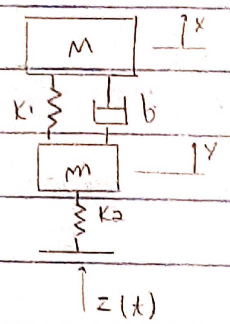
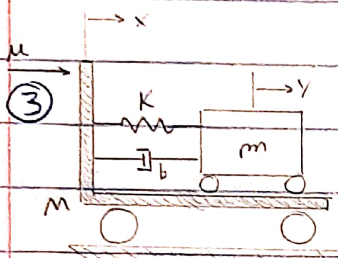
Gabriel Barbosa Paganini - 10722539 - Modelagem 01 e 06/10

② 

$$\begin{cases} M\ddot{x} + K_1(x-y) + b(\dot{x}-\dot{y}) = 0 \\ m\ddot{y} - K_1(x-y) - b(\dot{x}-\dot{y}) + K_2(y-z) = 0 \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_1}{M} & K_1/M & -b/M & b/M \\ K_1/m & -K_1-K_2/m & b/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_2}{m} \end{bmatrix} z(t)$$

$$\therefore \boxed{\dot{u} = Au + Bz} \longrightarrow u = [x, y, \dot{x}, \dot{y}]^T$$

③ 

$$\begin{cases} m\ddot{y} + k(y-x) + b(\dot{y}-\dot{x}) = 0 \\ M\ddot{x} - k(y-x) - b(\dot{y}-\dot{x}) = u \end{cases}$$

$$z = [x, y, \dot{x}, \dot{y}]^T \longrightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/M & K/M & -b/M & b/M \\ K/m & -K/m & b/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} u(t)$$

$$\therefore \boxed{\dot{z} = Az + Bu}$$

④. Definimos do problema : $X = [x_1, x_2, x_G, \theta, \dot{x}_1, \dot{x}_2, \dot{x}_G, \dot{\theta}]^T$

• Equações lineares :

$$\begin{cases} m_1\ddot{x}_1 + k(x_1 - z) - k_1(x_G - x_1 + l\theta) - b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) \\ m_2\ddot{x}_2 + k(x_2 - z') - k_2(x_G - x_2 - l\theta) - b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) \\ M\ddot{x}_G + k_1(x_G - x_1 + l\theta) + k_2(x_G - x_2 - l\theta) + b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta} \\ J_G\ddot{\theta} + k_1 l(x_G - x_1 + l\theta) - k_2 l(x_G - x_2 - l\theta) + b_1 l(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2 l(\dot{x}_G - \dot{x}_2 - l\dot{\theta} \end{cases}$$

• Vetor de estados : $\dot{X} = AX + BU$, onde $U = [z(t), z(t-\alpha)]$ e

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(k_1 k_1)}{m_1} & 0 & \frac{K_1}{m_1} & \frac{K_1 l}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 l}{m_1} \\ 0 & \frac{-(K_1 K_2)}{m_2} & \frac{K_2}{m_2} & \frac{-K_2 l}{m_2} & 0 & \frac{-b_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2 l}{m_2} \\ K_1/M & K_2/M & -(K_1+K_2)/M & l(K_2-K_1)/M & \frac{b_1}{M} & \frac{b_2}{M} & \frac{-(b_1+b_2)}{M} & \frac{l(b_2-b_1)}{M} \\ K_1 l/J & -K_2 l/J & l(K_2-K_1)/J & -\frac{l^2(K_2+K_1)}{J} & \frac{b_1 l}{J} & \frac{-b_2 l}{J} & \frac{l(b_2-b_1)}{J} & \frac{-l^2(b_2-b_1)}{J} \end{bmatrix}$$

⑤ Equações Linearizadas :

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u \\ J\ddot{\theta} + ml\ddot{x} - mlg\theta = 0 \end{cases}$$

$$X = [x, \theta, \dot{x}, \dot{\theta}]^T \longrightarrow \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-m^2 l^2}{J(M+m) - m^2 l^2} & 0 & 0 \\ 0 & \dfrac{g\, m\, l\,(M+m)}{J(M+m) - m^2 l^2} & 0 & 0 \end{bmatrix}}_{A} ; \underbrace{\begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{M+m - \dfrac{m^2 l^2}{J}} \\ \dfrac{-g\, m\, l}{J(M+m) - m^2 l^2} \end{bmatrix}}_{B}$$

$$\therefore \boxed{\dot{X} = AX + Bu}$$

⑥ Equações do sistema :

$$\begin{cases} m\ddot{x} = mg - KI^2/x^2 \\ L\dot{I} + RI = V \end{cases}$$

• Definindo : $X = [x, \dot{x}, I]^T$ e $U = V$, podemos escrever A como :

$$A = \begin{bmatrix} \dfrac{\partial \dot{x}}{\partial x} & \dfrac{\partial \dot{x}}{\partial \dot{x}} & \dfrac{\partial \dot{x}}{\partial I} \\ \dfrac{\partial \ddot{x}}{\partial x} & \dfrac{\partial \ddot{x}}{\partial \dot{x}} & \dfrac{\partial \ddot{x}}{\partial I} \\ \dfrac{\partial \dot{I}}{\partial x} & \dfrac{\partial \dot{I}}{\partial \dot{x}} & \dfrac{\partial \dot{I}}{\partial I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{2KI_0^2}{m x_0^3} & 0 & -\dfrac{2KI_0}{m x_0^2} \\ 0 & 0 & -\dfrac{R}{L} \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{L} \end{bmatrix} \rightarrow \underline{\underline{\dot{X} = AX + BU}}$$