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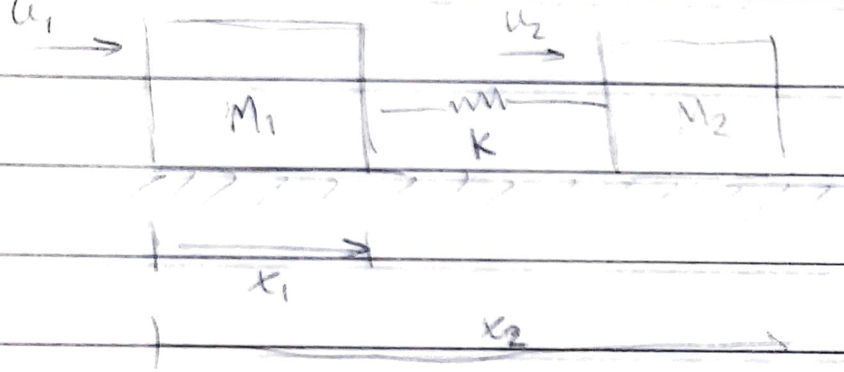

$$m_1 \ddot{x}_1 = k(x_1 - x_2) + u_1$$

$$M_2 \ddot{x}_2 = k(x_2 - x_1) + u_2$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad ; \quad \delta =$$

$$\delta = x_1 - x_2 \qquad M = m_1 + m_2$$

$$\ddot{\bar{x}} = \frac{m_1\ddot{x}_1}{m_1+m_2} + \frac{m_2\ddot{x}_2}{m_1+m_2} \Rightarrow \ddot{\bar{x}} = \frac{k\delta+u_1}{m_1+m_2} + \frac{(-k\delta+u_2)}{m_1+m_2} = \frac{u_1+u_2}{M}$$

$$\ddot{\delta} = \ddot{x}_1 - \ddot{x}_2 = \frac{k\delta+u_1}{m_1} + \frac{k\delta-u_2}{m_2} = \frac{u_1}{m_1} - \frac{u_2}{m_2} + \frac{(m_1+m_2)k\delta}{m_1 m_2} = \frac{u_1}{m_1} - \frac{u_2}{m_2} + \frac{Mk\delta}{m_1 m_2}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ 1/m_1 & -1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$A = \frac{MK}{m_1 m_2} \qquad x_1 = \delta + x_2$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{M} = \frac{m_1\delta + x_2(m_1+m_2)}{M}$$

$$\Rightarrow \bar{x} - \frac{m_1\delta}{M} = x_2 \Rightarrow x_1 = \bar{x} + \frac{(M-m_1)\delta}{M} \Rightarrow$$

$$\Rightarrow x_1 = \bar{x} + \frac{m_2\delta}{M} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & m_2/M \\ 1 & -m_1/M \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$