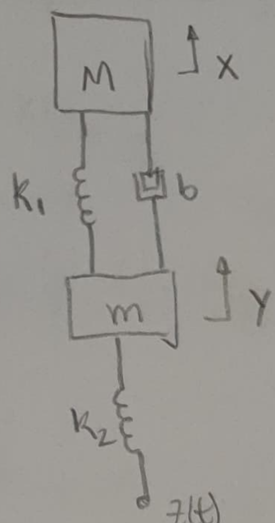
JOÃO PEDRO DIAS NUNES - 10705846 - 17110

Ex 2. Suspensão 1/4 carro



TMB (M): $M\ddot{x} = -k_1(x-y) - b(\dot{x}-\dot{y})$

TMB (m): $m\ddot{y} = -k_2(y-z) + k_1(x-y) + b(\dot{x}-\dot{y})$

Seja $x = x_1$, $y = x_2$

$$\begin{cases} \ddot{x}_1 = (-k_1 x_1 + k_1 x_2 - b\dot{x}_1 + b\dot{x}_2)M^{-1} \\ \ddot{x}_2 = (-k_2 x_2 + k_2 z + k_1 x_1 - k_1 x_2 + b\dot{x}_1 - b\dot{x}_2)m^{-1} \end{cases}$$

Seja $\dot{x}_1 = x_3$ e $\dot{x}_2 = x_4$ e $\underset{\sim}{X} = [x_1, x_2, x_3, x_4]^T$

$\dot{x}_1 = x_3$; $\dot{x}_3 = (-k_1 x_1 + k_1 x_2 - b x_3 + b x_4) M^{-1}$

$\dot{x}_2 = x_4$; $\dot{x}_4 = (-k_2 x_2 + k_2 z + k_1 x_1 + b x_3 - b x_4) m^{-1}$

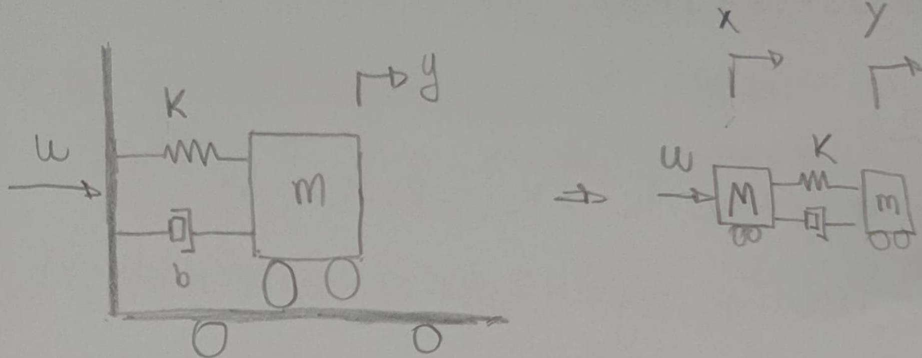
na forma matricial

$$\dot{\underset{\sim}{X}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/M & k_1/M & -b/M & b/M \\ k_1/m & -k_2/m & b/m & -b/m \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\underset{\sim}{X}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ k_2/m \end{bmatrix}}_{B} \underset{\sim}{z}_{\,w}$$

$$\underset{\sim}{Y} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}^{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$D = [0]$.

Ex(3): Carreta sem massa



$$\text{Caso } M=0: \quad u = K(x-y) + b(\dot{x}-\dot{y})$$

TMB (M): $M\ddot{x} = u - K(x-y) - b(\dot{x}-\dot{y})$

TMB (m): $m\ddot{y} = -K(y-x) - b(\dot{y}-\dot{x})$

Adotando $x = x_1$, $\dot{x} = x_3$, $y = x_2$, $\dot{y} = x_4$ (caso $M \neq 0$)

$$\begin{cases} \dot{x}_3 = \dfrac{u}{M} - \dfrac{K}{M}x_1 + \dfrac{K}{M}x_2 - \dfrac{b}{M}x_3 + \dfrac{b}{M}x_4 \\ \dot{x}_4 = -\dfrac{K}{m}x_2 + \dfrac{K}{m}x_1 - \dfrac{b}{m}x_4 + \dfrac{b}{m}x_3 \end{cases}$$

