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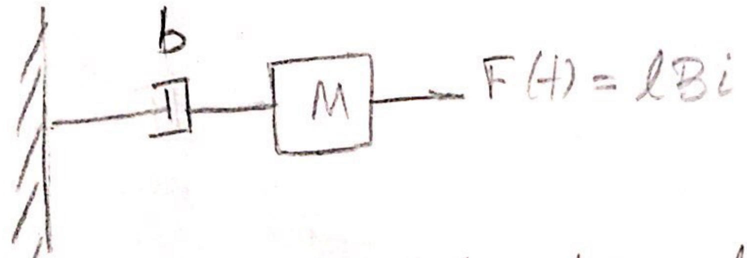
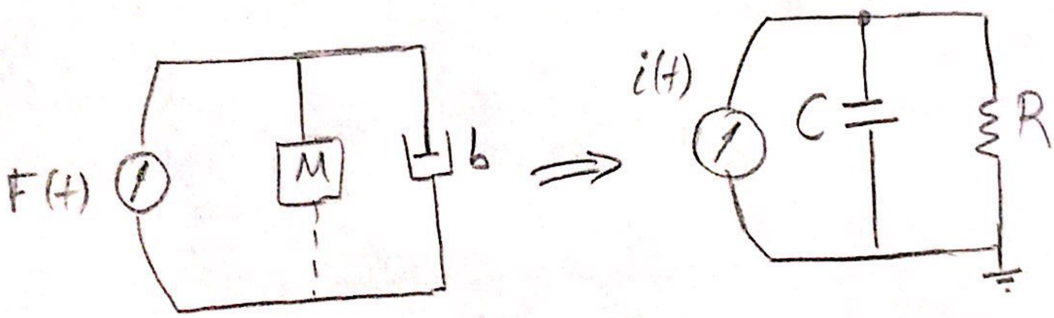
Modelagem de Sistemas Dinâmicos

Exercícios aula dia 10/08/2020

(1) O modelo físico para o movimento do cone de ar do alto falante é:



O circuito elétrico equivalente, pela analogia do tipo 2 fica:



$$V\left(CD + \frac{1}{R}\right) = i(t)$$

Pela analogia $\vec{f} \to i$, $\vec{v} \to V$:

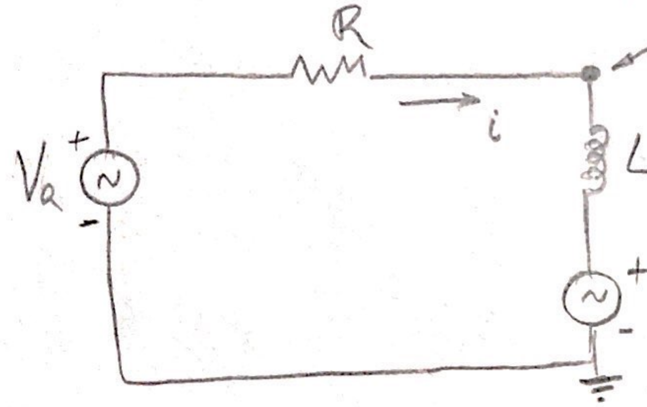
$$v(MD + b) = F(t) = lBi$$

$$\boxed{M\ddot{x} + b\dot{x} = lBi}$$

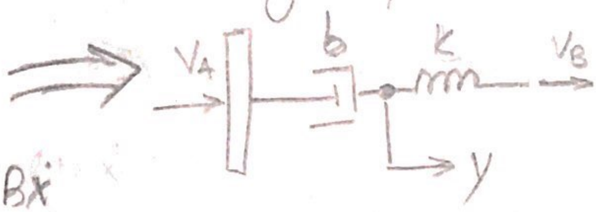
Scanned with CamScanner

O circuito elétrico do alto falante é:

Nó de tensão desconhecida



Analogia tipo 2:



Obtemos as equações do sistema pela analogia tipo 2 e usando as equações de Lagrange.

