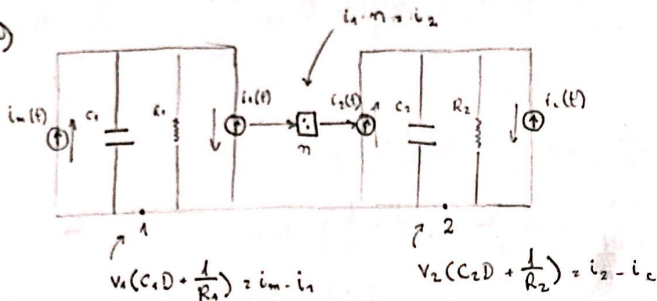


EXERCÍCIO DA AULA DE 15/09 - MODELAGEM

Pedro Pires Sulzer - 107069410

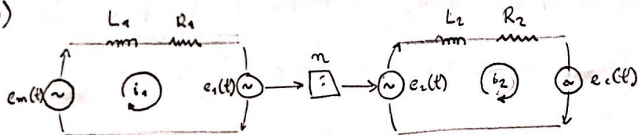
1 a)



ARREQUANDO A UM SISTEMA MECÂNICO

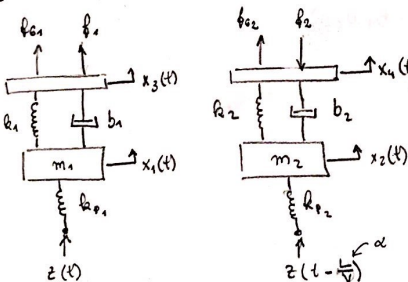
$$\begin{cases} \omega_1 (J_1 D + B_1) = T_m - T_1 \\ \omega_2 (J_2 D + B_2) = T_2 - T_c \\ T_2 = n T_1 \end{cases} \Rightarrow \begin{cases} J_1 \ddot{\theta}_1 + B_1 \dot{\theta}_1 = T_m - T_1 \\ J_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2 = T_2 - T_c \\ \dot{\theta}_2 = \frac{\dot{\theta}_1}{n} \end{cases}$$

b)



$$\begin{cases} e_m(t) = (L_1 D + R_1) i_1 + e_1(t) \\ e_2(t) = (L_2 D + R_2) i_2 + e_c(t) \\ e_2(t) = n e_1(t) \end{cases} \Rightarrow \begin{cases} (J_1 D + B_1) \omega_1 = T_m - T_1 \\ (J_2 D + B_2) \omega_2 = T_2 - T_c \\ T_2 = n T_1 \end{cases} \Rightarrow \begin{cases} J_1 \ddot{\theta}_1 + B_1 \dot{\theta}_1 = T_m - T_1 \\ J_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2 = T_2 - T_c \\ \dot{\theta}_2 = \frac{\dot{\theta}_1}{n} \end{cases}$$

2.



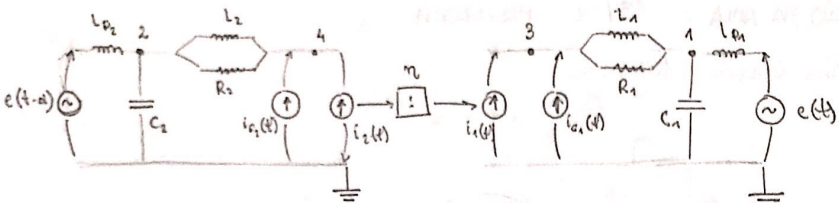
$$\begin{cases} f_{s1}(t) = k_1 (x_0 - x_1) \\ f_{d1}(t) = b_1 (\dot{x}_0 - \dot{x}_1) \\ f_{s2}(t) = k_2 (x_1 - x_2) + b_2 (\dot{x}_1 - \dot{x}_2) \end{cases}$$

ARRAÇAMENTO

$P_1 = P_2$ ← POTÊNCIAS DEVIDO A ARRANJAMENTO

$f_1 v_1 = f_2 v_2$

$f_1 l_1 \dot{\theta} = f_2 l_2 \dot{\theta} \Rightarrow \frac{f_2}{f_1} = \frac{l_1}{l_2} = n$



$$V_1 \left(C_1 D + \frac{1}{R_1} + \frac{1}{L_1 D} + \frac{1}{L_{p1} D} \right) - e(t) \frac{1}{L_{p1} D} - V_3 \left(\frac{1}{R_1} + \frac{1}{L_1 D} \right) = 0$$

$$V_2 \left(C_2 D + \frac{1}{R_2} + \frac{1}{L_2 D} + \frac{1}{L_{p2} D} \right) - e(t-\alpha) \frac{1}{L_{p2} D} - V_4 \left(\frac{1}{R_2} + \frac{1}{L_2 D} \right) = 0$$

$$V_3 \left(\frac{1}{R_1} + \frac{1}{L_1 D} \right) = i_1(t) + i_{G1}(t)$$

$$V_4 \left(\frac{1}{R_2} + \frac{1}{L_2 D} \right) = -i_2(t) + i_{G2}(t)$$

$$\Rightarrow \begin{cases} V_1 \left(C_1 D + \frac{1}{R_1} + \frac{1}{L_1 D} + \frac{1}{L_{p1} D} \right) = e(t) \frac{1}{L_{p1} D} + i_1(t) + i_{G1}(t) \\ V_2 \left(C_2 D + \frac{1}{R_2} + \frac{1}{L_2 D} + \frac{1}{L_{p2} D} \right) = e(t-\alpha) \frac{1}{L_{p2} D} - i_2(t) + i_{G2}(t) \end{cases}$$

PI SISTEMA MECÂNICO:

$$\begin{cases} m_1 \ddot{x}_1 + b_1 \dot{x}_1 + (k_1 + k_{p1}) x_1 = k_{p1} z(t) + f_1(t) + f_{G1}(t) \\ m_2 \ddot{x}_2 + b_2 \dot{x}_2 + (k_2 + k_{p2}) x_2 = k_{p2} z(t-\alpha) - f_2(t) + f_{G2}(t) \end{cases} \Rightarrow \begin{cases} f_1 + f_{G1} - f_2 + f_{G2} = M \ddot{x}_c \\ (f_1 + f_{G1}) l_1 + (f_{G2} - f_2) l_2 = J_G \ddot{\theta} \end{cases}$$

③ $\begin{cases} J_1 \dot{\omega}_1 + B_1 \omega_1 + T_1 = T_m \\ J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c = T_2 \end{cases} \quad P_1 = P_2 \Rightarrow T_1 \omega_1 = T_2 \omega_2 \Rightarrow T_2 = \frac{T_1 \omega_1}{\omega_2} = \eta T_1$

$$\begin{cases} J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c = \eta T_1 \\ T_1 = T_m - J_1 \dot{\omega}_1 - B_1 \omega_1 \end{cases} \Rightarrow J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c = \eta (T_m - J_1 \dot{\omega}_1 - B_1 \omega_1)$$

sendo $\omega_1 = \eta \omega_2$, $\dot{\omega}_1 = \eta \dot{\omega}_2$

$$J_2 \dot{\omega}_2 + B_2 \omega_2 + T_c = \eta (T_m - J_1 \eta \dot{\omega}_2 - B_1 \omega_2 \eta)$$

$$\dot{\omega}_2 \underbrace{(J_1 \eta^2 + J_2)}_{J_{eq}} + \omega_2 \underbrace{(B_1 \eta^2 + B_2)}_{B_{eq}} + T_c = T_m \cdot \eta$$