Caso 1: $M=0$: $\dot{x}_4 = u/m$

$$\dot{X} = \begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_2 \\ x_4 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{B} u$$

e $Y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}$ $\quad [0] = D$

Caso 2: $M \neq 0$:

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/m & +K/M & -b/M & +b/M \\ +K/m & -K/m & b/m & -b/m \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix}}^{B} u$$

$$Y = \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}^{C} X \qquad D = [0].$$

Ex ④. Inclinação do Chassi

Temos a modelagem feita do exercício já passado:
sem a hipótese de pequenos deslocamentos:

$$m_1\ddot{x}_1 + K(x_1) - K(z) - K_1(l-e)\,\text{sen}\theta + k_1 x_1 - b_1\dot{x}_g - b_1\cos\theta\dot{\theta}(l-e) + b_1\dot{x}_1 = 0$$

$$m_2\ddot{x}_2 + k(x_2) - kz' - k_2 x_g + k_2 e\,\text{sen}\theta + k_2 x_2 - b_2\dot{x}_g + b_2 e\cos\theta\dot{\theta} + b_2\dot{x}_2 = 0$$

$$M\ddot{x}_G + k_2 x_g - k_2 e\,\text{sen}\theta - k_2 x_2 + k_1 x_g + k_1(l-e)\,\text{sen}\theta - k_1 x_1 + b_2\dot{x}_g - b_2 e\cos\theta\dot{\theta} - b_2\dot{x}_2 + \cdots$$

$$+\cdots\ b_1\dot{x}_g + b_1(l-e)\cos\theta\dot{\theta} - b_1\dot{x}_1 = 0$$

$$J_0\ddot{\theta} + k_2 e(x_g - e\,\text{sen}\theta - x_2)\cos\theta + k_1(l-e)(x_g + (l-e)\,\text{sen}\theta - x_1)\cos\theta + b_2(\dot{x}_g - \cdots$$

$$\cdots - e\cos\theta\dot{\theta} - \dot{x}_2)e\cos\theta + b_1(\dot{x}_g + (l-e)\cos\theta\dot{\theta} - \dot{x}_1)(l-e)\cos\theta = 0$$

Adotando a mudança de coordenada:

$x_1 = x_1;\ x_2 = x_2;\ x_g = x_3;\ \theta = x_4;\ \dot{x}_1 = x_5;\ \dot{x}_2 = x_6;\ \dot{x}_g = x_7;\ \dot{\theta} = x_8;$ podemos

reescrever o sistema acima (NÃO LINEAR)

$$\begin{cases}
\dot{x}_1 = x_5;\ \dot{x}_2 = x_6;\ \dot{x}_3 = x_7;\ \dot{x}_4 = x_8; \\
\dot{x}_5 = -\dfrac{1}{m_1}\left[kx_1 - kz - k_1(l-e)\,\text{sen}(x_4) + k_1 x_1 - b_1 x_7 - b_1\cos(x_4)x_8(l-e) + b_1 x_5\right]; \\
\dot{x}_6 = -\dfrac{1}{m_2}\left[kx_2 - kz(t-d) - k_2 x_3 + k_2 e\,\text{sen}\,x_4 + k_2 x_2 - b_2 x_7 + b_2 e\cos x_4 x_8 + b_2 x_6\right]; \\
\dot{x}_7 = -\dfrac{1}{M}\left[k_2 x_3 - k_2 e\,\text{sen}(x_4) - k_2 x_2 + k_1 x_3 + k_1(l-e)\,\text{sen}(x_4) - k_1 x_1 + b_2 x_7 - b_2 e\cos(x_4)x_8 - \cdots\right. \\
\qquad \left.\cdots - b_2 x_6 + b_1 x_7 + b_1(l-e)\cos(x_4)\,x_8 - b_1 x_5\right]; \\
\dot{x}_8 = -\dfrac{1}{J_0}\left[k_2 e(x_3 - e\,\text{sen}(x_4) - x_2)\cos(x_4) + k_1(l-e)(x_3 + (l-e)\,\text{sen}(x_4) - x_1)\cos(x_4) + \cdots\right. \\
\qquad \left.\cdots + b_2(x_7 - e\cos x_4 \cdot x_8 - x_6)e\cos(x_4) + b_1(x_7 + (l-e)\cos(x_4)x_8 - x_5)(l-e)\cos(x_4)\right].
\end{cases}$$

