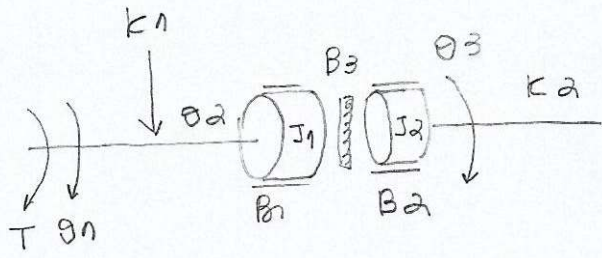
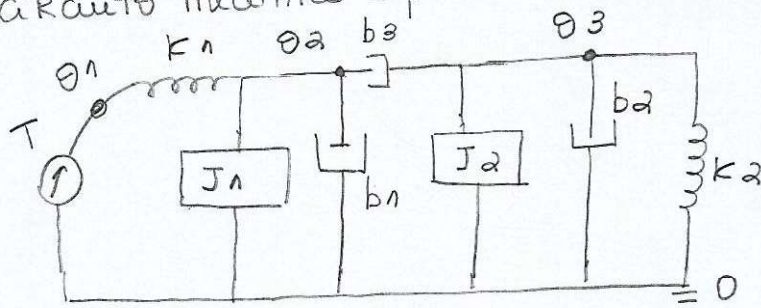


Ex. 1

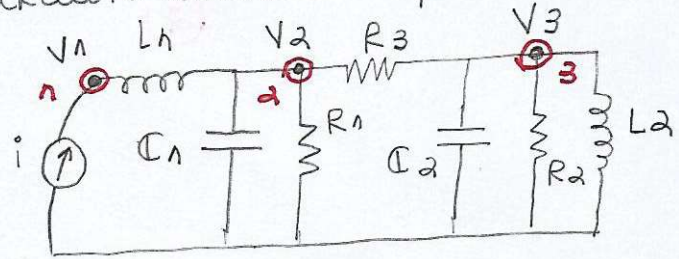
Modelo matemático por analogia para o mod físico do sistema



Circuito mecânico equivalente



Circuito elétrico equivalente



Resolução circuito

Nó 2:

$$V_2 \left(\frac{1}{L_1 D} + \frac{1}{R_1} + C_1 D + \frac{1}{R_3} \right) - V_3 \left(\frac{1}{R_3} \right) = i$$

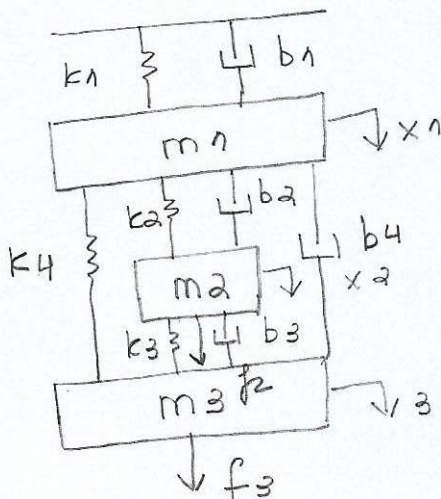
Nó 3

$$V_3 \left(\frac{1}{R_3} + \frac{1}{R_2} + C_2 D + \frac{1}{L_2 D} \right) - V_2 \left(\frac{1}{R_3} \right) = 0$$

Equações matemáticas utilizando analogia:

$$\begin{cases} \theta_2 k_1 + \theta_2 b_1 + J_1 \ddot{\theta}_2 + \theta_2 b_3 - \theta_3 b_3 = T(t) \\ \theta_3 b_2 + \theta_3 k_2 + J_2 \ddot{\theta}_3 + \theta_3 k_2 - \theta_2 b_3 = 0 \end{cases}$$

Ex 2.



1) Lagrange

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{m_3 \dot{x}_3^2}{2}$$

$$V = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_2 - x_1)^2}{2} + \frac{k_3 (x_3 - x_2)^2}{2} + \frac{k_4 (x_3 - x_1)^2}{2}$$

$$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b_3 (\dot{x}_3 - \dot{x}_2)^2}{2} + \frac{b_4 (\dot{x}_3 - \dot{x}_1)^2}{2}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \frac{\partial W}{\partial q_i}$$

$$\begin{cases} i = x/LD \\ i = v \times CD \end{cases}$$

para $q_i = x_1$:

$$m \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) + k_4 (x_1 - x_3) + b_1 \dot{x}_1 + b_2 (\dot{x}_1 - \dot{x}_2) + b_4 (\dot{x}_1 - \dot{x}_3) = 0$$

