PME 3380 - Modelagem dos Sistemas Dinâmicos

Exercício Aula 08/09/2020

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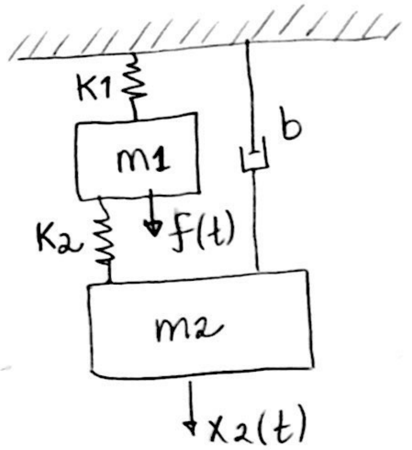
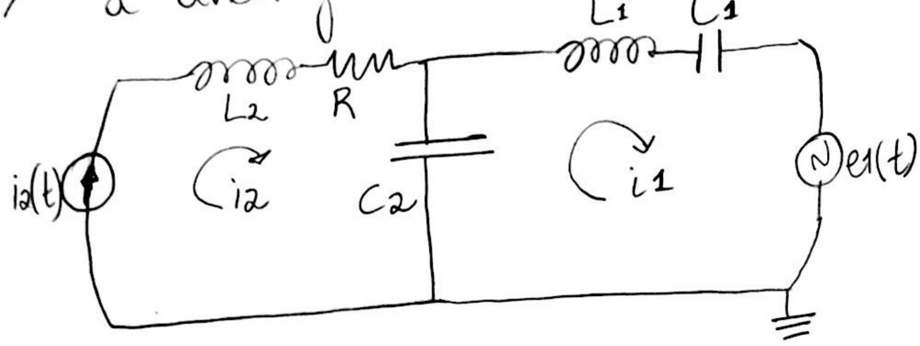
Ex 2.



Dado o sistema mecânico tem-se a analogia elétrica:



Utilizando o método prático:

$$e_1(t) = \left(L_1 D + \frac{1}{C_1 D} \right) i_1 + \frac{1}{C_2 D} (i_2 - i_1)$$

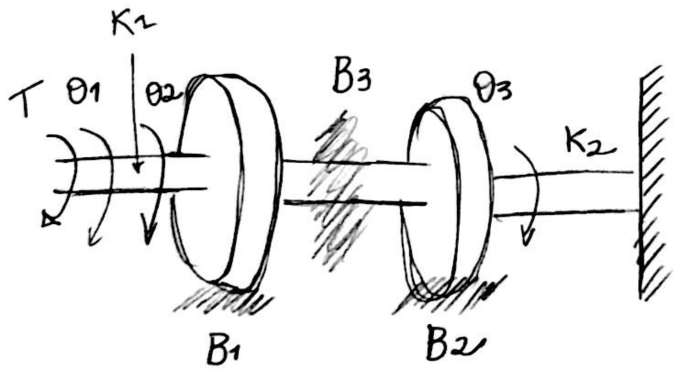
A equação para o sistema mecânico fica:

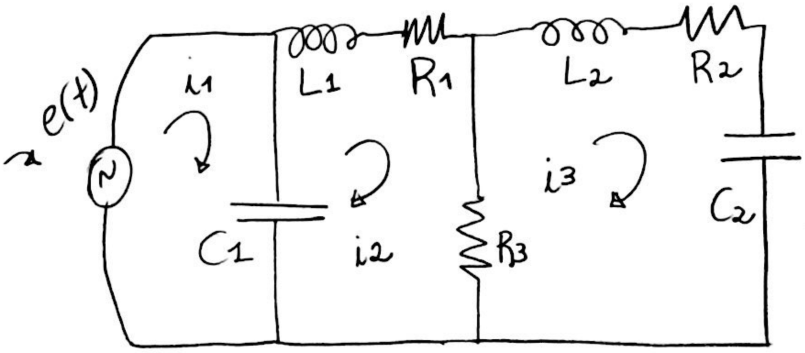
$$\left(m_1 D + \frac{K_1}{D} \right) V_1 + \frac{K_2}{D} (V_1 - V_2) = f_1(t)$$

$$\boxed{m_1 \ddot{x}_1 + (K_1 + K_2) x_1 = f_1(t) + K_2 x_2}$$

Ex 3.

