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$$\begin{cases} \ddot{\bar{x}} = (u_1 + u_2)/M \\ \ddot{\delta} = -\frac{KM}{M_1 M_2}\delta + \frac{u_1}{M_1} - \frac{u_2}{M_2} \end{cases}$$

$z = [\bar{x} \quad \delta \quad \dot{\bar{x}} \quad \dot{\delta}]$

$\dot{z} = [\dot{\bar{x}} \quad \dot{\delta} \quad \ddot{\bar{x}} \quad \ddot{\delta}]$

$$\dot{z} = \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -KM/M_1M_2 & 0 & 0 \end{pmatrix}}_{A} z + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ 1/M_1 & -1/M_2 \end{pmatrix}}_{B} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$y = [x_1 \quad x_2]$

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \begin{bmatrix} M_1/M & \\ 1 & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} & /M \\ & -1 \end{bmatrix}^{-1} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} & M_2/M \\ & M_1/M \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

$$y = \underbrace{\begin{pmatrix} & M_2/M & 0 & 0 \\ & M_1/M & 0 & 0 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{pmatrix}}_{z}$$

② $$\begin{cases} M\ddot{x} + b(\dot{x} - \dot{y}) + k_1(x-y) = 0 \\ m\ddot{y} - b(\dot{x} - \dot{y}) - k_1(x-y) - k_2(y-z) = 0 \end{cases}$$

$X = [x \quad y \quad \dot{x} \quad \dot{y}]^T \qquad \dot{X} = AX + Bz$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} & & 1 & \\ & & & 1 \\ -k_1/M & k_1/M & -b/M & b/M \\ k_1/m & \frac{-k_1-k_2}{m} & b/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} \\ \\ \\ k_2/m \end{bmatrix} z$$

③ $$\begin{cases} m\ddot{y} + b(\dot{y} - \dot{x}) + k(y-x) = 0 \\ M\ddot{x} - b(y-x) - k(\dot{y} - \dot{x}) = u \end{cases}$$

$X = [x \quad y \quad \dot{x} \quad \dot{y}] \qquad \dot{X} = AX + Bu$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} & & 1 & \\ & & & 1 \\ -k/M & k/M & -b/M & b/M \\ k/m & -k/m & b/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} \\ \\ 1/M \\ \\ \end{bmatrix} u(t)$$

④

$m_1 \ddot{x}_1 - b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) - k_1(x_G - x_1 + l\theta) + k(x_1 - z) = 0$

$m_2 \ddot{x}_2 - b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) - k_2(x_G - x_2 - l\theta) + k(x_2 - z') = 0$

$M\ddot{x}_G + b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) +$
$\quad + k_1(x_G - x_1 + l\theta) + k_2(x_G - x_2 - l\theta) = 0$

$J_G\ddot{\theta} + b_1 l(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2 l(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) +$
$\quad + k_1 l(x_G - x_1 + l\theta) - k_2 l(x_G - x_2 + l\theta) = 0$

$\vec{x} = [x_1 \quad x_2 \quad x_G \quad \theta]^T \therefore X = [\vec{x} \quad \dot{\vec{x}}]^T$

$\dot{X} = AX + BU$, com $U = [z(t), z(t-d)]$

$$A = \begin{bmatrix} 0_{4\times4} & I_{4\times4} \\ \bar{A}_{4\times4} & \bar{\bar{A}}_{4\times4} \end{bmatrix} \quad B = \begin{bmatrix} 0_{2\times1} \\ k \\ k \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} -\frac{k_1+k}{m_1} & & k_1/m_1 & k_1 l/m_1 \\ & -\frac{k_2+k}{m_2} & k_2/m_2 & -k_2 l/m_2 \\ k_1/M & k_2/M & -\frac{k_1+k_2}{M} & l\frac{k_2-k_1}{M} \\ k_1 l/J & -k_2 l/J & l\frac{k_2-k_1}{J} & -l^2\frac{k_2+k_1}{J} \end{bmatrix}$$

$$\bar{\bar{A}} = \begin{bmatrix} -b_1/m_1 & & b_1/m_1 & b_1 l/m_1 \\ & -b_2/m_2 & b_2/m_2 & -b_2 l/m_2 \\ b_1/M & M & -\frac{b_1+b_2}{M} & -l\frac{b_1-b_2}{M} \\ b_1 l/J & -b_1 l/J & l\frac{b_1-b_2}{J} & l^2\frac{b_1-b_2}{J} \end{bmatrix}$$

Obs: $\bar{A}$ e $\bar{\bar{A}}$ são, na representação (M, C, K), $M^{-1}K$ e $M^{-1}C$, respectivamente

⑥
$$\begin{cases} m\ddot{x} = mg - kI^2/x^2 \\ L\dot{I} + RI = V \end{cases}$$

$$\begin{cases} \ddot{x} = \dfrac{2i_0}{mx_0^2} I - \dfrac{2i_0^2}{mx_0^3} x \\ \dot{I} = (-R/L)I + V/L \end{cases}$$

$X = [\, x \quad \dot{x} \quad I \,]$ $\qquad \dot{X} = AX + BV$

$$A = \begin{bmatrix} & 1 & \\ -2i_0^2/mx_0^3 & & 2i_0/mx_0^2 \\ & & -R/L \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}$$

$$Y = CX + DV \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad D = 0_{3\times1}$$

⑤
$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u \\ J\ddot{\theta} + ml\ddot{x} - mlg\theta = 0 \end{cases} \quad \ddot{\theta} = \ldots, \quad \ddot{x} = \ldots$$

$$\ddot{x} = \frac{g[J(m+M)+1]}{m^2l^2-(m+M)J}\theta - \frac{J}{m^2l^2-(m+M)J}u$$

$$\ddot{\theta} = -\frac{mlg(m+M)}{m^2l^2-(m+M)J}\theta + \frac{ml}{m^2l^2-(m+M)J}u$$

Escrevemos $\ddot{x} = c_1\theta + d_1 u$
$\ddot{\theta} = c_2\theta + d_2 u$

$X = [\, x \quad \theta \quad \dot{x} \quad \dot{\theta} \,]^T \qquad \dot{X} = AX + Bu$

$$A = \begin{bmatrix} 0_{2\times2} & & I_{2\times2} \\ 0 & c_1 & 0_{2\times2} \\ 0 & c_2 & \end{bmatrix} \quad B = \begin{bmatrix} 0_{2\times1} \\ d_1 \\ d_2 \end{bmatrix}$$