As equações apresentadas anteriormente poderiam ser resolvidas numericamente pelo método de RK, mas sem funções prontas lineares do Scilab/Matlab.

Linearizando o sistema, obtemos:

$$
\begin{cases}
\dot{x}_5 = \dfrac{-1}{m_1}\left[k x_1 - kz - k_1 x_3 - k_1(l-e)x_4 + k_1 x_1 - b_1 x_7 - b_1(l-e)x_8 + b_1 x_5\right]; \\[2ex]
\dot{x}_6 = -\dfrac{1}{m_2}\left[k_? x_2 - kz(t-d) - k_2 x_3 + k_2 e x_4 + k_2 x_2 - b_2 x_7 + b_2 e x_8 + b_2 x_6\right]; \\[2ex]
\dot{x}_7 = \dfrac{-1}{M}\left[k_2 x_3 + k_2 e x_4 - k_2 x_2 + k_1 x_3 + k_1(l-e)x_4 - k_1 x_1 + b_2 x_7 - b_2 e x_8 - b_2 x_6 + b_1 x_7 + b_1(l-e) \dots \right. \\
\qquad \left. \dots x_8 - b_1 x_5\right]; \\[2ex]
\dot{x}_8 = -\dfrac{1}{J_0}\left[k_2(x_3 - e x_4 - x_2)e + k_1(x_3 + (l-e)x_4 - x_1)(l-e) + b_2(x_7 - e x_8 - x_6)e + b_1(x_7 + (l-e)x_8 - x_5)(l-e)\right]
\end{cases}
$$

Sendo $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T$

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\dfrac{-k-k_1}{m_1} & 0 & \dfrac{k_1}{m_1} & \dfrac{k_1(l-e)}{m_1} & -\dfrac{b_1}{m_1} & 0 & \dfrac{b_1}{m_1} & \dfrac{b_1(l-e)}{m_1} \\[2ex]
0 & \dfrac{-k-k_2}{m_2} & \dfrac{k_2}{m_2} & \dfrac{-k_2 e}{m_2} & 0 & \dfrac{-b_2}{m_2} & \dfrac{+b_2}{m_2} & \dfrac{-b_2 e}{m_2} \\[2ex]
-k_1/M & k_2/M & \dfrac{-k_1-k_2}{M} & \dfrac{(k_2 e + k_1(l-e))}{-M} & \dfrac{-b_1}{M} & \dfrac{b_2}{M} & \dfrac{-b_2-b_1}{M} & \dfrac{-b_2 e + b_1(l-e)}{-M} \\[2ex]
\dfrac{k_1(l-e)}{J_0} & \dfrac{k_2 e}{J_0} & \dfrac{k_2 e + k_1(l-e)}{-J_0} & \dfrac{-k_2 e^2 + k_1(l-e)^2}{-J_0} & \dfrac{+b_1(l-e)}{J_0} & \dfrac{b_2 e}{J_0} & \dfrac{b_2 e + b_1(l-e)}{J_0} & \dfrac{-b_2 e^2 + b_1(l-e)^2}{-J_0}
\end{bmatrix}
$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & k/m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k/m_2 & 0 & 0 \end{bmatrix}^T$$

