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PME 3380 - Modelagem de Sistemas Dinâmicos

Exercícios de aula 17/09/2020

(1) Linearizar $f(x) = \cos x$ em torno de $\bar{x} = 0$ e $\bar{x} = \frac{\pi}{4}$.

$$f(x) = f(\bar{x}) + \left.\frac{df}{dx}\right|_{x=\bar{x}} (x - \bar{x}) + O^2$$

$$f(x) \approx f(\bar{x}) + \left.\frac{df}{dx}\right|_{x=\bar{x}} = \cos \bar{x} - \operatorname{sen} \bar{x}\,(x - \bar{x})$$

a) $\bar{x} = 0$

$$\Rightarrow f(x) \approx \cos 0 - \cancel{\operatorname{sen} 0}^{\,0} \cdot (x - 0) = 1 \Rightarrow \boxed{f(x) \approx 1}$$

b) $\bar{x} = \frac{\pi}{4}$

$$\Rightarrow f(x) \approx \cos \frac{\pi}{4} - \operatorname{sen} \frac{\pi}{4}\left(x - \frac{\pi}{4}\right)$$

$$\Rightarrow \boxed{f(x) \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)}$$

(2) Linearizar $m\dot{v} = F(t) - mru + mx\dot{r}$ por expansão de Taylor.

No equilíbrio, $\dot{\bar{v}} = \bar{r} = \dot{\bar{r}} = 0$

Temos $f(x, u, r, \dot{r}, \dot{v}) = mx\dot{r} - mru - m\dot{v} = -F(t)$

Linearizando, devemos ter:

$$f(x,u,r,\dot{r},\dot{v}) \approx f(\bar{x}, \bar{u}, \bar{r}, \dot{\bar{r}}, \dot{\bar{v}}) + \left.\frac{\partial f}{\partial x}\right|_{eq} (x - \bar{x}) + \left.\frac{\partial f}{\partial u}\right|_{eq} (u - \bar{u}) +$$

$$+ \left.\frac{\partial f}{\partial r}\right|_{eq} (r - \bar{r}) + \left.\frac{\partial f}{\partial \dot{r}}\right|_{eq} (\dot{r} - \dot{\bar{r}}) + \left.\frac{\partial f}{\partial \dot{v}}\right|_{eq} (\dot{v} - \dot{\bar{v}})$$

Calculando os termos da expansão de ordem 1:

$$f(\bar{x}, \bar{u}, \bar{r}, \dot{\bar{r}}, \dot{\bar{v}}) = 0$$

$$\left.\frac{\partial f}{\partial x}\right|_{eq} = m\,\cancel{\dot{\bar{r}}}^{\,0} = 0 \qquad (\dot{\bar{r}} = 0)$$

$$\left.\frac{\partial f}{\partial u}\right|_{eq} = -m\,\cancel{\bar{r}}^{\,0} = 0 \qquad (\bar{r} = 0)$$

$$\left.\frac{\partial f}{\partial r}\right|_{eq} = -m\bar{u}$$

$$\left.\frac{\partial f}{\partial \dot{r}}\right|_{eq} = m\bar{x}$$

$$\left.\frac{\partial f}{\partial \dot{v}}\right|_{eq} = -m$$

Finalmente:

$$f(x, u, r, \dot{r}, \dot{v}) \approx -m\bar{u}(r - \cancel{\bar{r}}^{\,0}) + m\bar{x}(\dot{r} - \cancel{\dot{\bar{r}}}^{\,0}) - m(\dot{v} - \cancel{\dot{\bar{v}}}^{\,0})$$

$$\Rightarrow -F(t) = -m\bar{u}r + m\bar{x}\dot{r} - m\dot{v}$$

$$\Rightarrow \boxed{m\dot{v} = F(t) - m\bar{u}r + m\bar{x}\dot{r}}$$