{3}{4} mg \sin w_2 \cos w_2)\left(M + m + \frac{3}{4} m \cos^2 w_2\right) \end{bmatrix}$$

6 a.
$$\begin{cases} m\ddot{x} - mg + kI^2/x^2 = 0 \\ L\dot{I} + RI = V(t) \end{cases}$$

$$W(t) = [x \ \dot{x} \ I]^T = [w_1 \ \ w_2 \ \ w_3]^T$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} = \begin{bmatrix} w \\ g - (kw_3^2/mw_1^2) \\ V(t)/L - Rw_3/L \end{bmatrix}$$

b.
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2k\bar{x}_3^2/m\bar{x}_1^2 & 0 & -2k\bar{x}_3/m\bar{x}_1^2 \\ 0 & 0 & -R/L \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}$$

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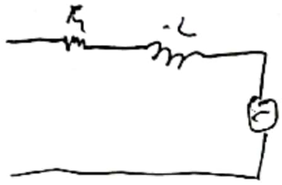

$T = K_1(t) \Rightarrow V_a - R_1 - Li - K\dot{\theta}_i = 0$

$$TMQM \begin{cases} Jm\ddot{\theta}_1 + B_m\dot{\theta}_1 - K_1(t) - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2) = 0 \\ J_D\ddot{\theta}_2 - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2) + 2\dot{x}^3 r = 0 \end{cases}$$

Linearizando

$$f(x) = 2\dot{x}^3 \Rightarrow f_1(x) = 2\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2(\dot{x} - \bar{\dot{x}}) = -4\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2\dot{x}$$

$$V_a - R_1 - LI - K\dot{\theta}_1 = 0$$
$$Jm\ddot{\theta}_1 + Bm\dot{\theta}_1 - K_1(t) - f_2(\theta_1, \theta_2) = 0$$
$$J_p\ddot{\theta}_2 - f_2(\theta_1, \theta_2) - 4\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2\dot{x} = 0$$
$$m\ddot{x} + 4\bar{\dot{x}}^3 - 6\bar{\dot{x}}^2\dot{x} + b\dot{x} + Kx = 0$$

$$\begin{array}{ccccccccc} X(t) = [& \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 & x & \dot{x} & i &]^T \\ & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & \end{array}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \\ \dot{x} \\ \ddot{x} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} w_2 \\ [Kw_7 - Bm w_2 - f_2(w_1, w_3)]/J_m \\ w_4 \\ [f_2(w_1, w_3) - r(4\bar{x}_4^3 - 6\bar{x}_4^2 x_4)]/J_p \\ w_6 \\ [r(4\bar{x}_4^3 - 6\bar{x}_4^2 x_4) - Kw_5 - bw_6)]/m \\ (V_0 - Rw_7 - Kw_2)/L \end{bmatrix}$$

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$$J_x\ddot{\theta}_x + 2b\dot{\theta}_x + Kl\theta_x = JW\theta_z$$

$$\begin{array}{cccc} w(t) = [& \theta_x & \dot{\theta}_x &]^T \\ & w_1 & w_2 & \end{array}$$

$$\dot{w}(t) = \begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} w_1 \\ (-JW\theta z - 2bw_1 - Klw_1)/J_x \end{bmatrix}$$

$$\dot{w}(t) = Aw(t) + Bu \qquad A = \begin{bmatrix} 0 & 1 \\ -Kl/J_x & -2b/J_x \end{bmatrix} \ ; \ B = \begin{bmatrix} 0 \\ -J/J_x \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$