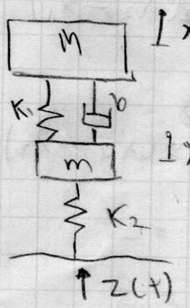
Enzo C Zuglimi - 10333741

$\boxed{\text{Ex 2}}$



- $x > y$; $\dot{x} > \dot{y}$
- M: $M\ddot{x} = -K_1(x-y) - b(\dot{x}-\dot{y}) \Rightarrow$

$$\ddot{x} = -\frac{K_1}{M}x + \frac{K_1}{M}y - \frac{b}{M}\dot{x} + \frac{b}{M}\dot{y}$$

- m: $m\ddot{y} = K_1(x-y) + b(\dot{x}-\dot{y}) - K_2(y-z) \Rightarrow$

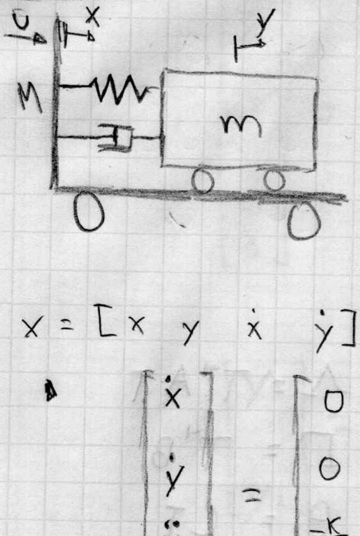
$$\ddot{y} = \frac{K_1}{m}x - \frac{(K_1+K_2)}{m}y + \frac{b}{m}\dot{x} - \frac{b}{m}\dot{y} + \frac{K_2}{m}z$$

- $X = [x \quad y \quad \dot{x} \quad \dot{y}]$
- $Y = [x \quad \dot{y}]$; $U = z$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \ddot{x}\\ \ddot{y}\end{bmatrix} = \overset{A}{\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\frac{K_1}{M} & \frac{K_1}{M} & -\frac{b}{M} & \frac{b}{M}\\ \frac{K_1}{m} & -\frac{(K_1+K_2)}{m} & \frac{b}{m} & -\frac{b}{m}\end{bmatrix}} \cdot \begin{bmatrix}x\\ y\\ \dot{x}\\ \dot{y}\end{bmatrix} + \overset{B}{\begin{bmatrix}0\\ 0\\ 0\\ \frac{K_2}{m}\end{bmatrix}} \cdot U$$

$$\begin{bmatrix}x\\ \dot{y}\end{bmatrix} = \overset{C}{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}} \cdot \begin{bmatrix}x\\ y\\ \dot{x}\\ \dot{y}\end{bmatrix}$$

$\boxed{\text{Ex 3}}$ (3.2)



- Equacões:
  - $m\ddot{y} = K(x-y) + b(\dot{x}-\dot{y}) \Rightarrow$

    $\Rightarrow \ddot{y} = \frac{K}{m}x - \frac{K}{m}y + \frac{b}{m}\dot{x} - \frac{b}{m}\dot{y}$
  - $M\ddot{x} = U - K(x-y) - b(\dot{x}-\dot{y}) \Rightarrow$

    $\Rightarrow \ddot{x} = \frac{U}{M} - \frac{K}{M}x + \frac{K}{M}y - \frac{b}{M}\dot{x} + \frac{b}{M}\dot{y}$

- $X = [x \quad y \quad \dot{x} \quad \dot{y}]$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \ddot{x}\\ \ddot{y}\end{bmatrix} = \overset{A}{\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\frac{K}{M} & \frac{K}{M} & -\frac{b}{M} & \frac{b}{M}\\ \frac{K}{M} & -\frac{K}{M} & \frac{b}{M} & -\frac{b}{M}\end{bmatrix}} \cdot \begin{bmatrix}x\\ y\\ \dot{x}\\ \dot{y}\end{bmatrix} + \overset{B}{\begin{bmatrix}0\\ 0\\ \frac{1}{M}\\ 0\end{bmatrix}} U$$

· Com $y = y$

$$C = [0 \quad 1 \quad 0 \quad 0], \qquad D = [0]$$

Ex 4.