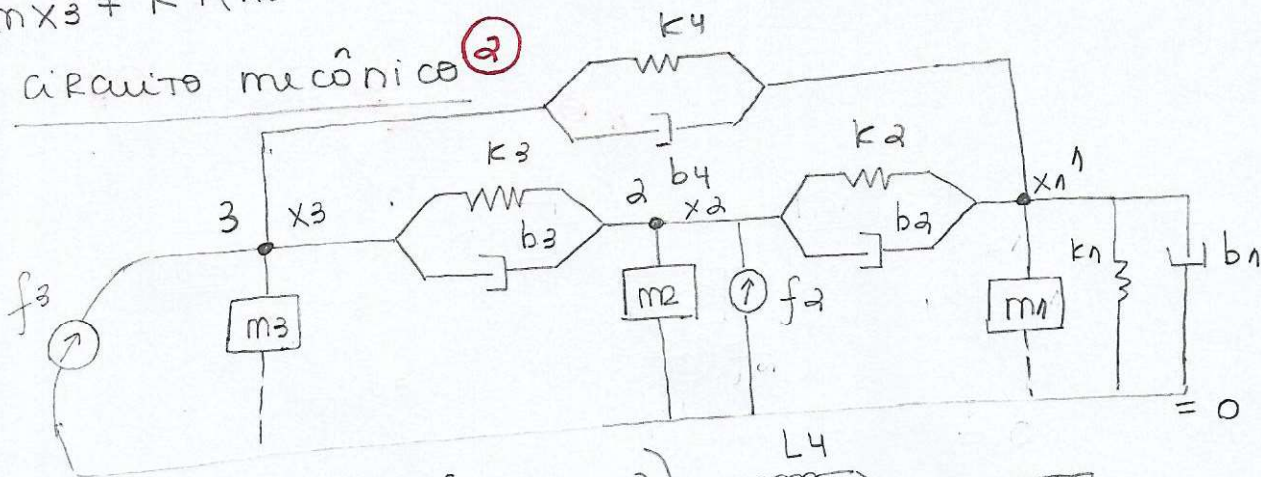
para $q_i = x_2$:

$$m \ddot{x}_2 + k_2 (x_2 - x_1) + k_3 (x_2 - x_3) + b_2 (\dot{x}_2 - \dot{x}_1) + b_3 (\dot{x}_2 - \dot{x}_3) = f_2$$

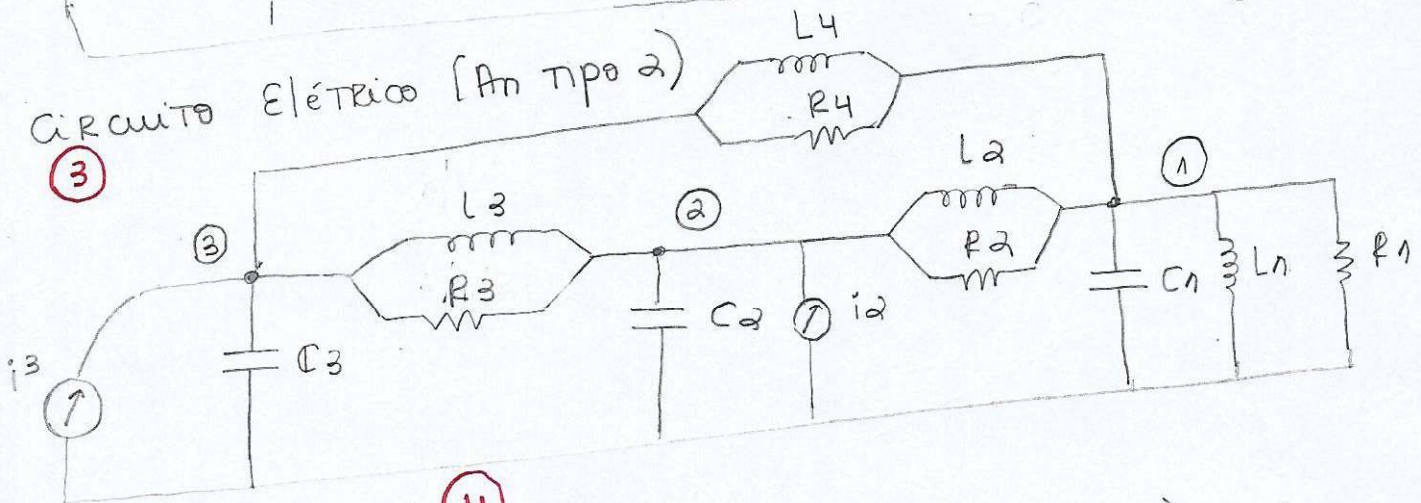
para $q_i = x_3$:

$$m \ddot{x}_3 + k_4 (x_3 - x_1) + k_3 (x_3 - x_2) + b_3 (\dot{x}_3 - \dot{x}_2) + b_4 (\dot{x}_3 - \dot{x}_1) = f_3$$

Circuito mecânico (2)



Circuito Elétrico (An tipo 2)



Equacionamento (4)

Nó 3

$$V_3 \left(C_3 D + \frac{1}{R_3} + \frac{1}{L_3 D} \right) - V_2 \left(\frac{1}{R_3} + \frac{1}{L_3 D} \right) - V_1 \left(\frac{1}{L_4 D} + \frac{1}{R_4} \right) = i_3$$