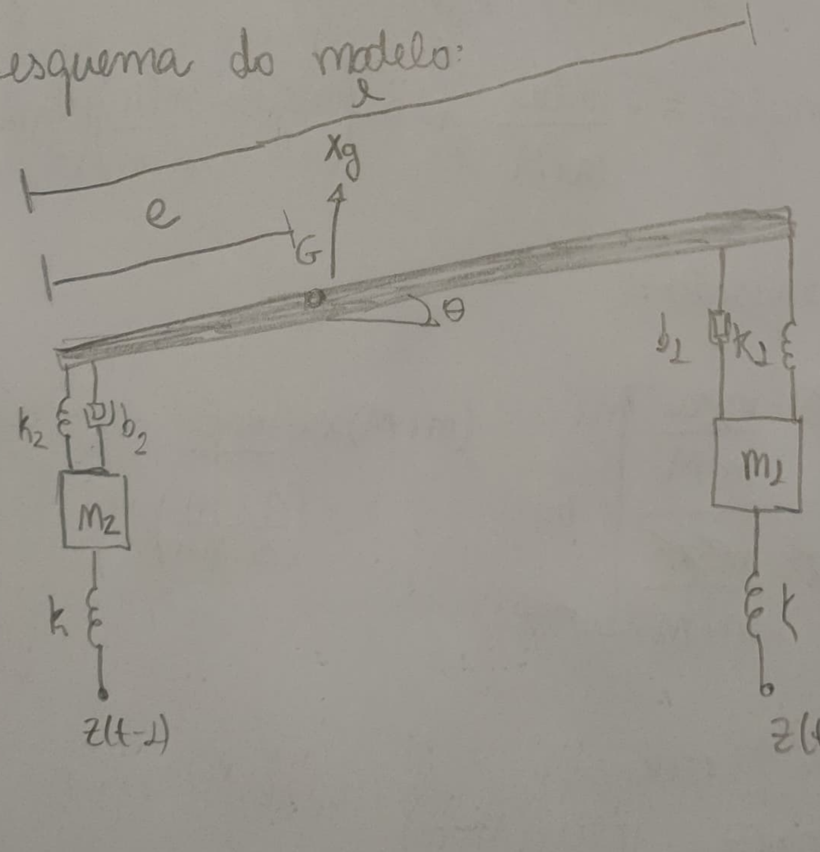
$$u = \begin{bmatrix} z(t) \\ z(t-\alpha) \end{bmatrix}$$

Portando: $\dot{X} = AX + Bu$

$Y = CX$ tal que

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad D = [0].$$

Obs: esquema do modelo:



Exercício 5: Pêndulo invertido

Para o pêndulo invertido, também já temos as equações:

NA FORMA NÃO LINEAR

$$\begin{cases} (M+m)\ddot{x} + ml(\cos\theta\,\ddot{\theta} - \operatorname{sen}\theta\,\dot{\theta}^2) = u \\ \frac{4}{3}ml^2\theta + ml\ddot{x}\cos\theta = mgl\operatorname{sen}\theta \end{cases}$$

NA FORMA LINEAR: $(\theta \approx 0°)$

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u \\ \frac{4}{3}ml^2\ddot{\theta} + ml\ddot{x} = mgl\theta \end{cases}$$

"Desacoplando" as equações:

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{(M+m)} \Rightarrow \frac{4}{3}ml^2\ddot{\theta} + ml\left(\frac{u - ml\ddot{\theta}}{M+m}\right) = mgl\theta \Rightarrow$$

$$\Rightarrow \left(\frac{4}{3}ml^2 - \frac{m^2l^2}{m+M}\right)\ddot{\theta} - mgl\theta = -\frac{mlu}{m+M} \Rightarrow \ddot{\theta} = \left(mgl\theta - \frac{mlu}{m+M}\right)\left(\frac{4}{3}ml^2 - \frac{m^2l^2}{m+M}\right)^{-1}$$

