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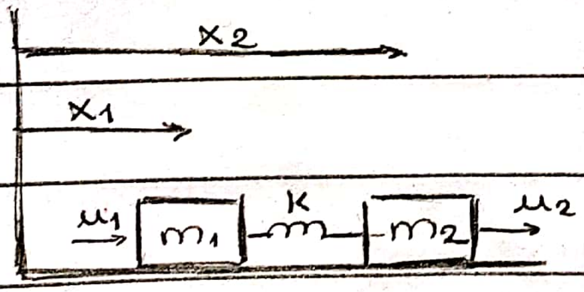
Exercícios Aula 01/10



Do centro de massa: $\bar{X} = X_G = \frac{M_1X_1+M_2X_2}{M_1+M_2} = \frac{M_1X_1+M_2X_2}{M}$

$$\delta = X_2 - X_1$$

Pelo TMA: $\ddot{X}_G = \frac{U_1+U_2}{M}$

$$\ddot{\delta} = -\frac{KM}{M_1M_2}\delta + \frac{U_1}{M_1} - \frac{U_2}{M_2}$$

Definindo os vetores: $z = [\bar{x}\ \delta\ \dot{\bar{x}}\ \dot{\delta}]^T$

$U = [U_1\ U_2]^T$

Espaço de estados:

$$\dot{z} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{KM}{M_1M_2} & 0 & 0 \end{bmatrix}}_{\bar{A}} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & -\frac{1}{M_2} \end{bmatrix}}_{\bar{B}} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Portanto: $\dot{z} = \bar{A}z + \bar{B}U$

Para $y = [x_1\ x_2]^T$:

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{M_1}{M} & \frac{M_2}{M} \\ 1 & -1 \end{bmatrix}}_{P^{-1}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow P = \frac{-1}{\frac{M_1+M_2}{M}} \begin{bmatrix} -1 & -\frac{M_2}{M} \\ -1 & \frac{M_1}{M} \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} \\ 1 & -\frac{M_1}{M} \end{bmatrix}$$

Assim:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} \\ 1 & -\frac{M_1}{M} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

Vetor de estados:

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{M_2}{M} & 0 & 0 \\ 1 & -\frac{M_1}{M} & 0 & 0 \end{bmatrix}}_{\bar{C}} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$

Portanto:

$$\boxed{\begin{cases} \dot{z} = \bar{A}z + \bar{B}U \\ y = \bar{C}z \end{cases}}$$