Modelagem de Sistemas Dinâmicos - PME 3380

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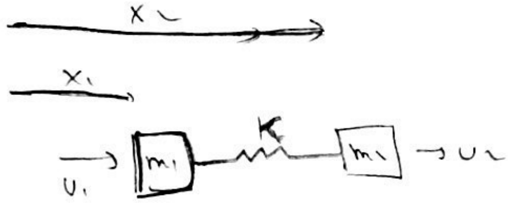
Exercícios aula 01/10/2020



$$\bar{x} = X_G = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad ; \quad m_1 + m_2 = M$$

$$\delta = x_1 - x_2$$

Equações obtidas via 1ª Lei => $u_1 + u_2 = M\ddot{\bar{x}}$; $u_2 + K(x_2 - x_1) = m_2 \ddot{x}_2$; $u_1 - K(x_2 - x_1) = m_1 \ddot{x}_1$

$$\Rightarrow \ddot{\bar{x}} = \frac{u_1 + u_2}{m_1 + m_2} \quad ; \quad \ddot{\delta} = -\frac{KM}{m_1 m_2}\delta + \frac{u_1}{m_1} - \frac{u_2}{m_2}$$

Definindo:

$$U = \begin{vmatrix} u_1 \\ u_2 \end{vmatrix} , \quad z = \begin{vmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{vmatrix} , \quad \dot{z} = \begin{vmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{vmatrix}$$

Montamos o Espaço de Estados: $\boxed{\dot{z} = Az + BU}$

$$\begin{vmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{KM}{m_1 m_2} & 0 & 0 \end{vmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{m_1} & -\frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \frac{u_1}{m_1} + \frac{u_2}{m_2} \\ -\frac{K}{m_1 m_2}\delta + \frac{u_1}{m_1} - \frac{u_2}{m_2} \end{bmatrix}$$

Para $y = [x_1 \; x_2]^t$, escrevemos x_1 e x_2 em função de δ e $\bar{x}$

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{m_1}{M} & \frac{m_2}{M} \\ 1 & -1 \end{bmatrix}}_{L^{-1}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad ; \quad L L^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & \frac{m_2}{M} \\ 1 & -\frac{m_1}{M} \end{bmatrix}$$

$$y = L[\bar{x} \; \delta] \Rightarrow \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 1 & \frac{m_2}{M} \\ 1 & -\frac{m_1}{M} \end{vmatrix} \begin{vmatrix} \bar{x} \\ \delta \end{vmatrix} \Rightarrow y = C' \begin{vmatrix} \bar{x} \\ \delta \end{vmatrix}$$

Para escrever em função de $z = \begin{vmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{vmatrix}$, ficamos com

$$y = \underbrace{\begin{vmatrix} 1 & \frac{m_2}{M} & 0 & 0 \\ 1 & \frac{-m_1}{M} & 0 & 0 \end{vmatrix}}_{C'} \begin{vmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{vmatrix} \rightarrow y = Cz$$

O sistema fica:

$$\begin{cases} \dot{z} = Az + Bu \\ y = Cz \end{cases}$$

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