Substituindo na primeira equação:

$$(M+m)\ddot{x} + \cancel{ml}\left[\frac{mg\cancel{l}\theta - \frac{m\cancel{l}u}{m+M}}{\frac{4}{3}\cancel{ml^2} - \frac{m^2l^2}{m+M}}\right] = u \Rightarrow (m+M)\ddot{x} + \frac{mg\theta}{\left(\frac{4}{3} - \frac{m}{M+m}\right)} - u = u$$

Portanto, o sistema fica:

$$\begin{cases} \left(\frac{4}{3} - \frac{m}{m+M}\right)ml^2\ddot{\theta} - mgl\theta = -mlu/(M+m) \\ (M+m)\ddot{x} + mg\left(\frac{4}{3} - \frac{m}{m+M}\right)^{-1}\theta = 2u \end{cases}$$

Adotando $\theta = x_1$; $x = x_2$; $\dot{\theta} = x_3$; $\dot{x} = x_4$:

$$\begin{cases} J_{eq}\ddot{\theta} - mg\ell\theta = m\ell u/(m+M) \\ (m+M)\ddot{x} + K_{eq}\theta = 2u \end{cases}$$

com $J_{eq} = \left(\frac{4}{3} - \frac{m}{m+M}\right)$ e

$K_{eq} = mg\left(\frac{4}{3} - \frac{m}{m+M}\right)^{-1}$

Temos:

$$\begin{cases} \dot{x}_3 = [m\ell u/(m+M) + mg\ell x_1]/J_{eq} \\ \dot{x}_4 = [2u - K_{eq}x_1]/(m+M) \end{cases}$$

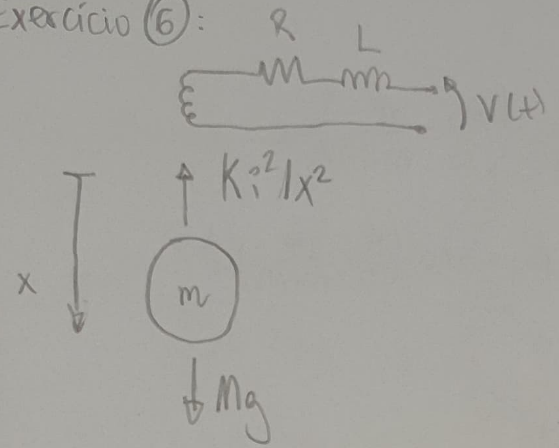
Definindo

$\mathcal{X} = [x_1, x_2, x_3, x_4]^T$

$$\dot{X} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{mg\ell}{J_{eq}} & 0 & 0 & 0 \\ -\frac{K_{eq}}{m+M} & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ m\ell/[(M+m)J_{eq}] \\ 2/(M+m) \end{bmatrix}}_{B} u$$

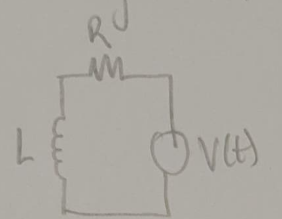
$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{C} X$$

Exercício 6:



TMB esfera: $m\ddot{x} = mg - Ki^2/x^2$

Lei das malhas:



$L\dot{i} + Ri = V$

Portanto, temos:

$$\begin{cases} \ddot{x} = g - \dfrac{Ki^2}{mx^2} \\ \dot{i} = \dfrac{V - Ri}{L} \end{cases}$$

→ termo não linear

Usando a definição de variável $x = x_1$; $\dot{x} = x_2$; $i = x_3$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g - \dfrac{Kx_3^2}{mx_1^2} \\ \dot{x}_3 = \dfrac{V}{L} - \dfrac{Rx_3}{L} \end{cases}$$

