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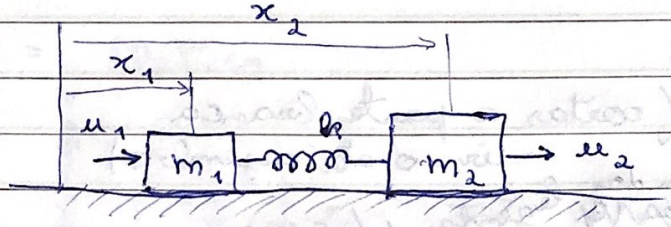
- Exercícios do dia 1º/10 -



$$\begin{cases} \bar{x} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\ = \dfrac{m_1 x_1 + m_2 x_2}{M} \\ \delta = x_1 - x_2 \end{cases}$$

Sistema dinâmico:

$$\ddot{\bar{x}} = \frac{u_1 + u_2}{M} \qquad \ddot{\delta} = \frac{u_1}{m_1} - \frac{u_2}{m_2} - \frac{k M}{m_1 m_2}\delta$$

Vetor de estados: $z = [\bar{x}, \delta, \dot{\bar{x}}, \dot{\delta}]^T \Rightarrow \dot{z} = [\dot{\bar{x}}, \dot{\delta}, \ddot{\bar{x}}, \ddot{\delta}]^T$

Entradas: $u = [u_1, u_2]^T$

Espaço de estados:

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{kM}{m_1 m_2} & 0 & 0 \end{bmatrix}}^{\bar{A}} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \overbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{m_1} & \frac{1}{m_2} \end{bmatrix}}^{\bar{B}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\dot{z} = \bar{A} z + \bar{B} u$$

Para $y = [x_1, x_2]^T$: $\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \overbrace{\begin{bmatrix} \frac{m_1}{M} & \frac{m_2}{M} \\ 1 & -1 \end{bmatrix}}^{L^{-1}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Como $L \cdot L^{-1} = I \Rightarrow L = \begin{bmatrix} \frac{M}{m_1+m_2} & -\frac{m_2}{M} \\ \frac{M}{m_1+m_2} & -\frac{m_1}{M} \end{bmatrix} = \begin{bmatrix} 1 & \frac{m_2}{M} \\ 1 & -\frac{m_1}{M} \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{m_2}{M} \\ 1 & -\frac{m_1}{M} \end{bmatrix} \begin{bmatrix} x \\ \delta \end{bmatrix}$$

Vetor de estados:

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{m_1}{M} & 0 & 0 \\ 1 & \frac{m_2}{M} & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x \\ \delta \\ \dot{x} \\ \dot{\delta} \end{bmatrix} \Rightarrow y = Cz$$

$\therefore$ $\begin{cases} \dot{z} = Az + Bu \\ y = Cz \end{cases}$