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Ex 2

$$M\ddot{x} + b(\dot{x}-\dot{y}) + K_1(x-y) = 0$$
$$m\ddot{y} + b(\dot{y}-\dot{x}) + K_1(y-x) + K_2(y-z) = 0$$

$x_1 = x \qquad \dot{x}_1 = x_3$

$x_2 = y \qquad \dot{x}_2 = x_4$

$x_3 = \dot{x} \qquad \dot{x}_3 = [-b(\dot{x}-\dot{y}) - K_1(x-y)]/M = [-b(x_3-x_4) - K_1(x_1-x_2)]/M$

$x_4 = \dot{y} \qquad \dot{x}_4 = [-b(\dot{y}-\dot{x}) - K_1(y-x) - K_2(y-z)]/m = [-b(x_4-x_3) - K_1(x_2-x_1) - K_2(x_2-z)]/m$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_1}{M} & \frac{K_1}{M} & \frac{-b}{M} & \frac{b}{M} \\ \frac{K_1}{m} & \frac{-2K_2}{m} & \frac{b}{m} & \frac{-b}{m} \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_2}{m} \end{bmatrix} ; \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} ; \quad D = [0]$$

Ex 3

3.1. $\left.\begin{matrix} m\ddot{y} + b(\dot{y}-\dot{x}) + K(y-x) = 0 \\ M\ddot{x} + b(\dot{x}-\dot{y}) + K(x-y) = u \end{matrix}\right\}$ x é o deslocamento da carreta

$x_1 = x \qquad \dot{x}_1 = x_3$

$x_2 = y \qquad \dot{x}_2 = x_4$

$x_3 = \dot{x} \qquad \dot{x}_3 = [-b(x_3-x_4) - K(x_1-x_2) + u]/M \Rightarrow u = b(x_3-x_4) + K(x_1-x_2) = m\ddot{y}$

$x_4 = \dot{y} \qquad \dot{x}_4 = [-b(x_4-x_3) - K(x_2-x_1)]/m$

$$Ax = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_2 \\ x_4 \end{bmatrix} ; \quad Bu = \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u ; \quad Cx = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_2 \\ x_4 \end{bmatrix}$$

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3.2 $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/m & K/M & -b/M & b/M \\ K/m & -K/m & b/m & -b/m \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ 0 \end{bmatrix} u$; $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Ex 5:

$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u$

$\frac{4ml^2}{3}\ddot{\theta} + m\ddot{x}l\cos\theta - mgl\sin\theta = 0$

$\ddot{x} = (ml\dot{\theta}^2\sin\theta - ml\ddot{\theta}\cos\theta)/(M+m)$

$\ddot{\theta} = (mgl\sin\theta - m\ddot{x}l\cos\theta)/\frac{4ml^2}{3}$

$(M+m)\ddot{x} + ml\cos\theta(mgl\sin\theta - m\ddot{x}l)/(\frac{4ml^2}{3}) - ml\dot{\theta}^2\sin\theta = u$

$\ddot{x}\left[(M+m) - \frac{m^2l^2\cos\theta}{\frac{4ml^2}{3}}\right] + \frac{m^2l^2g\cos\theta\sin\theta}{4ml^2/3} - ml\dot{\theta}^2\sin\theta = u$

$\ddot{x}$

$\ddot{x} = \left[u + ml\dot{\theta}^2\sin\theta - \frac{3}{4}mg\cos\theta\sin\theta\right] / \left[M+m - \frac{3}{4}m\cdot\cos\theta\right]$

$\frac{4ml^2}{3}\ddot{\theta} + ml\cos\theta(ml\dot{\theta}^2\sin\theta - ml\ddot{\theta}\cos\theta)/(M+m) - mgl\sin\theta = 0$

$\ddot{\theta}\left[\frac{4ml^2}{3} - \frac{m^2l^2\cos^2\theta}{M+m}\right] + \frac{m^2l^2\dot{\theta}^2\cos\theta\sin\theta}{M+m} - mgl\sin\theta = 0$

$\ddot{\theta}$ $x_1 = x$ $\quad \dot{x}_1 = x_3$

$x_2 = \theta$ $\quad \dot{x}_2 = x_4$

$x_3 = \dot{x}$ $\quad \dot{x}_3 = \left[u + mlx_4^2\sin x_2 - \frac{3}{8}mg\sin 2x_2\right] / \left[M+m - \frac{3}{4}m\cdot\cos x_2\right]$

