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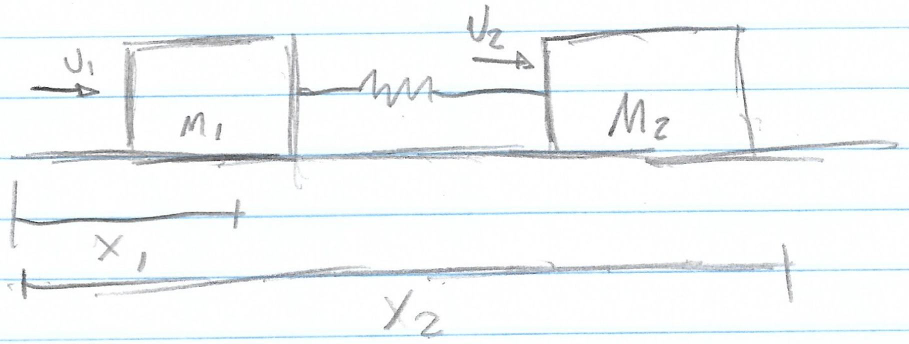

$$M_1 \ddot{x}_1 = k(x_1 - x_2) + U_1$$
$$M_2 \ddot{x}_2 = k(x_2 - x_1) + U_2$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad ; \quad \delta = x_1 - x_2 \quad ; \quad m = m_1 + m_2$$

$$\ddot{\bar{x}} = \frac{m_1 \ddot{x}_1}{m_1 + m_2} + \frac{m_2 \ddot{x}_2}{m_1 + m_2}$$

$$\ddot{\bar{x}} = \frac{k\delta + U_1}{m_1 + m_2} + \frac{-k\delta + U_2}{m_1 + m_2} = \frac{U_1 + U_2}{m_1 + m_2} = \frac{U_1 + U_2}{m}$$

$$\ddot{\delta} = \ddot{x}_1 - \ddot{x}_2 = \frac{k\delta + U_1}{m_1} + \frac{k\delta - U_2}{m_2}$$

$$\ddot{\delta} = \frac{U_1}{m_1} - \frac{U_2}{m_2} + \frac{(m_1 + m_2)}{m_1 m_2} k\delta = \frac{U_1}{m_1} - \frac{U_2}{m_2} + \frac{mk\delta}{m_1 M_2}$$

$$[x_1, x_2, \dot{x}_1, \dot{x}_2]^T \Rightarrow [\bar{x}, \delta, \dot{\bar{x}}, \dot{\delta}]^T$$

$$\dot{z} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & 1/m \\ 1/m_1 & -1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$A = \frac{mk}{m_1 m_2}$$

Retornando a x_1 e x_2

$$x_1 = \delta + x_2$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m} = \frac{m_1 \delta + m_1 x_2 + m_2 x_2}{m}$$

$$\Rightarrow \bar{x} - \frac{m_1 \delta}{m} = x_2$$

$$\Rightarrow x_1 = \bar{x} + \frac{m\delta - m_1\delta}{m}$$

$$x_1 = \bar{x} + \frac{m_2 \delta}{m}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & m_2/m \\ 1 & -m_1/m \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$