Vamos linearizar $\dot{x}_2$ em torno de (x_{3eq}, x_{1eq})

$$\dot{x}_2 - \dot{x}_{2eq} = -\frac{2Kx_{3eq}}{mx_{1eq}^2}(x_3 - x_{3eq}) + \frac{2Kx_{3eq}^2}{mx_{1eq}^3}(x_1 - x_{1eq})$$

Usando apenas as variações, temos

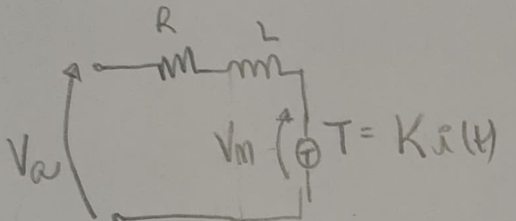
$$\begin{cases} \delta\dot{x}_1 = \delta x_2 \\ \delta\dot{x}_2 = \dfrac{2Kx_{3eq}^2}{mx_{1eq}^3}\delta x_1 + \left(-\dfrac{2Kx_{3eq}}{mx_{1eq}^2}\right)\delta x_3 \\ \delta\dot{x}_3 = -\dfrac{R}{L}\delta x_3 + \dfrac{1}{L}\delta V \end{cases} \Rightarrow \overbrace{\begin{bmatrix} \delta\dot{x}_1 \\ \delta\dot{x}_2 \\ \delta\dot{x}_3 \end{bmatrix}}^{\delta\dot{X}} = \overbrace{\begin{bmatrix} 0 & 1 & 0 \\ \dfrac{2Kx_{3eq}^2}{mx_{1eq}^3} & 0 & -\dfrac{2Kx_{3eq}}{mx_{1eq}^2} \\ 0 & 0 & -R/L \end{bmatrix}}^{\delta A} + \overbrace{\begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{L} \end{bmatrix}}^{B}\delta V$$

Finalmente, supondo interesse apenas em x e i

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{C.} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Ex 7:

· Modelo do motor



$V_a - Ri - L\dot{i} - V_m = 0$

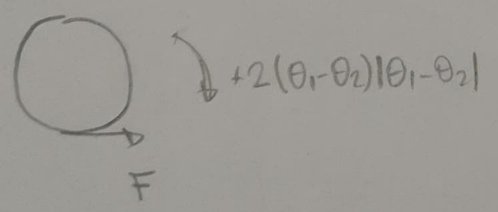
e mas $V_m = K\dot{\theta}_1$

$\boxed{V_a - Ri - L\dot{i} - K\dot{\theta}_1 = 0}$ I

TMAM motor

$\boxed{J_m\ddot{\theta}_1 = Ki(t) - B_m\dot{\theta}_1 - 2|\theta_1-\theta_2|(\theta_1-\theta_2)}$ II

TMAM Pinhão



$J_p\ddot{\theta}_2 = 2(\theta_1-\theta_2)|\theta_1-\theta_2| - F.R$

mas $F = 2\dot{x}^3$

$\boxed{J_p\ddot{\theta}_2 = 2|\theta_1-\theta_2|(\theta_1-\theta_2) - 2\dot{x}^3 r}$ III

TMB Bloco

$m\ddot{x} = F - b\dot{x} - kx \Rightarrow$ $\boxed{m\ddot{x} = 2\dot{x}^3 - b\dot{x} - kx}$ IV

Das 4 equações linearizamos apenas

- $f(x) = 2\dot{x}^3 \Rightarrow$ Portanto: $f_L(x) \simeq 2\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2(\dot{x} - \bar{\dot{x}}) \Rightarrow$

