# Modelagem 20/10

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2)

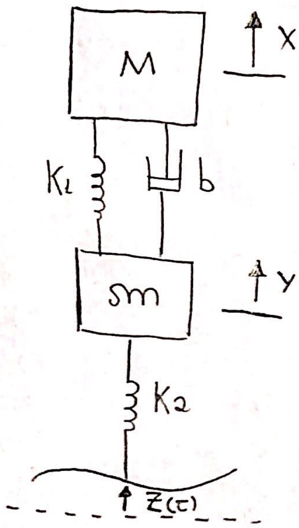
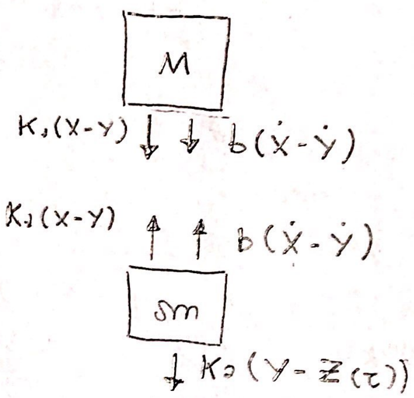


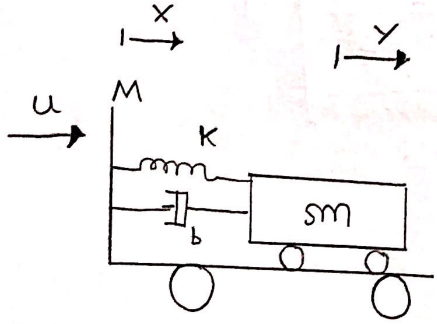

Diagrama de corpo livre:




$$\begin{cases} M\ddot{x} = -k_1(x-y) - b(\dot{x}-\dot{y}) \\ sm\ddot{y} = k_1(x-y) + b(\dot{x}-\dot{y}) - k_2(y-z(t)) \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1/M & -b/M & k_1/M & b/M \\ 0 & 0 & 0 & 1 \\ k_1/sm & -b/sm & \frac{-k_1-k_2}{sm} & -b/sm \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_2/sm \end{bmatrix} z(t)$$

3)




$$\begin{cases} sm\ddot{y} + k(y-x) + b(\dot{y}-\dot{x}) = 0 \\ M\ddot{x} - k(y-x) - b(\dot{y}-\dot{x}) = u(t) \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/M & -b/M & k/M & b/M \\ 0 & 0 & 0 & 1 \\ k/sm & b/sm & -k/sm & -b/sm \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 0 \end{bmatrix} u(t)$$

4)

Como o problema já foi resolvido em listas anteriores, será analisada apenas a resposta do sistema a deslocamentos dos graus de liberdade:

$$\begin{cases} m_1\ddot{x}_1 + K(x_1 - z) - K_1(x_G - x_1 + l\theta) - b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) = 0 \\ m_2\ddot{x}_2 + K(x_2 - z) - K_2(x_G - x_2 - l\theta) - b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) = 0 \\ M\ddot{x}_G + K_1(x_G - x_1 + l\theta) + K_2(x_G - x_2 - l\theta) + b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) = 0 \\ J_G\ddot{\theta} + K_1 l(x_G - x_1 + l\theta) - K_2 l(x_G - x_2 - l\theta) + b_1 l(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2 l(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) = 0 \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_G \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{(K+K_1)}{m_1} & 0 & K_1/m_1 & K_1 l/m_1 & -b_1/m_1 & 0 & b_1/m_1 & b_1 l/m_1 \\ 0 & -\frac{(K+K_2)}{m_2} & K_2/m_2 & -K_2 l/m_2 & 0 & -b_2/m_2 & b_2/m_2 & -b_2 l/m_2 \\ K_1/M & K_2/M & -(K_1+K_2)/M & l(K_2-K_1)/M & b_1/M & b_2/M & -(b_1+b_2)/M & \frac{l(b_2-b_1)}{M} \\ K_1 l/J & -K_2 l/J & l(K_2-K_1)/J & -l^2(K_2+K_1)/J & b_1 l/J & -b_2 l/J & l(b_2-b_1)/J & -\frac{l^2(b_2-b_1)}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_G \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \end{bmatrix}$$

5)

$$\begin{cases} (M+m)\ddot{x} + m l\ddot{\theta} = u \\ J\ddot{\theta} + m l\ddot{x} - m l g\theta = 0 \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m^2 l^2}{J(M+m) - m^2 l^2} & 0 & 0 \\ 0 & \frac{g m l(M+m)}{J(M+m) - m^2 l^2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M + m - \frac{m^2 l^2}{J}} \\ \frac{-g m l}{J(M+m) - m^2 l^2} \end{bmatrix} u$$

6)

$$\begin{cases} m\ddot{x} = mg - KI^2/x^2 \\ L\dot{I} + RI = V \end{cases}$$

- Linearizando o sistema:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2KI_0^2}{m x_0^3} & 0 & -\frac{2KI_0}{m x_0^2} \\ 0 & 0 & -R/L \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} V$$