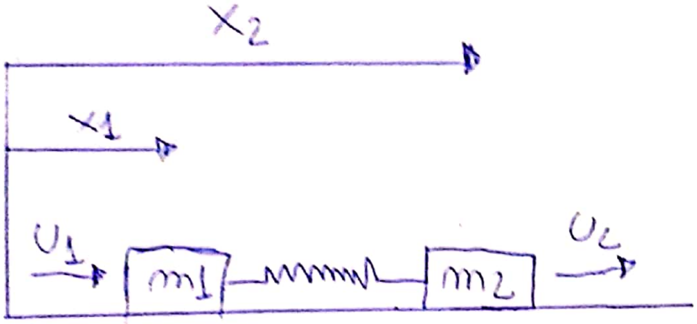
Nome: Paulo Mateus Corrêa Vianna - PME3380

Nusp: 10772741 - Modelagem de Sistemas Mecânicos



$$\delta = x_1 - x_2$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$m_1 + m_2 \to m \therefore \bar{X} = \frac{m_1 x_1 + m_2 x_2}{m}$$

$$\begin{cases} U_1 + U_2 = m\ddot{\bar{x}} \longrightarrow \ddot{\bar{x}} = \dfrac{U_1 + U_2}{m} \\ U_2 + k(x_2 - x_1) = m_2 \ddot{x}_2 \longrightarrow \ddot{x}_2 = \dfrac{U_2 + k(x_2 - x_1)}{m_2} \\ U_1 - k(x_2 - x_1) = m_2 \ddot{x}_1 \longrightarrow \ddot{x}_1 = \dfrac{U_1 - k(x_2 - x_1)}{m_1} \end{cases}$$

$$\delta = x_1 - x_2 \to \ddot{\delta} = \ddot{x}_1 - \ddot{x}_2 \to \ddot{\delta} : \frac{U_1 - k(x_2 - x_1)}{m_1} + \frac{U_2 + k(x_2 - x_1)}{m_2} \to$$

$$\to \ddot{\delta} = \frac{U_1}{m_1} + \frac{U_2}{m_2} + \frac{k m_1 (x_2 - x_1) - k m_2 (x_2 - x_1)}{m_1 m_2}$$

$$\to \ddot{\delta} = \frac{U_1}{m_1} + \frac{U_2}{m_2} - \frac{k M \delta}{M_1 m_2}$$

Portanto, temos o seguinte espaço de estados:

$$\underbrace{\begin{bmatrix} \dot{\bar{X}} \\ \dot{\delta} \\ \ddot{\bar{X}} \\ \ddot{\delta} \end{bmatrix}}_{(\dot{z})} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-k_m}{m_1 m_2} & 0 & 0 \end{bmatrix}}_{(A)} \underbrace{\begin{bmatrix} \bar{X} \\ \delta \\ \dot{\bar{X}} \\ \dot{\delta} \end{bmatrix}}_{(z)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ 1/m_1 & -1/m_2 \end{bmatrix}}_{(B)} \underbrace{\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}}_{(U)}$$

Portanto, $\dot{z} = Az + Bu$

Pela análise das variáveis podemos obter:

$$\begin{bmatrix} \bar{X} \\ \delta \end{bmatrix} = \begin{bmatrix} m_1/M & m_2/M \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$y = \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{(y)} = \underbrace{\begin{bmatrix} 1 & m_2/M & 0 & 0 \\ 1 & -m_1/M & 0 & 0 \end{bmatrix}}_{(C)} \underbrace{\begin{bmatrix} \bar{X} \\ \delta \\ \dot{\bar{X}} \\ \dot{\delta} \end{bmatrix}}_{(z)} \rightarrow y = Cz$$

Assim, temos: $\begin{cases} \dot{z} = Az + Bu \\ y = Cz \end{cases}$