PME 3380 - Exc 01/10

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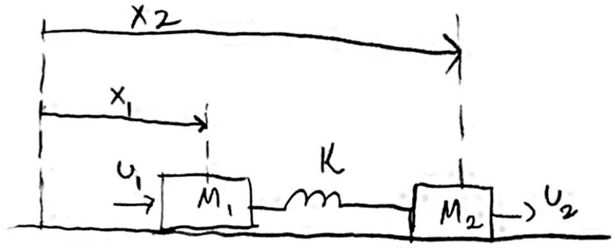


$$\bar{X} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2} = \frac{M_1 x_1 + M_2 x_2}{M}$$

$$\delta = x_1 - x_2$$

Equações obtidas:

$$\ddot{\bar{X}} = \frac{U_1 + U_2}{M} \quad ; \quad \ddot{\delta} = \frac{-KM}{M_1 M_2}\delta + \frac{U_1}{M_1} - \frac{U_2}{M_2}$$

Define-se

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad e \quad Z = \begin{bmatrix} \bar{X} \\ \delta \\ \dot{\bar{X}} \\ \dot{\delta} \end{bmatrix}$$

Escrevemos:

$$\dot{Z} = \begin{bmatrix} \dot{\bar{X}} \\ \dot{\delta} \\ \ddot{\bar{X}} \\ \ddot{\delta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{KM}{M_1 M_2} & 0 & 0 \end{bmatrix}}_{\bar{A}} \begin{bmatrix} \bar{X} \\ \delta \\ \dot{\bar{X}} \\ \dot{\delta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & -\frac{1}{M_2} \end{bmatrix}}_{\bar{B}} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Assim:

$$\boxed{\dot{Z} = \bar{A} Z + \bar{B} U}$$

Para $y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, obtemos:

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{M_1}{M} & \frac{M_2}{M} \\ 1 & -1 \end{bmatrix}}_{P^{-1}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$P = \frac{-1}{\frac{M_1 + M_2}{M}} \begin{bmatrix} -1 & -\frac{M_2}{M} \\ -1 & \frac{M_1}{M} \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} \\ 1 & -\frac{M_1}{M} \end{bmatrix}$$

Ou seja:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{M_2}{M} \\ 1 & -\frac{M_1}{M} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

Escrevendo com vetor de estados:

$$y = \begin{bmatrix} 1 & \frac{M_2}{M} & 0 & 0 \\ 1 & -\frac{M_1}{M} & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} \Rightarrow \boxed{y = \bar{C} z}$$

Logo chegamos:

$$\begin{cases} \dot{z} = \bar{A} z + \bar{B} u \\ y = \bar{C} z \end{cases}$$