Energia cinética: $T = \frac{M\dot{x}^2}{2}$

Energia potencial: $V = \frac{\lambda^2}{2L}$

$\mathcal{L} = T - V$

Dissipação de Rayleigh: $R = \frac{b\dot{x}^2}{2} + \frac{\dot{\lambda}^2}{2R}$

Grau de liberdade x:

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = M\dot{x} \Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = M\ddot{x}$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \qquad \frac{\partial R}{\partial \dot{x}} = b\dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} + \frac{\partial R}{\partial \dot{x}} = F(t) = \ell B i$$

$$\boxed{M\ddot{x} + b\dot{x} = \ell B i}$$

Grau de liberdade λ:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\lambda}}\right) = 0 \qquad \frac{\partial \mathcal{L}}{\partial \lambda} = -\frac{\lambda}{L} \qquad \frac{\partial R}{\partial \dot{\lambda}} = \frac{\dot{\lambda}}{R}$$

Forças externas análogas: $bv_A \Rightarrow \frac{V_a}{R}$; $\frac{kV_B}{D} \Rightarrow \frac{V_b}{LD}$

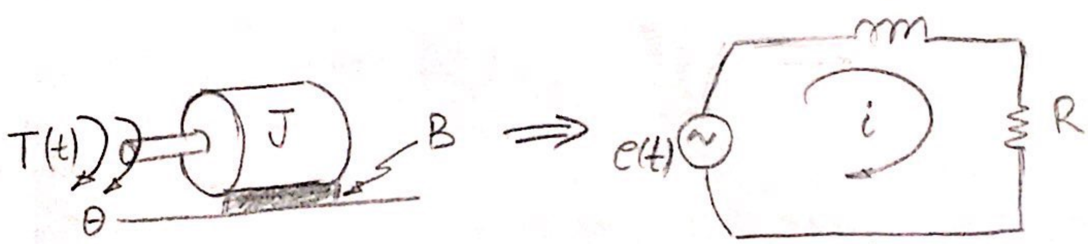
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\lambda}}\right) - \frac{\partial L}{\partial \lambda} + \frac{\partial R}{\partial \dot{\lambda}} = \frac{V_a}{R} + \frac{V_B}{LD}$$

$$\frac{\lambda}{L} + \frac{\dot{\lambda}}{R} = \frac{V_a}{R} + \frac{\ell B \dot{x}}{LD}$$

$$\boxed{\frac{\lambda}{L} + \frac{\dot{\lambda}}{R} = \frac{V_a}{R} + \frac{\ell B x}{L}}$$

onde $\lambda = \int V dt$ e V é a tensão no nó incógnita.

② Modelo mecânico e circuito equivalente pela analogia tipo 1:



$$\Downarrow$$

$$(JD + B)\omega = T(t) = K i_a(t) \Leftarrow e(t) = (LD + R)i$$

$$\boxed{J\ddot{\theta} + B\dot{\theta} = K i_a(t)}$$

Obtendo as equações do modelo por analogia tipo 1 e Lagrange:

Energia cinética: $T = \frac{J\omega^2}{2} + \frac{L_a i_a^2}{2} = \frac{J\dot{\theta}^2}{2} + \frac{L_a \dot{q}_a^2}{2}$

Energia potencial: $V = 0$ (sem mola e sem capacitor)

Dissipação de Rayleigh: $R = \frac{B\omega^2}{2} + \frac{R_a i_a^2}{2} = \frac{B\dot{\theta}^2}{2} + \frac{R_a \dot{q}_a^2}{2}$

Lagrangeano: $L = T - V$

Grau de liberdade θ:

$$\frac{\partial L}{\partial \dot{\theta}} = J\dot{\theta} \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = 0 \qquad \frac{\partial R}{\partial \dot{\theta}} = B\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = T(t) = K i_a = K \dot{q}_a$$

$$\boxed{J\ddot{\theta} + B\dot{\theta} = K\dot{q}_a}$$

Grau de liberdade q_a:

$$\frac{\partial L}{\partial \dot{q}_a} = L_a \dot{q}_a \longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_a}\right) = L_a \ddot{q}_a$$

$$\frac{\partial L}{\partial q_a} = 0 \qquad \frac{\partial R}{\partial \dot{q}_a} = R_a \dot{q}_a$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_a}\right) - \frac{\partial L}{\partial q_a} + \frac{\partial R}{\partial \dot{q}_a} = e_a(t) - \underbrace{e_b(t)}_{K_b(t)\,\dot{\theta}(t)}$$

