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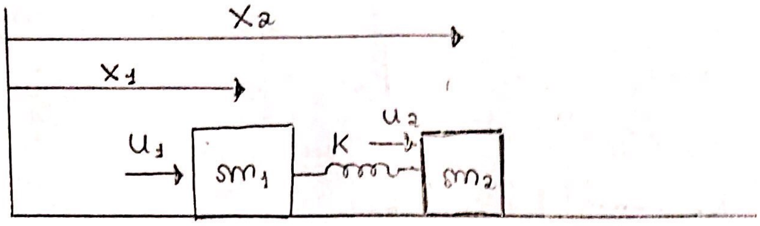
Exercícios da aula 01/10



Método alternativo: Definir o movimento do sistema pelo movimento do centro de massa.

$$\bar{X} = X_G = \frac{M_1 X_1 + M_2 X_2}{M_1 + M_2} = \frac{M_1 X_1 + M_2 X_2}{M}$$

$$\delta = X_2 - X_1$$

Do TMA:

$$U_1 + U_2 = M\ddot{X}_G \rightarrow \boxed{\ddot{X}_G = \frac{U_1 + U_2}{M}} \quad (I)$$

$$U_2 + K(X_2 - X_1) = M_2 . \ddot{X}_2 \quad e \quad U_1 - K(X_2 - X_1) = M_1 \ddot{X}_1$$

*Mas: $\ddot{\delta} = \ddot{X}_2 - \ddot{X}_1 \rightarrow \ddot{X}_1 = \ddot{X}_2 - \ddot{\delta}$

$$\ominus \begin{cases} U_2 + K\delta = M_2 \ddot{X}_2 \\ U_1 - K\delta = M_1 \ddot{X}_1 \end{cases} \rightarrow \frac{U_2}{M_2} + \frac{K\delta}{M_2} - \frac{U_1}{M_1} + \frac{K}{M_1}\delta = \ddot{X}_2 - \ddot{X}_1$$

$$\boxed{\ddot{\delta} = \frac{U_2}{M_2} - \frac{U_1}{M_1} + \frac{(\overset{M}{M_1 + M_2})}{M_1 M_2} . K\delta} \quad (II)$$

Definindo: $\bar{Z} = [\bar{x} \ \delta \ \dot{\bar{x}} \ \dot{\delta}]^T$

$$\begin{cases} \dot{\bar{Z}} = \bar{A} Z + \bar{B} u \\ Y = \bar{C} Z \end{cases}$$

$$\dot{\bar{Z}} = \begin{bmatrix} \dot{\bar{X}} \\ \dot{\delta} \\ \ddot{\bar{X}} \\ \ddot{\delta} \end{bmatrix}$$, Usando (I) e (II):

$$\dot{\bar{Z}} = \overset{A}{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{MK}{M_1 M_2} & 0 & 0 \end{bmatrix}} \bar{Z} + \overset{B}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ -1/M_1 & 1/M_2 \end{bmatrix}} u$$

• Retornando para as variáveis iniciais:

$$\begin{bmatrix} \bar{X} \\ \delta \end{bmatrix} = \begin{bmatrix} M_1/M & M_2/M \\ -1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \rightarrow \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{1}{\frac{M_1}{M} + \frac{M_2}{M}} \begin{bmatrix} 1 & -M_2/M \\ 1 & M_1/M \end{bmatrix} \begin{bmatrix} \bar{X} \\ \delta \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \overset{C}{\begin{bmatrix} 1 & -M_2/M & 0 & 0 \\ 1 & M_1/M & 0 & 0 \end{bmatrix}} \begin{bmatrix} \bar{X} \\ \delta \\ \dot{\bar{X}} \\ \dot{\delta} \end{bmatrix}$$

• Portanto:

$$\begin{cases} \dot{\bar{Z}} = A\bar{Z} + Bu \\ Y = CZ \end{cases}$$