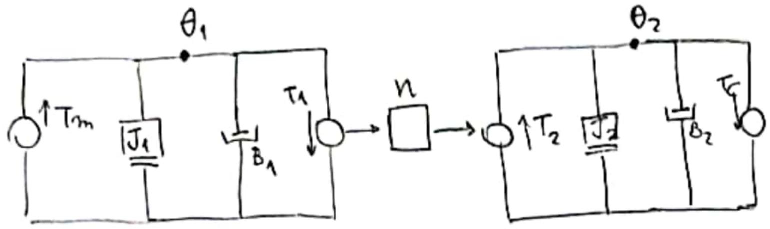
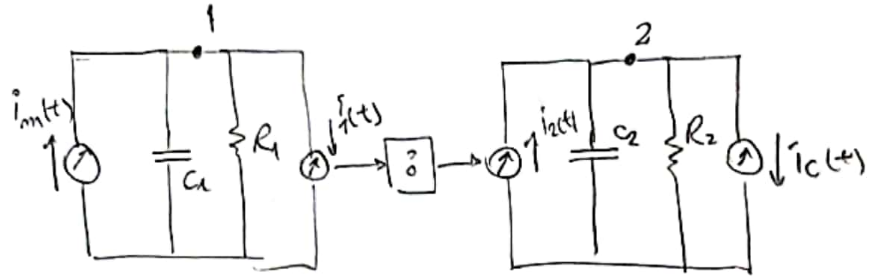
① ⓐ Analogia do Tipo 2



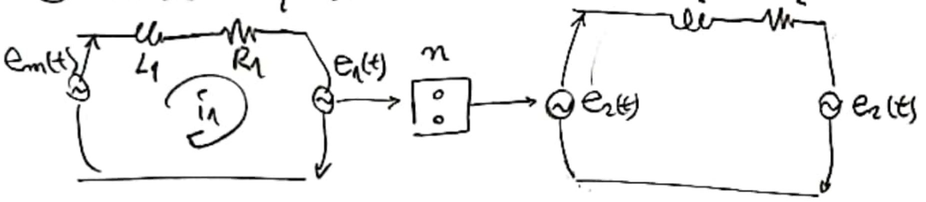


Nó 1: $V_1\left(C_1D+\frac{1}{R_1}\right) = i_m - i_1$ $\longrightarrow$ Nó 1: $W_1(J_1D+B_1) = T_m - T_1 \Rightarrow J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 = T_m - T_1$

Nó 2: $V_2\left(C_2D+\frac{1}{R_2}\right) = i_2 - i_c$ $\longrightarrow$ Nó 2: $W_2(J_2D+B_2) = T_2 - T_c \Rightarrow J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 = T_2 - T_c$

Transformador $\Rightarrow i_2 = n\cdot i_1$ Transformador $\Rightarrow T_2 = nT_1 \Rightarrow \dot{\theta}_2 = \frac{\dot{\theta}_1}{n}$