· Equações obtidas anteriormente (lineares):

$$\bullet\ m_1 \ddot{x}_1 + (K_p + K_1) x_1 + b_1 \dot{x}_1 = K_p \cdot z(t) + K_1 (x_G + b\theta) + b_1 (\dot{x}_G + b\dot{\theta}) \Rightarrow$$

$$\Rightarrow \ddot{x}_1 = -\frac{(K_p + K_1)}{m_1} x_1 - \frac{b_1}{m_1} \dot{x}_1 + \frac{K_p}{m_1} \cdot z(t) + \frac{K_1 x_G}{m_1} + \frac{1}{m_1} K_1 b \theta + b_1 \dot{x}_G \frac{1}{m_1} + b_1 b \dot{\theta} \frac{1}{m_1}$$

$$\bullet\ m_2 \ddot{x}_2 + (K_p + K_2) x_2 + b_2 \dot{x}_2 = K_p \cdot z(t - \tfrac{l}{v}) + K_2 (x_G - a\theta) + b_2 (\dot{x}_G - a\dot{\theta}) \Rightarrow$$

$$\Rightarrow \ddot{x}_2 = -\frac{(K_p + K_2)}{m_2} x_2 - \frac{b_2}{m_2} \dot{x}_2 + \frac{K_p}{m_2} \cdot z(t - \tfrac{l}{v}) + \frac{K_2}{m_2} x_G - \frac{K_2}{m_2} a \theta + \frac{b_2}{m_2} \dot{x}_G - \frac{b_2 a}{m_2} \dot{\theta}$$

$$\bullet\ M \ddot{x}_G + K_1 (x_G + b\theta) + b_1 (\dot{x}_G + b\dot{\theta}) + K_2 (x_G - a\theta) + b_2 (\dot{x}_G - a\dot{\theta}) = K_1 x_1 + b_1 \dot{x}_1 + K_2 x_2 + b_2 \dot{x}_2 \Rightarrow$$

$$\Rightarrow \ddot{x}_G = -\frac{K_1}{M} x_G - \frac{K_1 \cdot b}{M} \theta - \frac{b_1}{M} \dot{x}_G - \frac{b_1 b}{M} \dot{\theta} - \frac{K_2}{M} x_G + \frac{K_2 a}{M} \theta - \frac{b_2}{M} \dot{x}_G + \frac{b_2 a}{M} \dot{\theta} + \frac{K_1}{M} x_1 + \frac{K_2}{M} x_2 + \frac{b_1}{M} \dot{x}_1 + \frac{b_2}{M} \dot{x}_2$$

$$\bullet\ J \ddot{\theta} + K_1 b (x_G + b\theta) + b_1 b (\dot{x}_G + b\dot{\theta}) - K_2 a (x_G - a\theta) - b_2 a (\dot{x}_G - a\dot{\theta}) = K_1 x_1 b + b_1 \dot{x}_1 b - K_2 x_2 a - b_2 \dot{x}_2 a \Rightarrow$$

$$\Rightarrow \ddot{\theta} = \frac{1}{J}(K_2 a - K_1 b) x_G + \frac{1}{J}(b_2 a - b_1 b) \dot{x}_G - \frac{1}{J}(K_2 a^2 + K_1 b^2) \theta - \frac{1}{J}(b_2 a^2 + b_1 b^2) \dot{\theta} + \frac{1}{J} K_1 x_1 b + \frac{1}{J} b_1 \dot{x}_1 b - \frac{K_2}{J} x_2 a - \frac{b_2}{J} \dot{x}_2 a$$