$$\boxed{L_a \ddot{q}_a + R_a \dot{q}_a = e_a(t) - K_b(t)\dot{\theta}(t)}$$

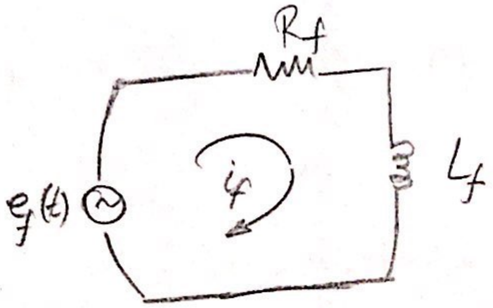
Se $L_a \approx 0$, então $\dot{q}_a = \dfrac{e_a - K_b\dot{\theta}}{R_a}$. Substituindo na 1ª equação:

$$J\ddot{\theta} + \left(B + K\frac{K_b}{R_a}\right)\dot{\theta} = K\frac{e_a}{R_a}$$

$$\ddot{\theta} + \underbrace{\left(\frac{BR_a + KK_b}{JR_a}\right)}_{\frac{1}{\tau_m}}\dot{\theta} = \underbrace{\frac{K}{R_aJ}}_{K_m} e_a \Rightarrow \boxed{\ddot{\theta} + \frac{1}{\tau_m}\dot{\theta} = K_m e_a}$$

b) Agora para o caso de eletroimã (caso b):

$i_a = cte$ $\qquad B(t) = k_f i_f(t) \Rightarrow T(t) = K i_f(t)$



→ Circuito do estator

Obtemos as equações por analogia do tipo I e equações de Lagrange.

Energia cinética: $T = \frac{J\omega^2}{2} + \frac{L_f i_f^2}{2} = \frac{J\dot{\theta}^2}{2} + \frac{L_f \dot{q}_f^2}{2}$

Energia potencial: $V = 0$ (Novamente sem mola nem capacitor)

Dissipação de Rayleigh: $R = \frac{B\omega^2}{2} + \frac{R_f i_f^2}{2} = \frac{B\dot{\theta}^2}{2} + \frac{R_f \dot{q}_f^2}{2}$

Grau de liberdade θ:

$$\frac{\partial L}{\partial \dot{\theta}} = J\dot{\theta} \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = 0 \qquad \frac{\partial R}{\partial \dot{\theta}} = B\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = T(t) = K i_f(t)$$

$$\boxed{J\ddot{\theta} + B\dot{\theta} = K\dot{q}_f(t)}$$

Grau de liberdade q_f:

$$\frac{\partial L}{\partial \dot{q}_f} = L_f \dot{q}_f \longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_f}\right) = L_f \ddot{q}_f$$

$$\frac{\partial L}{\partial q_f} = 0 \qquad \frac{\partial R}{\partial \dot{q}_f} = R_f \dot{q}_f$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_f}\right) - \frac{\partial L}{\partial \dot{q}_f} + \frac{\partial R}{\partial \dot{q}_f} = e_f(t)$$

$$\boxed{L_f \ddot{q}_f + R_f \dot{q}_f = e_f(t)} \Rightarrow \dot{q}_f = \frac{e_f(t)}{R_f + L_f D}$$

Se $L_f \approx 0$, então $\dot{q}_f \approx \dfrac{e_f(t)}{R_f}$.

Substituindo na primeira equação diferencial:

$$J\ddot{\theta} + B\dot{\theta} = K\frac{e_f(t)}{R_f}$$

$$\ddot{\theta} + \underbrace{\frac{B}{J}}_{\frac{1}{\tau_J}}\dot{\theta} = \underbrace{\frac{K}{JR_f}}_{K_m} e_f(t) \longrightarrow \boxed{\ddot{\theta} + \frac{1}{\tau_J}\dot{\theta} = K_m e_f(t)}$$

Se L_f não for desprezível:

$$J\ddot{\theta} + B\dot{\theta} = \left(\frac{K}{R_f + L_f D}\right) e_f(t)$$

$$\ddot{\theta} + \frac{B\dot{\theta}}{J} = \frac{K}{JR_f} \cdot \frac{1}{\left(1 + \frac{L_f}{R_f} D\right)} e_f(t)$$

$$\boxed{\ddot{\theta} + \frac{1}{\tau_J}\dot{\theta} = \frac{K_m}{1 + \tau_f D} e_f(t)}$$

onde $\tau_f = \dfrac{L_f}{R_f}$.