ⓑ Analogia do Tipo 1



Elétrico

$e_m(t) = i_1(L_1D+R_1) + e_1(t)$

$e_2(t) = i_2(L_2D+R_2) + e_c(t)$

$e_2(t) = n\cdot e_1(t)$

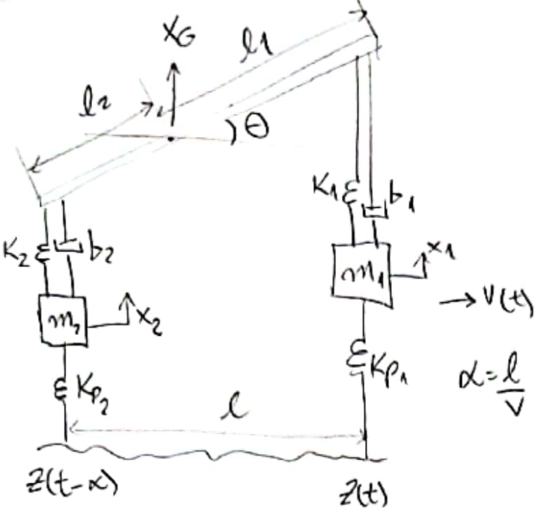
Mecânico

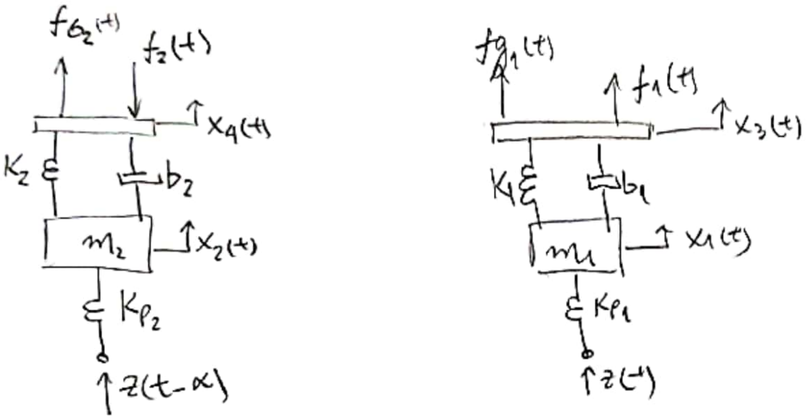
$W_1(J_1D+B_1) = T_m - T_1 \Rightarrow J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 = T_m - T_1$

$W_2(J_2D+B_2) = T_2 - T_c \Rightarrow J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 = T_2 - T_c$

$T_2 = nT_1 \Rightarrow \dot{\theta}_2 = \frac{\dot{\theta}_1}{n}$

(2)





$$\left.\begin{aligned} f_2(t) &= l_2(K_2\theta + b_2\dot{\theta}) \\ f_1(t) &= l_1(K_1\theta + b_1\dot{\theta}) \end{aligned}\right\} \text{ARFAGEM}$$

$$\left.\begin{aligned} f_{G_2}(t) &= K_2(x_G - x_2) + b_2(\dot{x}_G - \dot{x}_2) \\ f_{G_1}(t) &= K_1(x_G - x_1) + b_1(\dot{x}_G - \dot{x}_1) \end{aligned}\right\} \text{Oscilação vertical}$$

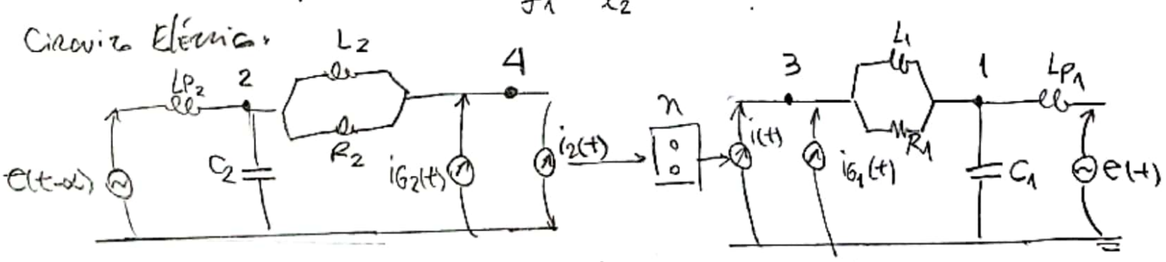
CONSIDERANDO TRANSFORMADOR IDEAL QUE INTRODUZ O MOVIMENTO DE ARFAGEM, ENTÃO AS POTÊNCIAS DEVEM SER IGUAIS.

$$P_1 = P_2$$

$$f_1 V_{1,arf} = f_2 V_{2,arf}$$

$$f_1 l_1 \dot{\theta} = f_2 l_2 \dot{\theta} \Rightarrow n = \frac{f_2}{f_1} = \frac{l_1}{l_2}$$

Circuito Elétrico:



