

Exercício da aula 01/10/2020

→ Equações: $\bar{x} = \frac{(M_1 x_1 + M_2 x_2)}{M} \frac{1}{M}$; $\ddot{\bar{x}} = \frac{(U_1 + U_2)}{M} \frac{1}{M}$; $\delta = x_1 - x_2$

$\ddot{\delta} = -\frac{KM}{M_1 M_2} \delta + \frac{U_1}{M_1} - \frac{U_2}{M_2}$; $z = [\bar{x} \ \delta \ \dot{\bar{x}} \ \dot{\delta}]^T$; $\dot{z} = [\dot{\bar{x}} \ \dot{\delta} \ \ddot{\bar{x}} \ \ddot{\delta}]^T$

→ Reescrevendo

• $\dot{\bar{x}} = 0 \cdot \bar{x} + 0 \cdot \delta + 1 \cdot \dot{\bar{x}} + 0 \cdot \dot{\delta} + 0 U_1 + 0 U_2$

• $\dot{\delta} = 0 \cdot \bar{x} + 0 \cdot \delta + 0 \cdot \dot{\bar{x}} + 1 \cdot \dot{\delta} + 0 U_1 + 0 U_2$

• $\ddot{\bar{x}} = 0 \cdot \bar{x} + 0 \cdot \delta + 0 \cdot \dot{\bar{x}} + 0 \cdot \dot{\delta} + \frac{1}{M} U_1 + \frac{1}{M} U_2$

• $\ddot{\delta} = 0 \cdot \bar{x} - \frac{KM}{M_1 M_2} \delta + 0 \cdot \dot{\bar{x}} + 0 \cdot \dot{\delta} + \frac{1}{M_1} U_1 - \frac{1}{M_2} U_2$

→ Encontrando x_1 e x_2 em função de \bar{x} e δ :

• $\bar{x} = \frac{(M_1 x_1 + M_2 (x_1 - \delta))}{M} = \frac{(M x_1 - M_2 \delta)}{M} \Rightarrow \bar{x} = x_1 - \frac{M_2}{M} \delta \Rightarrow \boxed{x_1 = \bar{x} + \frac{M_2}{M} \delta}$

• $\bar{x} = \frac{(M_1 (\delta + x_2) + M_2 x_2)}{M} = \frac{(M_1 \delta + M x_2)}{M} = \bar{x} \Rightarrow \bar{x} = x_2 + \frac{M_1}{M} \delta \Rightarrow \boxed{x_2 = \bar{x} + \frac{M_1}{M} \delta}$

→ Na forma matricial:

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{KM}{M_1 M_2} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & -\frac{1}{M_2} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} & 0 & 0 \\ 1 & -\frac{M_1}{M} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$