· Definindo:

$$x = [x_1 \quad x_2 \quad x_G \quad \theta \quad \dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_G \quad \dot{\theta}]^T$$

$$y = [x_G \quad \theta]^T$$

$$u = [z(t) \quad z(t - \tfrac{l}{v})]^T$$

- Sistema do tipo:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Onde:

$$A = \begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_G & \theta & \dot{x}_1 & \dot{x}_2 & \dot{x}_G & \dot{\theta} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(k_p+k_1)}{m_1} & 0 & \frac{k_1}{m_1} & \frac{k_1 b}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 b}{m_1} \\ 0 & \frac{-(k_p+k_2)}{m_2} & \frac{k_2}{m_2} & \frac{-k_2 a}{m_2} & 0 & \frac{-b_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2 a}{m_2} \\ \frac{k_1}{M} & \frac{k_2}{M} & \frac{-(k_1+k_2)}{M} & \frac{(k_2 a - k_1 b)}{M} & \frac{b_1}{M} & \frac{b_2}{M} & \frac{-(b_1+b_2)}{M} & \frac{(b_2 a - b_1 b)}{M} \\ \frac{k_1 b}{J} & \frac{-k_2 a}{J} & \frac{(k_2 a - k_1 b)}{J} & \frac{-(k_2 a^2 + k_1 b^2)}{J} & \frac{b_1 b}{J} & \frac{-b_2 a}{J} & \frac{(b_2 a - b_1 b)}{J} & \frac{-(b_2 a^2 - b_1 b^2)}{J} \end{bmatrix} \end{array}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_p}{m_1} & 0 \\ 0 & \frac{k_p}{m_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = 0_{(2 \times 2)}$$

Ex 5

- Equações obtidas anteriormente (linearizados):

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u & (I) \\ J\ddot{\theta} = lmg\theta - ml\ddot{x} & (II) \end{cases}$$

- Substituindo (II) em (I), isolado $\ddot{\theta}$:

$$(M+m)\ddot{x} + ml\left(\frac{lmg\theta}{J} - \frac{ml}{J}\ddot{x}\right) = u \Rightarrow$$

$$\Rightarrow (M+m)\ddot{x} + \frac{m^2l^2g}{J}\theta - \frac{m^2l^2}{J}\ddot{x} = u \Rightarrow$$

$$\Rightarrow \left(M+m-\frac{m^2l^2}{J}\right)\ddot{x} + \frac{m^2l^2}{J}g\theta = u \Rightarrow$$

$$\Rightarrow \ddot{x} = \left(\frac{-m^2l^2}{MJ+mJ-m^2l^2}\right)\theta + \frac{u}{M+m-\frac{m^2l^2}{J}}$$

- Substituindo (II) em (I), isolado $\ddot{x}$:

$$(M+m)\cdot\left(\frac{lmg\theta}{lm} - \frac{J\ddot{\theta}}{ml}\right) + ml\ddot{\theta} = u \Rightarrow$$

$$\Rightarrow (M+m)\cdot g\theta - \left((M+m)\cdot\frac{J}{ml} - ml\right)\ddot{\theta} = u \Rightarrow$$

$$\Rightarrow \left(-\frac{ml}{M+m} + \frac{J}{ml}\right)\ddot{\theta} = g\theta - \frac{u}{(M+m)}$$

$$\Rightarrow \ddot{\theta} = \left(\frac{gml(M+m)}{J(M+m)-m^2l^2}\right)\theta - \left(\frac{gml}{J(M+m)-m^2l^2}\right)u$$

- Definindo:

$$X = [\,x \quad \theta \quad \dot{x} \quad \dot{\theta}\,]^T$$

$$Y = [\,x \quad \theta\,]$$

$$u = u$$

· Sistema do tipo

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Onde:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \left(\dfrac{-m^2 l^2}{MJ + mJ - m^2 l^2}\right) & 0 & 0 \\ 0 & \left(\dfrac{g\,m\,l\,(M+m)}{J(M+m) - m^2 l^2}\right) & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \left(\dfrac{1}{M + m - \frac{m^2 l^2}{J}}\right) \\ \left(\dfrac{-g\,m\,l}{J(M+m) - m^2 l^2}\right) \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = 0\ (2 \times 1)$$

