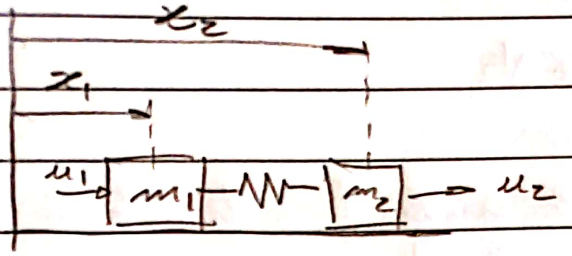
Ex 01/10 - Henrique Kuhlmann - 10772672



Tem-se que:

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + m_2 x_2}{M}$$

$$\delta = x_1 - x_2$$

2ª lei: $u_2 + u_1 = M \cdot \ddot{\bar{x}} \rightarrow \ddot{\bar{x}} = \frac{u_2 + u_1}{M}$

$$\ddot{\delta} = -\frac{KM}{m_1 m_2}\delta + \frac{u_1}{m_1} - \frac{u_2}{m_2}$$

$$z = \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$

$$\dot{z} = \bar{A}z + \bar{B}u$$
$$y = \bar{C}z$$

Organizando em forma matricial:

$$\underbrace{\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{KM}{m_1 m_2} & 0 & 0 \end{bmatrix}}_{\bar{A}} \underbrace{\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ 1/m_1 & 1/m_2 \end{bmatrix}}_{\bar{B}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u}$$

$$z' = \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

Para $y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, temos que $\bar{C}' = D^{-1}$, onde $z' = Dy$

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \begin{bmatrix} -1 & -m_2/M \\ -1 & m_1/M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Invertendo D:

$$D^{-1} = \bar{C}' = \begin{bmatrix} 1 & m_2/M \\ 1 & -m_1/M \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} \rightarrow \bar{C} = \begin{bmatrix} 1 & m_2/M & 0 & 0 \\ 1 & -m_1/M & 0 & 0 \end{bmatrix}$$