Nó 1: $V_1\left(C_1D + \frac{1}{R_1} + \frac{1}{L_1D} + \frac{1}{L_{1p}D}\right) - e(t)\cdot\frac{1}{L_{1p}D} - V_3\left(\frac{1}{R_1} + \frac{1}{L_1D}\right) = 0$

Nó 2: $V_2\left(C_2D + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{L_{2p}D}\right) - e(t-\alpha)\cdot\frac{1}{L_{2p}D} - V_4\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) = 0$

Nó 3: $V_3\left(\frac{1}{R_1} + \frac{1}{L_1D}\right) = i_1(t) + i_{G_1}(t)$

Nó 4: $V_4\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) = -i_2(t) + i_{G_2}(t)$

Substituindo 3 em 1, 4 em 2,

$$V_1\left(C_1D + \frac{1}{R_1} + \frac{1}{L_1D} + \frac{1}{L_{1p}D}\right) = \frac{e(t)}{L_{1p}D} + i_1(t) + i_{G1}(t)$$

$$V_2\left(C_2D + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{L_{2p}D}\right) = \frac{e(t-\alpha)}{L_{2p}D} + i_{G2}(t) - i_2(t)$$

$$\Rightarrow \begin{cases} V_1\left(m_1D + b_1 + \frac{K_1}{D} + \frac{K_{1p}}{D}\right) = \frac{\dot{z}(t)\cdot K_{1p}}{D} + f_1(t) + f_{G1}(t) \\ V_2\left(m_2D + b_2 + \frac{K_2}{D} + \frac{K_{2p}}{D}\right) = \frac{\dot{z}(t-\alpha)\cdot K_{2p}}{D} - f_2(t) + f_{G2}(t) \end{cases}$$

$$\Rightarrow \begin{cases} m_1\ddot{x}_1 + b_1\dot{x}_1 + (K_1 + K_{1p})x_1 = K_{1p}\cdot z(t) + f_1(t) + f_{G1}(t) \\ m_2\ddot{x}_2 + b_2\dot{x}_2 + (K_2 + K_{2p})x_2 = K_{2p}\cdot z(t-\alpha) - f_2(t) + f_{G2}(t) \end{cases}$$

$$f_1 + f_{G1} - f_2 + f_{G2} = M\ddot{x}_G$$

$$(f_1 + f_{G1})l_1 + (f_{G2} - f_2)l_2 = J_G\ddot{\theta}$$

3 Demonstração

$$\begin{cases} J_1\dot{w}_1 + B_1w_1 + T_1 = T_m & (1) \\ J_2\dot{w}_2 + B_2w_2 + T_c = T_2 & (2) \end{cases}$$

$$P_1 = P_2 \Rightarrow T_1w_1 = T_2w_2 \Rightarrow T_2 = \frac{T_1w_1}{w_2} = nT_1 \quad (3)$$

De (3) em (2): $J_2\dot{w}_2 + B_2w_2 + T_c = nT_1$ (4)

De (1): $T_1 = T_m - J_1\dot{w}_1 - B_1w_1$ (5)

De (5) em (4):

$$J_2\dot{w}_2 + B_2w_2 + T_c + n(J_1\dot{w}_1 + B_1w_1) = nT_m$$

Mas $w_1 = n\cdot w_2 \Leftrightarrow \dot{w}_1 = n\cdot\dot{w}_2$, então,

$$J_2\dot{w}_2 + B_2w_2 + T_c + n(J_1n\dot{w}_2 + B_1nw_2) = nT_m$$

$$\underbrace{(J_2 + n^2J_1)}_{J_{eq2}}\dot{w}_2 + \underbrace{(B_2 + n^2B_1)}_{B_{eq2}}w_2 + T_c = nT_m$$

$$\boxed{J_{eq_2}\dot{w}_2 + B_{eq_2}w_2 + T_c = nT_m}$$

onde: $\begin{cases} J_{eq} = J_2 + n^2J_1 \\ B_{eq} = B_2 + n^2B_1 \end{cases}$