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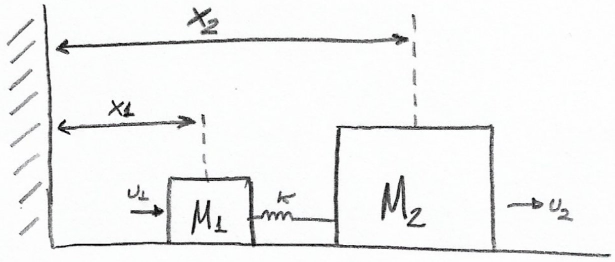
# PME3380 - Modelagem de Sistemas Dinâmicos

## Exercício da Aula do dia 01/10/2020

1)




$$\begin{cases} X_G = \bar{x} = \dfrac{M_1 x_1 + M_2 x_2}{M_1 + M_2} \therefore X_G = \dfrac{M_1 x_1 + M_2 x_2}{M} \\ \delta = x_1 - x_2 \therefore \ddot{\delta} = \ddot{x}_1 - \ddot{x}_2 \end{cases}$$

→ Montando o sistema de equações a partir da 1ª Lei de Newton:

$$\begin{cases} M\ddot{\bar{x}} = u_1 + u_2 \\ M_1 \ddot{x}_1 = u_1 - k(x_2 - x_1) \\ M_2 \ddot{x}_2 = u_2 + k(x_2 - x_1) \end{cases} \Rightarrow \begin{cases} \ddot{\bar{x}} = \dfrac{u_1 + u_2}{M} \\ \ddot{\delta} = \dfrac{u_1}{M_1} - \dfrac{u_2}{M_2} - \dfrac{kM\delta}{M_1 M_2} \end{cases}$$

→ Vetores:

$$z = \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} ; \quad \dot{z} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} ; \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

I)

→ Espaço de Estados: $\boxed{\dot{z} = Az + Bu}$ → $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{kM}{M_1 M_2} & 0 & 0 \end{bmatrix}$

$$\rightarrow B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & -\frac{1}{M_2} \end{bmatrix}$$

II) $\rightarrow Y = [x_1 \;\; x_2]^T$

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{M_1}{M} & \frac{M_2}{M} \\ 1 & -1 \end{bmatrix}}_{L^{-1}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow Y = L \cdot [\bar{x} \;\; \delta] \therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} \\ 1 & -\frac{M_1}{M} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

$\rightarrow$ Por fim: $\boxed{Y = CZ}$ $\rightarrow$ $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} & 0 & 0 \\ 1 & -\frac{M_1}{M} & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$