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PME 3380 - EXERCÍCIOS 01/10 E 06/10

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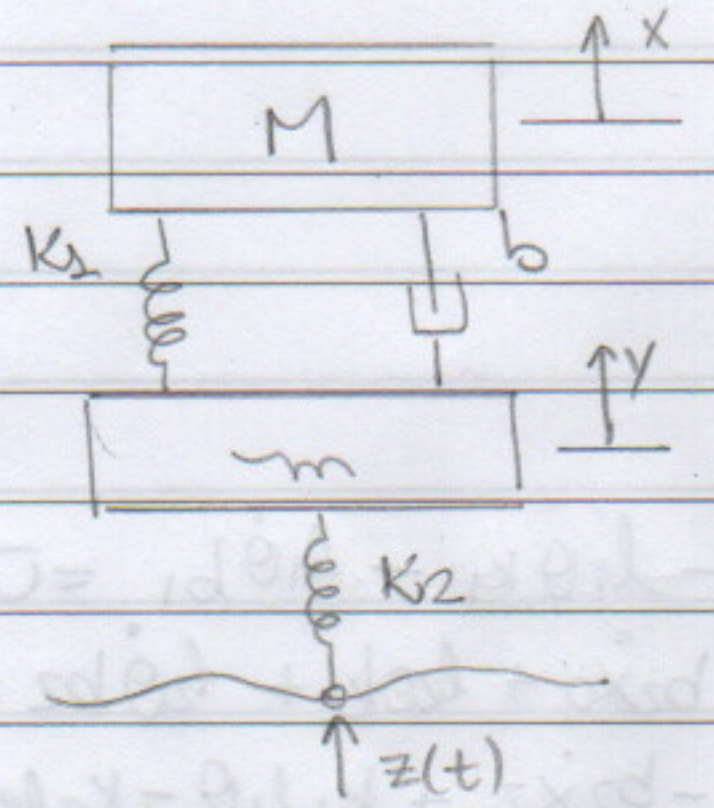
2.



$$\begin{cases} M\ddot{x} + b\dot{x} + k_1 x - b\dot{y} - k_1 y = 0 \\ m\ddot{y} + b\dot{y} + (k_1+k_2)y - b\dot{x} - k_1 x - k_2 z = 0 \end{cases}$$

espaço de estados:

$$w(t) = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \qquad \dot{w}(t) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix}$$

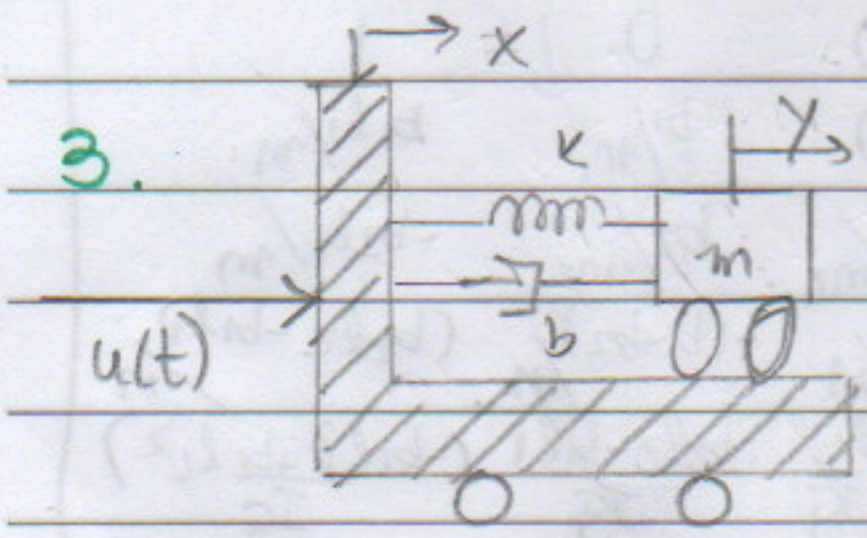
$$\dot{w}(t) = A\,w(t) + B z(t)$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/M & k_1/M & -b/M & b/M \\ k_1/m & -(k_1+k_2)/m & b/m & -b/m \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_2/m \end{bmatrix}$$

$$r = C w + D \cdot z$$

$$r(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \Rightarrow C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

3.