Sistema Rotativo: $\vec{f} \to \vec{T} \to V$, $\vec{v} \to \vec{w} \to i$





$$\begin{cases} e(t) = \dfrac{1}{C_1 D}(i_1 - i_2) \\ (L_1 D + R_1)\, i_2 + \dfrac{1}{C_1 D}(i_2 - i_1) + R_3(i_2 - i_3) = 0 \\ \left(L_2 D + R_2 + \dfrac{1}{C_2 D}\right) i_3 + R_3(i_3 - i_2) = 0 \end{cases}$$

Usando Analogia do tipo 1

$$T(t) = \frac{K_1}{D}(\omega_1 - \omega_2) \longrightarrow \boxed{T(t) = K_1(\theta_1 - \theta_2)}$$

$$(J_1 D + B_1)\,\omega_2 + \frac{K_1}{D}(\omega_2 - \omega_1) + B_3(\omega_2 - \omega_3) = 0$$

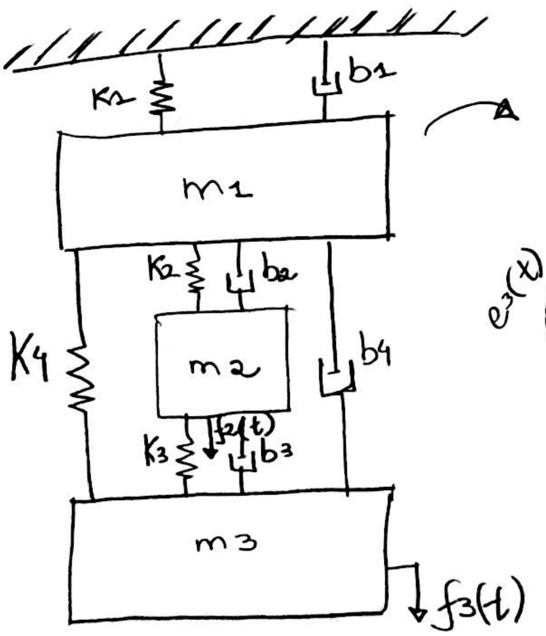
$$\boxed{J_1 \ddot{\theta}_2 + (B_1 + B_3)\dot{\theta}_2 + K_1 \theta_2 = K_1 \theta_1 + B_3 \dot{\theta}_3}$$

$$\left(J_2 D + B_2 + \frac{K_2}{D}\right)\omega_3 + B_3(\omega_3 - \omega_2) = 0$$

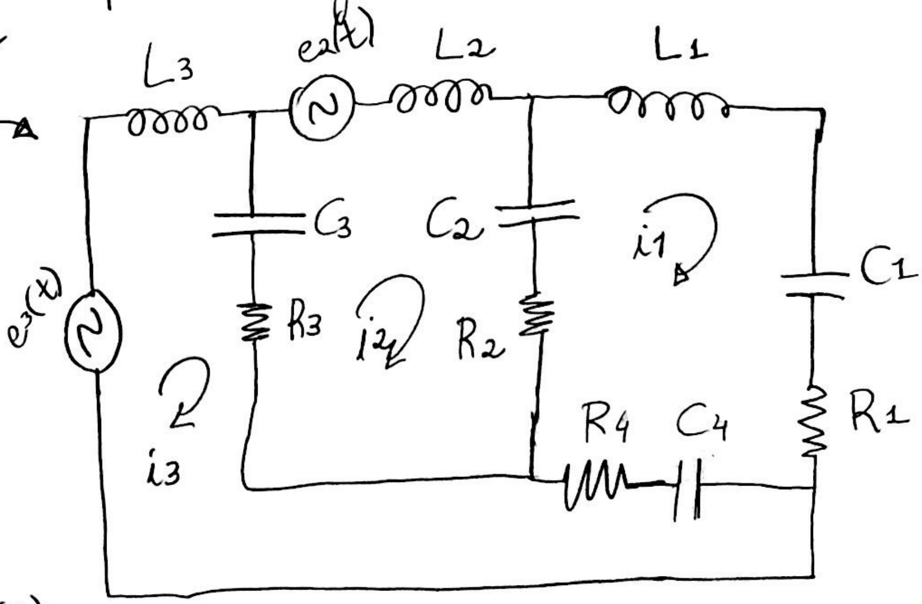
$$\boxed{J_2 \ddot{\theta}_3 + (B_2 + B_3)\dot{\theta}_3 + K_2 \theta_3 = B_3 \dot{\theta}_2}$$

