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*PME 3380 – Exercício do dia 01/10

Dados: $\bar{x} = x_G = \dfrac{M_1x_1 + M_2x_2}{M}$; $\delta = x_1 - x_2$

$$\ddot{\bar{x}} = \frac{u_1+u_2}{M} \; ; \; \ddot{\delta} = -\frac{KM}{M_1M_2}\delta + \frac{u_1}{M_1} - \frac{u_2}{M_2}$$

$$z = \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} \; ; \; \begin{cases} \dot{z} = \bar{A}z + \bar{B}u \\ y = \bar{C}z \end{cases}$$

$$\Rightarrow \dot{z} = \bar{A}z + \bar{B}u \rightarrow \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \bar{A}\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \bar{B}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Olhando-se os dados fornecidos tem-se que:

$$\bar{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{KM}{M_1M_2} & 0 & 0 \end{bmatrix} \; ; \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & -\frac{1}{M_2} \end{bmatrix}$$

Para encontrar $\bar{C}$ toma-se que $y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ e, além disso:

$$(*) \; \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \bar{D}^{-1}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{M_1}{M} & \frac{M_2}{M} \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\bar{D}.\bar{D}^{-1} = I \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \frac{M_1}{M} & \frac{M_2}{M} \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{cases} a\frac{M_1}{M} + b = 1 \\ a\frac{M_2}{M} - b = 0 \\ c\frac{M_1}{M} + d = 0 \\ c\frac{M_2}{M} - d = 1 \end{cases}$$

$$\rightarrow a\frac{(M_1+M_2)}{M} = 1 \Rightarrow \boxed{a = 1} \;/\; b = 1 - \frac{M_1}{M} \Rightarrow \boxed{b = \frac{M_2}{M}} \;/\; c\frac{(M_1+M_2)}{M} = 1 \Rightarrow \boxed{c = 1} \;/\; \boxed{d = -\frac{M_1}{M}}$$

Logo multiplicando (*) por $\bar{D}$ à esquerda em ambos os membros tem-se:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} \\ 1 & -\frac{M_1}{M} \end{bmatrix}\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

→ Estendendo $\bar{D}$ para incluir todo o vetor de estados z, obtém-se $\bar{C}$ →

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{M_2}{M} & 0 & 0 \\ 1 & \frac{M_1}{M} & 0 & 0 \end{bmatrix}}_{\bar{C}}\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$