$f_L(x) = -4\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2\dot{x}$

$\cdot f(x) = |\theta_1 - \theta_2| (\theta_1 - \theta_2) 2$

em $\bar{\theta}_1$ e $\bar{\theta}_2$:

$$f_2(x) = \begin{cases} \text{Se } \theta_1 > \theta_2: \; 2(\bar{\theta}_1 - \bar{\theta}_2)^2 + 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_1 - \bar{\theta}_1) - 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_2 - \bar{\theta}_2) \\ \text{Se } \theta_1 < \theta_2: \; -2(\bar{\theta}_1 - \bar{\theta}_2)^2 - 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_1 - \bar{\theta}_1) + 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_2 - \bar{\theta}_2) \\ 0, \text{ se } \theta_1 = \theta_2 \end{cases}$$

↳ "linear por partes"

Finalmente, as equações que regem o sistema são:

$$\begin{cases} V_a - R i - L \dot{i} - K \dot{\theta}_1 = 0 \\ J_m \ddot{\theta}_1 = K i - B_m \dot{\theta}_1 - f_2(\theta_1, \theta_2) \\ J_p \ddot{\theta}_2 = f_2(\theta_1, \theta_2) - r 4 \bar{\dot{x}}^3 + r 6 \bar{\dot{x}}^2 \dot{x} \\ m \ddot{x} = 4 r \bar{\dot{x}}^3 - 6 \bar{\dot{x}}^2 \dot{x} - K x - b \dot{x} \end{cases}$$

linear por partes.

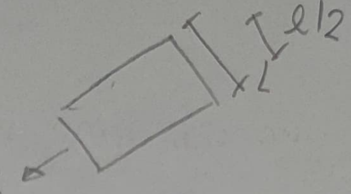
Sendo $\theta_1 = x_1$; $\dot{\theta}_1 = x_2$; $\theta_2 = x_3$; $\dot{\theta}_2 = x_4$; $x = x_5$; $\dot{x} = x_6$; $i = x_7$

$$\begin{cases} \dot{x}_7 = (V_a - R x_7 - K x_2) L^{-1} \\ \dot{x}_2 = (K x_7 - B_m x_2 - f_2(x_1, x_3)) J_m^{-1} \\ \dot{x}_4 = (f_2(\theta_1, \theta_2) - r 4 \bar{x}_6^3 + r 6 \bar{x}_6^2 x_6) J_p^{-1} \\ \dot{x}_6 = (4 r \bar{x}_6^3 - 6 \bar{x}_6^2 x_6 - K x_5 - b x_6) m^{-1} \\ \dot{x}_1 = x_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_5 = x_6 \end{cases}$$

Como $f_2(\theta_1, \theta_2)$ não é contínua, não se pode colocar na forma

$$\dot{X} = AX + Bu$$

Ex 8: Giroscópio: Obs:



$$J_x\ddot{\theta}_x + 2b\dot{\theta}_x + 2k\frac{l}{2}\theta_x = \text{"}J\dot{\vec{\omega}}\text{"}$$

mas $J\dot{\vec{\omega}} = J\omega\dot{\hat{j}}$ e $\dot{\hat{j}} = \underbrace{\vec{\omega}}_{\theta_z}\wedge\hat{j} \Rightarrow \dot{\hat{j}} = \theta_z\hat{k}\wedge\hat{j} = -\theta_z\hat{i}$

$$J_x\ddot{\theta}_x + 2b\dot{\theta}_x + kl\theta_x = -J\omega\theta_z$$

Entradas: θ_z. Saídas θ_x.

$\theta_x = x_1$; $\dot{\theta}_x = x_2$; $w = u_1$; $\theta_z = u_2$

$\dot{x}_1 = x_2$

$\dot{x}_2 = (-J u_1 u_2 - 2b x_2 - kl x_1) J_x^{-1}$

Seja $X = [x_1, x_2]^T$

$$\dot{X} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{kl}{J_x} & -\frac{2b}{J_x} \end{bmatrix}}_{A} X + \begin{bmatrix} 0 \\ -\frac{J}{J_x} \end{bmatrix} U_{eq} \qquad U_{eq} = U_1 \cdot U_2$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} X$$