$\boxed{\text{Ex 6}}$

· Equações do sistema, não linear:

· $m\ddot{x} = mg - \dfrac{Ki^2}{x^2}$

· $L\dot{i} + Ri = V$

· Não Linear:

· Definindo $x = [x, \dot{x}, i]^T$

$\dot{x} = [\dot{x}, \ddot{x}, \dot{i}]^T$

$y = [x]$

$u = V$

E.E.:

$$
\begin{aligned}
f_1 &= \left| \dot{x} = \dot{x} \right. \\
f_2 &= \left| \ddot{x} = g - \frac{K}{m} \cdot \frac{i^2}{x^2} \right. \\
f_3 &= \left| \dot{i} = \frac{U}{L} - \frac{R}{L} \cdot i \right.
\end{aligned}
\qquad g_1 = y = x
$$

- Lineari:
  - Sistema da forma $\dot{x} = Ax + BU$

    $y = Cx$

- $$A = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \dot{x}} & \frac{\partial f_1}{\partial i} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} & \frac{\partial f_2}{\partial i} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial \dot{x}} & \frac{\partial f_3}{\partial i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{2K i_0^2}{m x_0^3} & 0 & -\frac{2K i_0}{m x_0^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}$$

- $$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

  $$D = 0 \ (1 \times 1)$$

Ex 7

a) · Motor:

· Circuito da armadura:

$$L\dot{i} + Ri = V_a - e_b \cong V_a - K_b\Omega_1$$

· Parte mecânica:

$$J_m \cdot \dot{\Omega}_1 = T_i - B_m \cdot \Omega_1 - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2) \Rightarrow$$

$$\Rightarrow J_m \cdot \dot{\Omega}_1 = K \cdot i - B_m \cdot \Omega_1 - 2|\theta_1 - \theta_2| \cdot (\theta_1 - \theta_2)$$

· Carro:

$$m\ddot{x}_1 = 2|\theta_1 - \theta_2| \cdot (\theta_1 - \theta_2)\frac{1}{R} - 2\dot{x}_1^3 - Kx_1$$

· Vínculo cinemático:

· $x_1 = \theta_2 \cdot R$ · $\dot{x}_1 = \Omega_2 \cdot R$

↳ Sistema de quinta ordem

· Vetor de estados: $X = [i \ \theta_1 \ x_1 \ \dot{\theta}_1 \ \dot{x}_1]^T$

b) Termos não lineares

· $f_1 = 2|\theta_1 - \theta_2| \cdot (\theta_1 - \theta_2) = 2\left|\theta_1 - \frac{x_1}{R}\right|\left(\theta_1 - \frac{x_1}{R}\right)$

$$f_1 \cong f(0) + \left.\frac{\partial f}{\partial \theta_1}\right|_{eq}(\theta_1 - \theta_{10}) + \left.\frac{\partial f}{\partial \theta_2}\right|_{eq}(\theta_2 - \theta_{20})$$

$$f_1 \cong 2 \cdot \left|\theta_{10} - \frac{x_{10}}{R}\right|\left(\theta_{10} - \frac{x_{10}}{R}\right) + 4 \cdot \left|\theta_{10} - \frac{x_{10}}{R}\right| \cdot (\theta_1 - \theta_{10}) - 4\left|\frac{x_{10}}{R} - \theta_{10}\right|(\theta_2 - \theta_{20})$$

· Definindo $\left(\theta_{10} - \frac{x_{10}}{R}\right) = \delta_0$

· $(\theta_1 - \theta_{10}) = \theta_1$ $\rightsquigarrow f = 4\delta_0(\theta_1 - \theta_2) + \delta_0^2$

