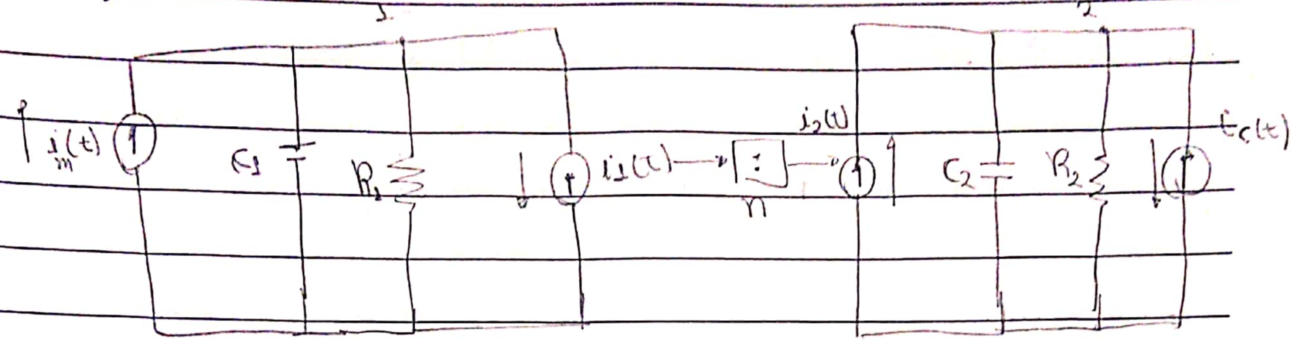
Exercícios 15/09

① a)



Pelo método dos nós:

nó 1: $V_1 \left(C_1 D + \frac{1}{R_1}\right) = i_m - i_1$

nó 2: $V_2 \left(C_2 D + \frac{1}{R_2}\right) = i_2 - i_c$

No transformador temos: $i_2 = n i_1$

Analogamente temos para o sistema mecânico:

1 → $\omega_1 (J_1 D + B_1) = T_m - T_1$

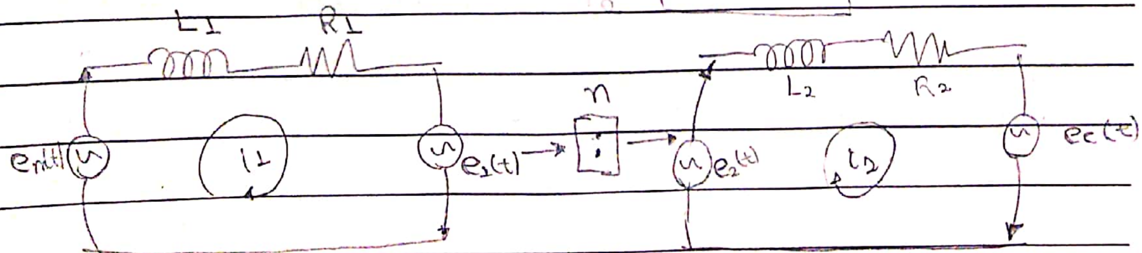
$$\boxed{J_1 \ddot{\theta}_1 + B_1 \dot{\theta}_1 = T_m - T_1}$$

2 → $\omega_2 (J_2 D + B_2) = T_2 - T_c$

$$\boxed{J_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2 = T_2 - T_c}$$

Transformador → $T_2 = n T_1 \Rightarrow \boxed{\dot{\theta}_2 = \frac{\dot{\theta}_1}{n}}$

b)



