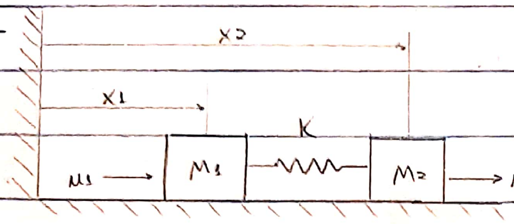
Gabriel Barbosa Paganini - 10772539 - Modelagem - ex. aula 01/10

ex



$$\left\{\begin{array}{l} \bar{x} = \dfrac{M_1 x_1 + M_2 x_2}{M_1 + M_2} = \dfrac{M_1 x_1 + M_2 x_2}{M} \\ \delta = x_1 - x_2 \end{array}\right.$$

• Sistema dinâmico: $\ddot{\bar{x}} = \dfrac{u_1 + u_2}{M}$, $\ddot{\delta} = \dfrac{u_1}{M_1} - \dfrac{u_2}{M_2} - \dfrac{KM\delta}{M_1 M_2}$

• Vetores: $u = [u_1, u_2]^T$; $z = [\bar{x}, \delta, \dot{\bar{x}}, \dot{\delta}]^T$; $\dot{z} = [\dot{\bar{x}}, \dot{\delta}, \ddot{\bar{x}}, \ddot{\delta}]^T$

• Espaço de estados:

$$\underbrace{\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-KM}{M_1 M_2} & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ 1/M_1 & -1/M_2 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u}$$

$\therefore$ $\boxed{\dot{z} = Az + Bu}$

• Para uma $y = [x_1, x_2]^T$: $\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{M_1}{M} & \frac{M_2}{M} \\ 1 & -1 \end{bmatrix}}_{L^{-1}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ → Achar L para escrever x_1, x_2 em função $\bar{x}, \delta$

$$L \cdot L^{-1} = I \rightarrow L = \begin{bmatrix} \frac{M}{M_1 + M_2} & \frac{-M_2}{M} \\ \frac{M}{M_1 + M_2} & \frac{-M_1}{M} \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} \\ 1 & \frac{-M_1}{M} \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & M_2/M \\ 1 & -M_1/M \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

• Vetor de estados: $y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{M_2}{M} & 0 & 0 \\ 1 & \frac{-M_1}{M} & 0 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}}_{z}$ → $\boxed{y = Cz}$

$\therefore$ Sistema final: $\left\{\begin{array}{l} \dot{z} = Az + Bu \\ y = Cz \end{array}\right.$, com A, B, C matrizes definidas