$$\begin{cases} m\ddot{y} + ky + b\dot{y} - kx - b\dot{x} = 0 \\ M\ddot{x} + kx + b\dot{x} - ky - b\dot{y} - u(t) = 0 \end{cases}$$

$$w(t) = [x \quad y \quad \dot{x} \quad \dot{y}]^t$$

$$\dot{w}(t) = [\dot{x} \quad \dot{y} \quad \ddot{x} \quad \ddot{y}]^t$$

$$\dot{w}(t) = A\,w(t) + B\,u(t)$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/m & k/m & -b/m & b/m \\ k/M & -k/M & b/M & -b/M \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M \end{bmatrix}$$

$$r = Cw + Du$$

$$r(t) = \begin{bmatrix} \dot{y} \\ \dot{x} \end{bmatrix} \Rightarrow C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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31) como $m \gg M$, $m\dot{y} = u(t)$

$\therefore \quad w(t) = [y \;\; \dot{y}]^t \quad ; \quad \dot{w} = Aw + Bu$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix} \qquad C = [1 \;\; 0]$$

4. para pequenos movimentos:

$$\begin{cases} m_1\ddot{x}_1 + b_1\dot{x}_1 + (k_{p1}+k_1)x_1 - k_{p1}z(t) - k_1x_G - b_1\dot{x}_G - l_1\theta k_1 - l_1\dot{\theta}b_1 = 0 \\ m_2\ddot{x}_2 + b_2\dot{x}_2 + (k_{p2}+k_2)x_2 - k_{p2}z(t-\alpha) - k_2x_G - b_2\dot{x}_G + l_2\theta k_2 + l_2\dot{\theta}b_2 = 0 \\ M\ddot{x}_G + (b_1+b_2)\dot{x}_G + (k_1+k_2)x_G - k_1x_1 - b_1\dot{x}_1 - k_2x_2 - b_2\dot{x}_2 + k_1l_1\theta - k_2l_2\theta \\ \qquad + b_1l_1\dot{\theta} - b_2l_2\dot{\theta} = 0 \\ J_G\ddot{\theta} + (b_1l_1^2 + b_2l_2^2)\dot{\theta} + (k_1l_1^2 + k_2l_2^2)\theta - k_1l_1x_1 - b_1l_1\dot{x}_1 + k_2l_2x_2 + b_2l_2\dot{x}_2 \\ \qquad + (k_1l_1 - k_2l_2)x_G + (b_1l_1 - b_2l_2)\dot{x}_G = 0 \end{cases}$$

$$w = [x_1, x_2, x_G, \theta, \dot{x}_1, \dot{x}_2, \dot{x}_G, \dot{\theta}]^t$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -(k_{p1}+k_2)/m_1 & 0 & k_1/m_1 & k_1l_1/m_1 & -b_1/m_1 & 0 & b_1/m_1 & b_1l_1/m \\ 0 & -\frac{(k_{p2}+k_2)}{m_2} & k_2/m_2 & -k_2l_2/m_2 & 0 & -b_2/m_2 & b_2/m_2 & -b_2l_2/m \\ k_1/M & k_2/M & -(k_1+k_2)/M & \frac{k_2l_2-k_1l_1}{M} & b_1/M & b_2/M & -(b_1+b_2)/M & (b_2l_2-b_1l_1)/M \\ k_1l_1/J_G & -\frac{k_2l_2}{J_G} & \frac{k_2l_2-k_1l_1}{J_G} & \frac{-(k_1l_1^2+k_2l_2^2)}{J_G} & \frac{b_1l_1}{J_G} & \frac{-b_2l_2}{J_G} & \frac{b_2l_2-b_1l_1}{J_G} & \frac{(-b_1l_1^2+b_2l_2^2)}{J_G} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & k_{p2}/m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{p1}/m_1 & 0 & 0 & 0 \end{bmatrix}^t$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5. $\begin{cases} M\ddot{x} + m\ddot{x} - ml\dot{\theta}^2 sen\theta - ml\ddot{\theta}\cos\theta = u \\ \dfrac{4ml^2\ddot{\theta}}{3} - mgl\,sen\theta + \ddot{x}\cos\theta = 0 \end{cases}$

espaço de estados : $w(t) = \begin{array}{c} \begin{matrix} w_1 & w_2 & w_3 & w_4 \end{matrix} \\ \begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} \end{bmatrix}^t \end{array}$

entrada : $u(t)$

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \end{bmatrix} = \begin{bmatrix} w_3 \\ w_4 \\ \left[\left(g - \frac{mlw_4^2}{M+m}\cos(w_2)\right) sen w_2 - \frac{u(t)\cos w_2}{M+m}\right]\left(\frac{4}{3}l + \frac{ml\cos^2 w_2}{M+m}\right)^{-1} \\ \left(u(t) + mlw_4^2 sen w_2 + \frac{3}{4} mg\, sen w_2 \cos w_2\right)\left(M + m + \frac{3}{4} m\cos^2 w_2\right) \end{bmatrix}$$