$e_m(t) = (L_1 D + R_1)\, i_1 + e_1(t)$

$e_2(t) = (L_2 D + R_2)\, i_2 + e_c(t)$

$e_2(t) = n(t)$

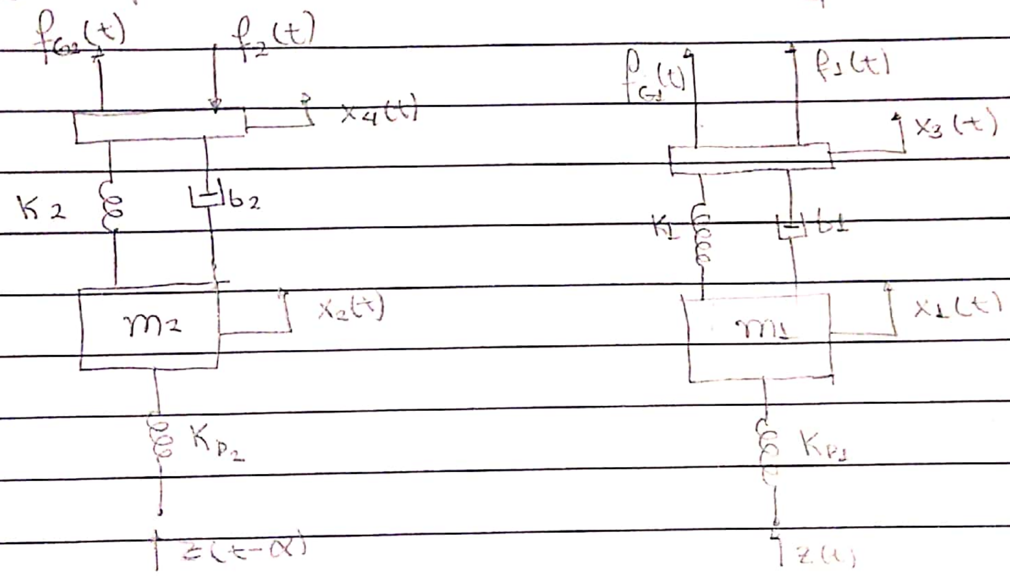
Novamente, por analogia temos

$(J_1 D + B_1)\,\omega_1 = T_m - T_1 \Rightarrow J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 = T_m - T_1$

$(J_2 D + B_2)\,\omega_2 = T_2 - T_c \Rightarrow J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 = T_2 - T_c$

$T_n = n\,T_1 \Rightarrow \dot{\theta}_2 = \dfrac{\dot{\theta}_1}{n}$

② Separando em 2 sistemas com $\frac{1}{4}$ do carro:



Assumindo pequenas amplitudes e deslocamentos:

$f_{G2}(t) = K_2(x_6 - x_2) + b_2(\dot{x}_6 - \dot{x}_2)$

$f_{G1}(t) = K_1(x_6 - x_1) + b_1(\dot{x}_6 - \dot{x}_1)$

$f_2(t) = l_2(K_2\theta + b_2\dot{\theta})$

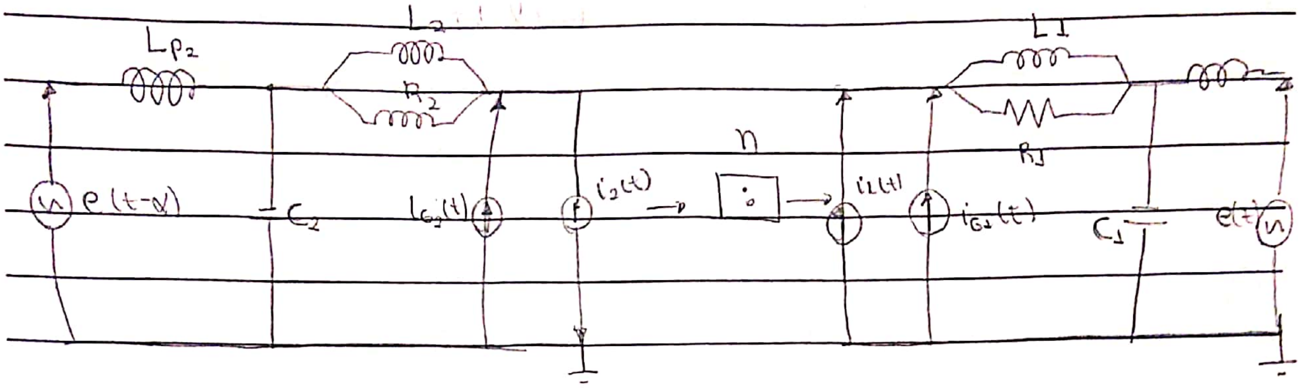
$f_1(t) = l_1(K_1\theta + b_1\dot{\theta})$

Sendo o transformador ideal, as potências devido à acoplagem devem ser iguais.