Exercícios da Lista Adicional

EX 3.



Por analogia do tipo 1:



$$\begin{cases} e_3(t) = L_3 D i_3 + \left(R_3 + \dfrac{1}{C_3 D}\right)(i_3 - i_2) + \left(R_4 + \dfrac{1}{C_4 D}\right)(i_3 - i_1) \\ e_2(t) = L_2 D i_2 + \left(R_2 + \dfrac{1}{C_2 D}\right)(i_2 - i_1) + \left(R_3 + \dfrac{1}{C_3 D}\right)(i_2 - i_3) \\ \left(L_1 D + R_1 + \dfrac{1}{C_1 D}\right) i_1 + \left(R_2 + \dfrac{1}{C_2 D}\right)(i_1 - i_2) + \left(R_4 + \dfrac{1}{C_4 D}\right)(i_1 - i_3) = 0 \end{cases}$$

Por analogia, para o sistema mecânico:

$$m_3 D V_3 + \left(b_3 + \frac{K_3}{D}\right)(V_3 - V_2) + \left(b_4 + \frac{K_4}{D}\right)(V_3 - V_1) = f_3(t)$$

$$\boxed{m_3 \ddot{x}_3 + (b_3 + b_4)\dot{x}_3 + (K_3 + K_4)x_3 = f_3(t) + b_3 \dot{x}_2 + K_3 x_2 + b_4 \dot{x}_1 + K_4 x_1}$$

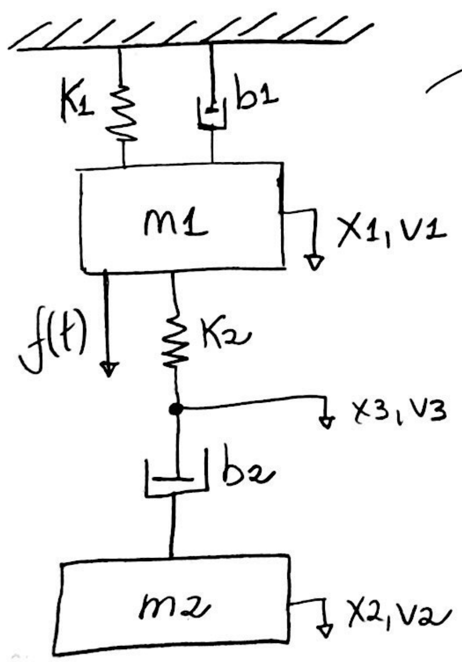
$$m_2 D V_2 + \left(b_2 + \frac{K_2}{D}\right)(V_2 - V_1) + \left(b_3 + \frac{K_3}{D}\right)(V_2 - V_3) = f_2(t)$$

$$\boxed{m_2 \ddot{x}_2 + (b_2 + b_3)\dot{x}_2 + (K_2 + K_3)x_2 = f_2(t) + b_2 \dot{x}_1 + b_3 \dot{x}_3 + K_2 x_1 + K_3 x_3}$$

