

Exercícios p/ 15/10:

$$\begin{cases} \bar{x} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2} \\ \delta = x_1 - x_2 \end{cases} \Rightarrow \begin{cases} \ddot{\bar{x}} = \frac{U_1 + U_2}{M} \\ \ddot{\delta} = -\frac{kM}{M_1 M_2} \delta + \frac{U_1}{M_1} - \frac{U_2}{M_2} \end{cases} \quad \text{com } M = M_1 + M_2$$

$$\underbrace{\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-kM}{M_1 M_2} & 0 & 0 \end{bmatrix}}_{\bar{A}} \underbrace{\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}}_z + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & -\frac{1}{M_2} \end{bmatrix}}_{\bar{B}} \underbrace{\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}}_u \Rightarrow \dot{z} = \bar{A}z + \bar{B}u \quad \textcircled{I}$$

Chega-se a

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \frac{U_1 + U_2}{M} \\ -\frac{kM}{M_1 M_2} \delta + \frac{U_1}{M_1} - \frac{U_2}{M_2} \end{bmatrix}$$

Reescreve-se em termos de x_1 e x_2 :

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \begin{bmatrix} M_1/M & M_2/M \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & M_2/M \\ 1 & -M_1/M \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

ou

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & M_2/M & 0 & 0 \\ 1 & -M_1/M & 0 & 0 \end{bmatrix}}_{\bar{C}} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} \Rightarrow y = \bar{C}z \quad \textcircled{II}$$

Chega-se, enfim, a:

$$\begin{cases} \dot{z} = \bar{A}z + \bar{B}u \\ y = \bar{C}z \end{cases}$$