$P_1 = P_2 \Rightarrow f_1\, v_{1,ap} = f_2\, v_{2,ap} \Rightarrow f_1\, l_1\dot{\theta} = f_2\, l_2\dot{\theta} \Rightarrow n = \dfrac{f_2}{f_1} = \dfrac{l_1}{l_2}$

PESQUISAR MAIS SOBRE ESSA MATÉRIA:

Por analogia tipo 2 temos:



Pelo Método dos nós:

Nó 1:

$$V_1\left(C_1D + \frac{1}{R_1} + \frac{1}{L_1D} + \frac{1}{L_{1p}D}\right) - e(t)\,\frac{1}{L_{1p}D} - V_3\left(\frac{1}{R_1} + \frac{1}{L_1D}\right) = 0$$

Nó 2:

$$V_2\left(C_2D + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{L_{2p}D}\right) - e(t-\alpha)\,\frac{1}{L_{2p}D} - V_4\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) = 0$$

Nó 3:

$$V_3\left(\frac{1}{R_1} + \frac{1}{L_1D}\right) = i_1(t) + i_{G1}(t)$$

Nó 4:

$$V_4\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) = -i_2(t) + i_{G2}(t)$$

Substituindo (3) em (1):

$$V_1\left(C_1D + \frac{1}{R_1} + \frac{1}{L_1D} + \frac{1}{L_{1p}D}\right) = \frac{e(t)}{L_{1p}D} + i_1(t) + i_{G1}(t)$$

$$V_1\left(m_1D + b_1 + \frac{K_1}{D} + \frac{K_{1p}}{D}\right) = \frac{\dot{z}(t)\,K_{1p}}{D} + f_1(t) + f_{G1}(t)$$

$$m_1\ddot{x}_1 + b_1\dot{x}_1 + (K_1 + K_{1p})\,x_1 = K_{1p}\,z(t) + f_1(t) + f_{G1}(t)$$

DATA __/__

Substituindo (4) em (2):

$$V_2\left(C_2 D + \frac{1}{R_2} + \frac{1}{L_2 D} + \frac{1}{L_{2p} D}\right) = \frac{e(t-\alpha)}{L_{2p} D} + (G_2(t) - i_2(t))$$

$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + (K_2 + K_{2p})x_2 = K_{2p} Z(t-\alpha) - f_2(t) + f_{G2}(t)$$

3

$$\begin{cases} J_1 \dot{\omega}_1 + B_1 \omega_1 + T_1 = T_m & (1) \\ J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c = T_2 & (2) \end{cases}$$

$P_1 = P_2 \Rightarrow T_1 \omega_1 = T_2 \omega_2 \Rightarrow T_2 = \frac{T_1 \omega_1}{\omega_2} = n T_1 \quad (3)$

$(3) \rightarrow (2): \quad J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c = n T_1 \quad (4)$

$(1) \rightarrow (4): \quad J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c + n(J_1 \dot{\omega}_1 + B_1 \omega_1) = n T_m$

Como $\omega_1 = n \omega_2 \rightarrow \dot{\omega}_1 = n \dot{\omega}_2$:

$$J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c + n(J_1 n \dot{\omega}_2 + B_1 n \omega_2) = n T_m$$

$$\underbrace{(J_2 + n^2 J_1)}_{J_{eq2}} \dot{\omega}_2 + \underbrace{(B_2 + n^2 B_1)}_{B_{eq2}} \omega_2 + T_c = n T_m$$

$$J_{eq2} \dot{\omega}_2 + B_{eq2} \omega_2 + T_c = n T_m$$