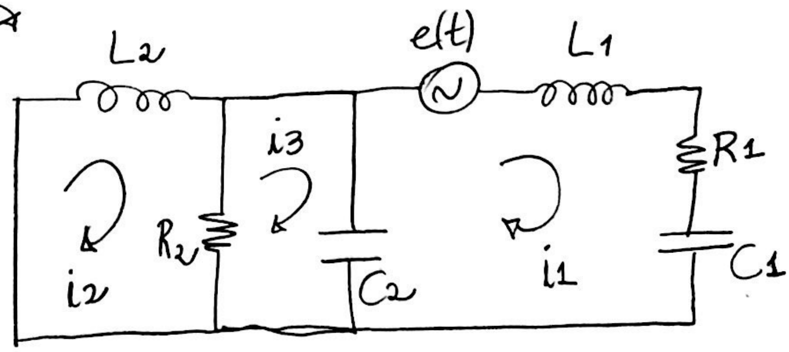
$$\left(m_1 D + b_1 + \frac{K_1}{D}\right) V_1 + \left(b_2 + \frac{K_2}{D}\right)(V_1 - V_2) + \left(b_4 + \frac{K_4}{D}\right)(V_1 - V_3) = 0$$

$$\boxed{m_1 \ddot{x}_1 + (b_1 + b_2 + b_4)\dot{x}_1 + (K_1 + K_2 + K_4)x_1 = b_2 \dot{x}_2 + K_2 x_2 + b_4 \dot{x}_3 + K_4 x_3}$$

Ex.6



Circuito Elétrico Equivalente



$$\begin{cases} \left(L_1 D + R_1 + \dfrac{1}{C_1 D}\right) i_1 + \dfrac{1}{C_2 D}(i_1 - i_3) = e(t) \\ L_2 D i_2 + R_2 (i_2 - i_3) = 0 \\ R_2 (i_3 - i_2) + \dfrac{1}{C_2 D}(i_3 - i_1) = 0 \end{cases}$$

Por analogia, para o sistema mecânico:

$$\left(m_1 D + b_1 + \frac{K_1}{D}\right) V_1 + \frac{K_2}{D}(V_1 - V_3) = f(t)$$