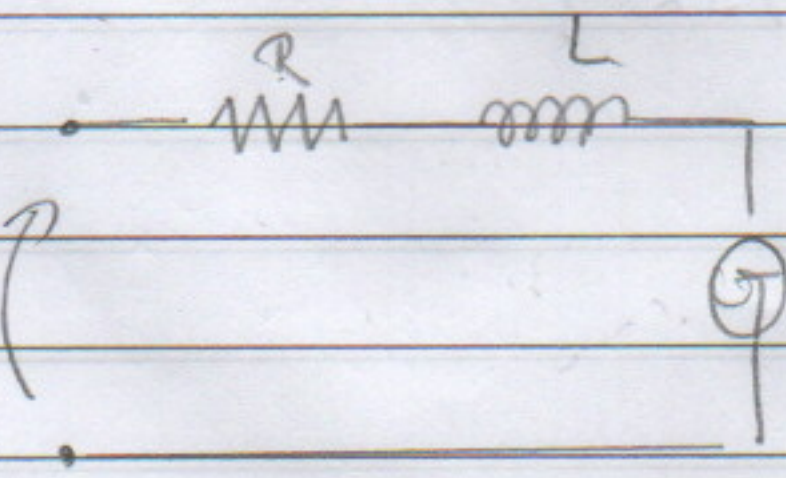
6. a. $\begin{cases} m\ddot{x} - mg + KI^2/x^2 = 0 \\ L\dot{I} + RI = V(t) \end{cases}$

$w(t) = [\,x \quad \dot{x} \quad I\,]^t = [\,w_1 \quad w_2 \quad w_3\,]^t$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} = \begin{bmatrix} w \\ g - (Kw_3^2/mw_1^2) \\ V(t)/L - Rw_3/L \end{bmatrix}$$

b. $A = \begin{bmatrix} 0 & 1 & 0 \\ 2Kx_3^2/mx_1^3 & 0 & -2Kx_3/mx_1^2 \\ 0 & 0 & -R/L \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}$

7.



$T = Ki(t) \Rightarrow Va - Ri - L\dot{i} - K\dot{\theta} = 0$

TMQM $\begin{cases} J_m\ddot{\theta}_1 + B_m\dot{\theta}_1 - k_1(t) - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2) = 0 \\ J_p\ddot{\theta}_2 - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2) + 2\dot{x}^3 r = 0 \end{cases}$

TMB $\rightarrow$ $m\ddot{x} = 2\dot{x}^3 - b\dot{x} - kx$

linearização :

$f(x) = 2\dot{x}^3 \Rightarrow f_1(x) = 2\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2(\dot{x} - \bar{\dot{x}}) = -4\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2\dot{x}$

$\therefore$ $$V_a - R_1 - Li - K\dot{\theta}_1 = 0$$
$$J_m\ddot{\theta}_1 + B_m\dot{\theta}_1 - \kappa_1(t) - f_2(\theta_1,\theta_2) = 0$$
$$J_p\ddot{\theta}_2 - f_2(\theta_1,\theta_2) - 4\bar{x}^3 + 6\bar{x}^2\dot{x} = 0$$
$$m\ddot{x} + 4\bar{x}^3 - 6\bar{x}^2\dot{x} + b\dot{x} + kx = 0$$

$$w(t) = \begin{array}{c} \left[\begin{array}{ccccccc} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 & x & \dot{x} & i \end{array} \right]^t \\ \begin{array}{ccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 \end{array} \end{array}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \\ \dot{x} \\ \ddot{x} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} w_2 \\ [Kw_7 - B_m w_2 - f_2(w_1,w_3)]/J_m \\ w_4 \\ [f_2(w_1,w_3) - r(4\bar{x}_g^3 - 6\bar{x}_g^2 x_g)]/J_p \\ w_6 \\ [r(4\bar{x}_g^3 - 6\bar{x}_g^2 x_g) - Kw_5 - bw_6]/m \\ (V_a - Rw_7 - Kw_2)/L \end{bmatrix}$$

8. EDO: $J_x\ddot{\theta}_x + 2b\dot{\theta}_x + k\ell\theta_x = J\omega\theta_z$

$$w(t) = \begin{array}{c} [\theta_x \;\; \dot{\theta}_x]^t \\ w_1 \;\; w_2 \end{array}$$

$$\dot{w}(t) = \begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} w_1 \\ (-J\omega\theta_z - 2bw_2 - k\ell w_1)/J_x \end{bmatrix}$$

$$\dot{w}(t) = A\,w(t) + Bu$$

$$A = \begin{bmatrix} 0 & 1 \\ -k\ell/J_x & -2b/J_x \end{bmatrix} \; ; \; B = \begin{bmatrix} 0 \\ -J/J_x \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$