PME3380 - Modelagem de Sistemas Dinâmicos

Exercícios da Aula 01/10/2020

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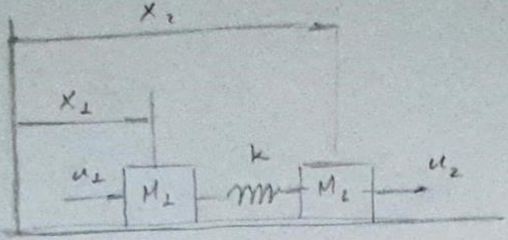

$$\bar{x} = x_G = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2} \qquad M = M_1 + M_2$$

$$\delta = x_1 - x_2$$

Equações da dinâmica:

$$\ddot{\bar{x}} = \frac{u_1 + u_2}{M}$$

$$\ddot{\delta} = -\frac{KM}{M_1 M_2}\delta + \frac{u_1}{M_1} - \frac{u_2}{M_2}$$

$z = [\bar{x} \;\; \delta \;\; \dot{\bar{x}} \;\; \dot{\delta}]^T$ e $u = [u_1 \;\; u_2]^T$

$$\dot{z} = \bar{A} z + \bar{B} u$$

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{KM}{M_1 M_2} & 0 & 0 \end{bmatrix}}_{\bar{A}} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & -\frac{1}{M_2} \end{bmatrix}}_{\bar{B}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \bar{C} z :$$

$$y \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \begin{bmatrix} \frac{M_1}{M} & \frac{M_2}{M} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \overset{\text{inversa}}{\iff} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{M_2}{M} \\ 1 & \frac{M_1}{M} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & -\frac{M_2}{M} & 0 & 0 \\ 1 & \frac{M_1}{M} & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$