$$\boxed{m_1 \ddot{x}_1 + b_1 \dot{x}_1 + (K_1 + K_2) x_1 = f(t) + K_2 x_3}$$

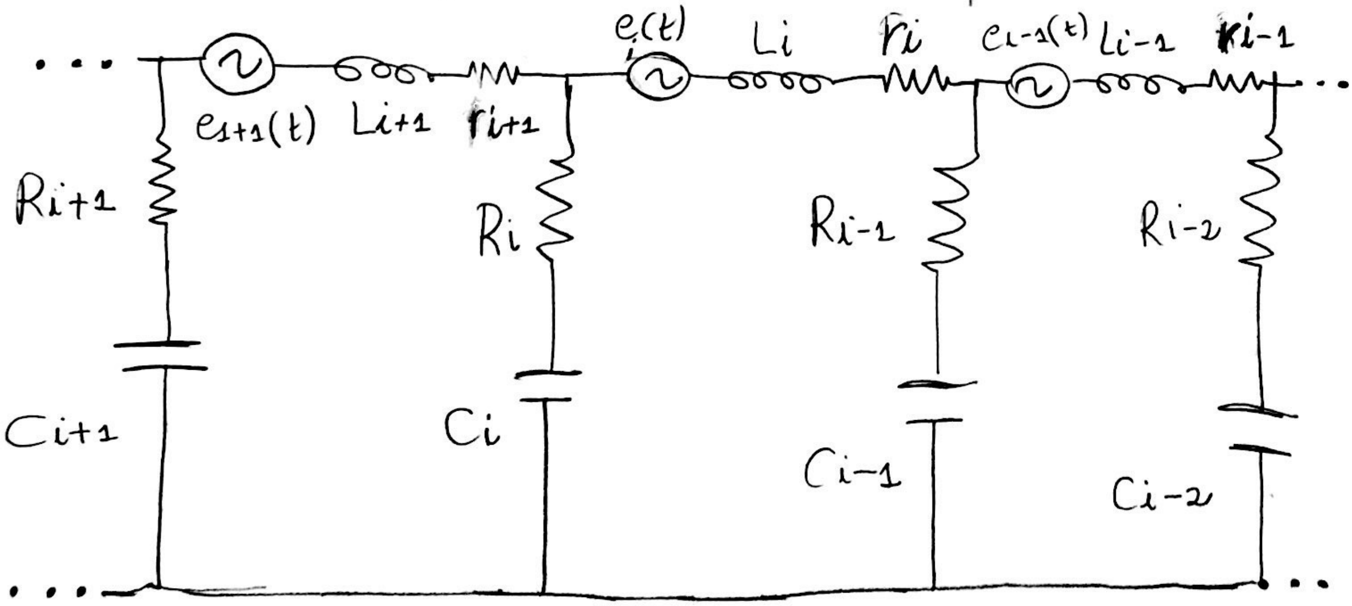
$$m_2 D V_2 + b_2 (V_2 - V_3) = 0$$

$$\boxed{m_2 \ddot{x}_2 + b_2 \dot{x}_2 = b_2 \dot{x}_3}$$

$$b_2 (V_3 - V_2) + \frac{K_2}{D}(V_3 - V_1) = 0$$

$$\boxed{b_2 \dot{x}_3 + K_2 x_3 = b_2 \dot{x}_2 + K_2 x_1}$$

Ex. 7 Usando a Analogia do Tipo 1, Dado o sistema mecânico, têm-se o circuito elétrico equivalente



Aplicando o método prático

$$e_i(t) = (L_i D + r_i)i_i + \left(R_{i-1} + \frac{1}{C_{i-1}D}\right)(i_i - i_{i-1}) + \left(R_i + \frac{1}{C_i D}\right)(i_i - i_{i+1})$$

$$e_i(t) = \left(L_i D + R_{i-1} + R_i + r_i + \frac{1}{C_{i-1}D} + \frac{1}{C_i D}\right)i_i - \left(R_{i-1} + \frac{1}{C_{i-1}D}\right)i_{i-1} - \left(R_i + \frac{1}{C_i D}\right)i_{i+1}$$

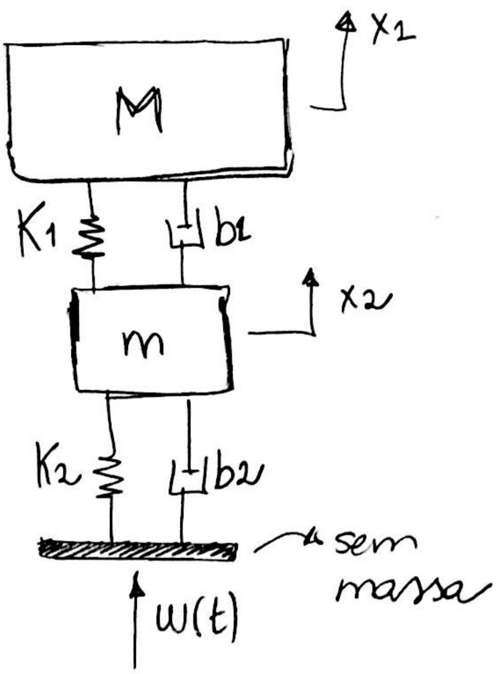
Sabendo que a força sobre cada vagão é:

$$u_i'(t) = u_i - m_i g \operatorname{sen}\theta_i$$

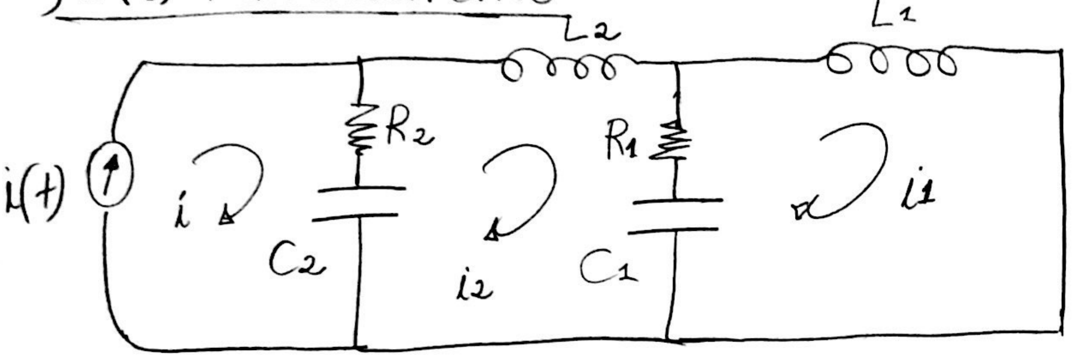
Usando a analogia para obter as equações no sistema mecânico, chega-se em:

$$\boxed{m_i \ddot{x}_i + (d_{i-1} + d_i + b_i)\dot{x}_i + (K_{i-1} + K_i)x_i = d_{i-1}\dot{x}_{i-1} + K_{i-1}x_{i-1} + d_i \dot{x}_{i+1} + K_i x_{i+1} + u_i - m_i g \operatorname{sen}\theta_i}$$

Ex. 8



a) w(t) → deslocamento



$$\begin{cases} L_1 D i_1 + \left(R_1 + \dfrac{1}{C_1 D}\right)(i_1 - i_2) = 0 \\ L_2 D i_2 + \left(R_1 + \dfrac{1}{C_1 D}\right)(i_2 - i_1) + \left(R_2 + \dfrac{1}{C_2 D}\right)(i_2 - i) = 0 \end{cases}$$

Sistema Mecânico:

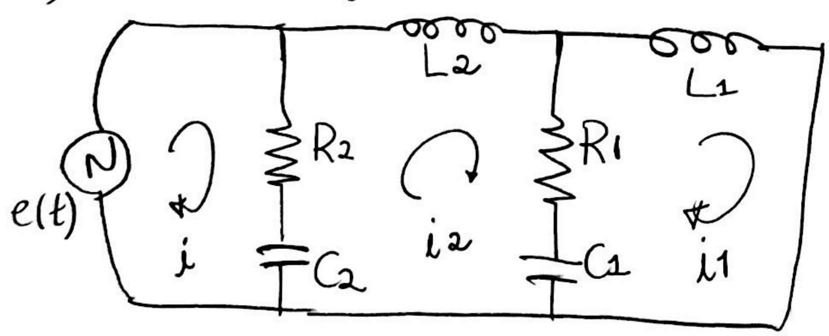
$$M D v_1 + \left(b_1 + \frac{K_1}{D}\right)(v_1 - v_2) = 0$$

$$\boxed{M\ddot{x}_1 + b_1 \dot{x}_1 + K_1 x = b_1 \dot{x}_2 + K_1 x_2}$$

$$m D v_2 + \left(b_1 + \frac{K_1}{D}\right)(V_2 - V_1) + \left(b_2 + \frac{K_2}{D}\right)(v_2 - \dot{w}(t)) = 0$$

$$\boxed{m\ddot{x}_2 + (b_1 + b_2)\dot{x}_2 + (K_1 + K_2)x_2 = b_1 \dot{x}_1 + K_1 x_1 + b_2 \dot{w}(t) + K_2 w(t)}$$

b) w(t) → Força Imposta pela Via



$$
\begin{cases}
\left(R_2 + \dfrac{1}{C_2 D}\right)(i - i_2) = e(t) \\
L_2 D i_2 + \left(R_1 + \dfrac{1}{C_1 D}\right)(i_2 - i_1) + \underbrace{\left(R_2 + \dfrac{1}{C_2 D}\right)(i_2 - i)}_{-e(t)} = 0 \\
L_1 D i_1 + \left(R_1 + \dfrac{1}{C_1 D}\right)(i_1 - i_2) = 0
\end{cases}
$$

Sistema Mecânico:

$$M D v_1 + \left(b_1 + \frac{K_1}{D}\right)(v_1 - v_2) = 0$$

$$\boxed{M\ddot{x}_1 + b_1\dot{x}_1 + K_1 x_1 = b_1\dot{x}_2 + K_1 x_2}$$

$$m_2 D v_2 + \left(b_1 + \frac{K_1}{D}\right)(v_2 - v_1) - w(t) = 0$$

$$\boxed{m_2\ddot{x}_2 + b_1\dot{x}_2 + K_1 x_2 = w(t) + b_1\dot{x}_1 + K_1 x_1}$$