$x_4 = \dot{\theta}$ $\quad \dot{x}_4 = \left[mgl\sin x_2 - \frac{m^2l^2x_4^2\sin 2x_2}{2(M+m)}\right] / \left[\frac{4ml^2}{3} - \frac{m^2l^2\cos^2\theta}{M+m}\right]$

Ex 6:

$$m\ddot{x} = mg - Ki^2/x^2 \qquad \dot{x}_1 = x_2$$

$$L\frac{\partial i}{\partial t} + Ri = V \qquad \dot{x}_2 = g - \frac{Kx_3^2}{mx_1^2}$$

$$\dot{x}_3 = -\frac{R}{L}x_3 + \frac{1}{L}V$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2Kx_3^2}{mx_1^3} & 0 & -\frac{2Kx_3}{mx_1^2} \\ 0 & 0 & \frac{-R}{L} \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

Ex 7:

a)
$$\left.\begin{aligned} L_a\frac{\partial i_a}{\partial t} + R_a i_a &= e_a - K_b(t)\,w(t) \\ J\ddot{\theta}_1 + B\dot{\theta}_1 &= K i_a - T_{NLK} \\ m\ddot{x} &= T_{NLK}\cdot R - Kx_1 - F_{at} \end{aligned}\right\} \begin{aligned} L\frac{\partial i}{\partial t} + Ri &= V_a(t) - K\Omega_1 \\ J_m\ddot{\theta}_1 + B_m\Omega_1 &= Ki - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2) \\ m\ddot{x}_1 &= 2R\cdot|\theta_1 - \theta_2|(\theta_1 - \theta_2) - Kx_1 \end{aligned}$$

b)
$$\begin{aligned} x_1 &= x_1 & \dot{x}_1 &= x_5 \\ x_2 &= \theta_1 & \dot{x}_2 &= x_6 \\ x_3 &= \theta_2 & \dot{x}_3 &= x_7 \\ x_4 &= \int i\,dt & \dot{x}_4 &= x_8 \\ x_5 &= \dot{x}_1 & \dot{x}_5 &= [2R|x_2 - x_3|(x_2 - x_3) - Kx_1]/m \\ x_6 &= \dot{\theta}_1 & \dot{x}_6 &= [Kx_8 - 2|x_2 - x_3|(x_2 - x_3) - B_m x_6]/J_m \\ x_7 &= \dot{\theta}_2 & \dot{x}_7 &= 0 \\ x_8 &= i & \dot{x}_8 &= [V_a(t) - Kx_6 - Rx_8]/L \end{aligned}$$

c)
$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \frac{-K}{m} & [2R|-x_3|(x_2 - x_3) + 2R|x_2 - x_3|(-x_3)]/m & [-2R|x_2|(x_2 - x_3) - 2R|x_2 - x_3|(x_2)]/m & \cdots \\ 0 & [-2|-x_3|(x_2 - x_3) - 2|x_2 - x_3|(-x_3)]/J_m & [2|x_2|(x_2 - x_3) + 2|x_2 - x_3|(x_2)]/J_m & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix}$$

$$
\begin{matrix} \cdots & 0 & 1 & 0 & 0 & 0 \\ \cdots & 0 & 0 & 1 & 0 & 0 \\ \cdots & 0 & 0 & 0 & 1 & 0 \\ \cdots & 0 & 0 & 0 & 0 & 1 \\ \cdots & 0 & 0 & 0 & 0 & 0 \\ \cdots & 0 & 0 & \frac{-B_m}{J_m} & 0 & \frac{K}{J_m} \\ \cdots & 0 & 0 & 0 & 0 & 0 \\ \cdots & 0 & 0 & \frac{-K}{L} & 0 & \frac{-R}{L} \end{matrix} \Bigg] ; B_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} V_a(t)
$$

Ex 8:

$$J_x \ddot{\theta}_x + 2b\dot{\theta}_x + 2K\ell \frac{\theta_x}{2} = J w \theta_z$$

$X_1 = \theta_x \quad X_1 = X_2$

$X_2 = \dot{\theta}_x \quad X_2 = (-J w \theta_z - 2b\, X_2 - K\ell\, X_1)/J_x$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{K\ell}{J_x} & -\frac{2b}{J_x} \end{bmatrix} ; \quad B_u = \begin{bmatrix} 0 \\ -\frac{J}{J_x} \end{bmatrix} \begin{bmatrix} w \\ \theta_z \end{bmatrix} ; C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$