· $(\theta_2 - \theta_{20}) = \theta_2$

· $f_2 = 2\dot{x}_1^3$

$$f_2 \cong f_{eq} + \left.\frac{\partial f}{\partial \dot{x}_1}\right|_{eq}(\dot{x}_1 - \dot{x}_{10})$$

$$f_2 = F_{00} + 6\dot{x}_{10}^2 \cdot (\dot{x}_1 - \dot{x}_{10})$$

· Definindo $(\dot{x}_1 - \dot{x}_{10}) = \dot{x}_{10}$

$\rightsquigarrow f = 6\dot{x}_{10}^2 \cdot \dot{x}_1 + F_{00}$

- Reescrevendo as equações linearizadas:

- $\dot{i} = \frac{V_a}{L} - \frac{K_b}{L} \cdot \dot{\theta}_1 - \frac{R}{L} i$

- $\ddot{\theta}_1 = \frac{K}{J_m} \cdot i - \frac{B_m}{J_m} \cdot \dot{\theta}_1 - \frac{4\delta_0}{J_m}\left(\theta_1 - \frac{x_1}{R}\right) - \frac{\delta_0^2}{J_m}$

- $\ddot{x}_1 = \frac{4\delta_0}{mR}\left(\theta_1 - \frac{x_1}{R}\right) + \frac{\delta_0^2}{mR} - 6\dot{x}_{10}^2 \cdot \dot{x}_1 + F_{a0} - Kx_1$

c) · Redefinindo $x = [i \;\; \theta_1 \;\; x_1 \;\; \dot{\theta}_1 \;\; \dot{x}_1]^T$ onde cada estado é a sua perturbação em torno de um valor do equilíbrio:

- Sistema do tipo $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$ onde:

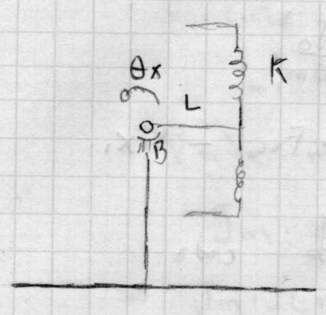
$$A = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & -\frac{K_b}{L} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{K}{J_m} & -\frac{4\delta_0}{J_m} & \frac{4\delta_0}{J_m R} & -\frac{B_m}{J_m} & 0 \\ 0 & \frac{4\delta_0}{mR} & \left(-\frac{4\delta_0}{mR^2} - K\right) & 0 & -6\dot{x}_{10}^2 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

saída: x_1

Ex 8 a)

· Vista Lateral



· $H_x = J_x \cdot w_x$ → Momento angular

· As Equações de Euler podem ser escritas como:

· $\dot{w}_x + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right)\cdot w_z = \frac{\tau_x}{J_d}$

· $\dot{w}_z + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right)\cdot w_x = \cancel{\frac{\tau_z}{J_d}}^{\,0}$

· $\tau_x = -\theta_x \cdot L \cdot 2K - Bw_x$ · Também, $J_y = J_z = J_d$

· Reescrevendo:

· $\dot{w}_x + H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right)\cdot w_z = -\frac{2LK}{J_d}\cdot \theta_x - \frac{B}{J_d} w_x$

· Definindo $x = [\theta_x \quad \dot{\theta}_x]$

e $\dot{\theta}_x = w_x$, $w_z = u$

▸ $\ddot{\theta}_x = -\frac{2LK}{J_d}\cdot\theta_x - \frac{B}{J_d}\dot{\theta}_x - H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right)\cdot u$

$\dot{\theta}_x = \dot{\theta}_x$

b) No EE:

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{2LK}{J_d} & -\frac{B}{J_d} \end{bmatrix} \cdot \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} + \begin{bmatrix} 0 \\ -H_x\left(\frac{1}{J_d} - \frac{1}{J_x}\right) \end{bmatrix} \cdot w_z$$