Modelagem

Ex. aula 01/10 ✓

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Espaço de estados

$$\ddot{x}_1 + \frac{k}{M_1}(x_1 - x_2) = \frac{u_1}{M_1}$$

$$\ddot{x}_2 + \frac{k}{M_2}(x_2 - x_1) = \frac{u_2}{M_2}$$

definimos:

$$\bar{x} = x_g = \frac{m_1 x_1 + m_2 x_2}{m}$$

$$\delta = x_1 - x_2$$

o vetor de estados

$$z = [\bar{x} \;\; \delta \;\; \dot{\bar{x}} \;\; \dot{\delta}]^T$$

$$\begin{cases} \dot{z} = \bar{A}z + \bar{B}u \longrightarrow \text{obter } \bar{A}, \bar{B}, \bar{C} \\ y = \bar{C}z \end{cases}$$

$$\dot{z} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} \qquad \ddot{\bar{x}} = \frac{u_1 + u_2}{M} = \frac{1}{m}u_1 + u_2\frac{1}{m}$$

$$\ddot{\delta} = -\frac{km}{m_1 m_2}\delta + \frac{u_1}{M_1} - \frac{u_2}{M_2}$$

assim:

$$\dot{z} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{km}{m_1 m_2} & 0 & 0 \end{bmatrix}}_{\bar{A} \checkmark \text{ok}} \underbrace{\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ 1/m_1 & -1/m_2 \end{bmatrix}}_{\bar{B} \checkmark \text{ok}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u}$$

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{m \cdot \bar{x}}{m} \Rightarrow (x_1 m_1 + m_2 x_2)/m$$

$$\frac{x_1 m_1 + m_2 x_2}{m} + \frac{m_2}{m}\delta = \frac{x_1 m_1 + m_2 x_1}{m} = x_1(m_1 + m_2)/m$$

$$x_1 M/m = x_1$$

$$\bar{x} - \delta \frac{m_1}{m} = \frac{x_1 m_1 + x_2 m_2}{m} - \frac{(x_1 + x_2) m_1}{m} = \frac{x_2 (m_2 + m_1)}{\cancel{m}} \quad \checkmark \text{ ok}$$

$$y = \begin{bmatrix} 1 & m_2/m & 0 & 0 \\ 1 & -m_1/M & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$