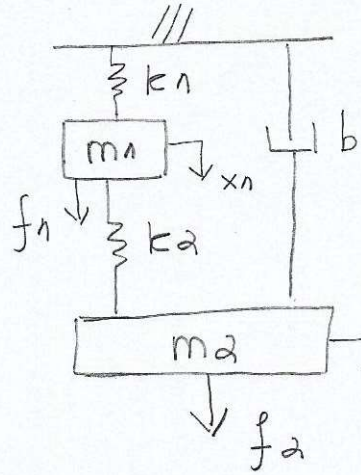
Nó 2

$$V_2 \left(C_2 D + \frac{1}{R_3} + \frac{1}{L_3 D} + \frac{1}{L_2 D} + \frac{1}{R_2} \right) - V_3 \left(\frac{1}{R_3} + \frac{1}{L_3 D} \right) - V_1 \left(\frac{1}{L_2 D} + \frac{1}{R_2} \right) = i_2$$

Nó 1

$$V_1 \left(C_1 D + \frac{1}{R_1} + \frac{1}{L_1 D} + \frac{1}{R_2} + \frac{1}{L_2 D} + \frac{1}{R_4} + \frac{1}{L_4 D} \right) - V_3 \left(\frac{1}{R_4} + \frac{1}{L_4 D} \right) - V_2 \left(\frac{1}{R_2} + \frac{1}{L_2 D} \right) = 0$$

$$v_2 (m_3 D + b_3 + k_3/D + k_2/D + b_2) - v_3 (b_3 + k_3/D) - v_1 (b_2 + k_2/D) = f_2$$
$$v_1 (m_3 D + b_1 + k_1/D + b_2 + k_2/D + b_4 + k_4/D) - v_3 (b_4 + k_4/D) - v_2 (b_2 + \frac{k_2}{D}) = c$$



$$L = T - V = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} - \frac{k_1 x_1^2}{2} - \frac{k_a (x_2 - x_1)^2}{2}$$

$$R = b \dot{x}_2^2 / 2$$

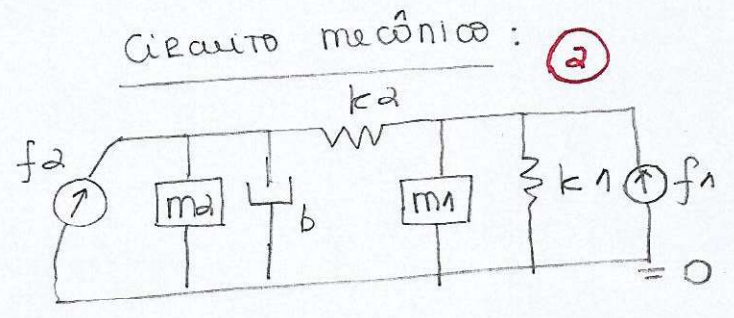
Equacionamento:

$$m_1 \ddot{x}_1 + k_1 x_1 + k_a (x_1 - x_2) = f_1$$

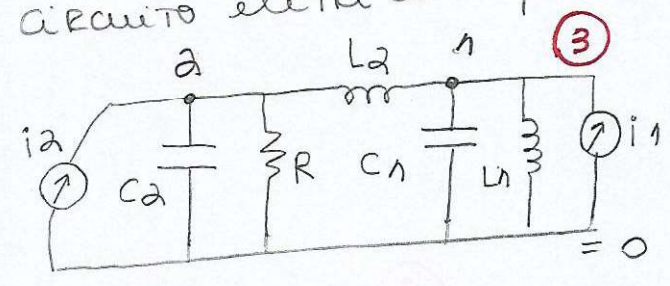
$$q = x_2$$

$$m_2 \ddot{x}_2 + k_a (x_2 - x_1) + b \dot{x}_2 = f_2$$

Circuito elétrico Equivalente:



Circuito mecânico: 2



Equacionamento:

$$\text{No 1: } v_2 \left(c_2 D + \frac{1}{R} + \frac{1}{L_2 D} \right) - v_1 \left(\frac{1}{L_2 D} \right) = i_2$$

$$\text{No 2: } v_1 \left(c_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D} \right) - v_2 \left(\frac{1}{L_1 D} \right) = i_1$$

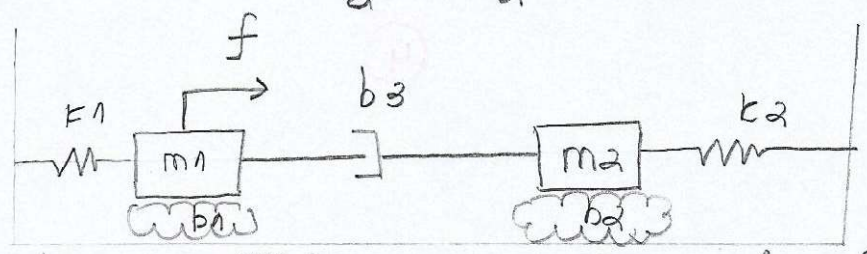
Equacionamento por analogia:

$$v_1 (m_1 D + k_1 / D + k_2 / D) - v_1 (k_1 / D) = f_1 \quad 5$$

$$v_2 (m_2 D + b + k_2 / D) - v_1 (k_2 / D) = f_2$$

ex 5.

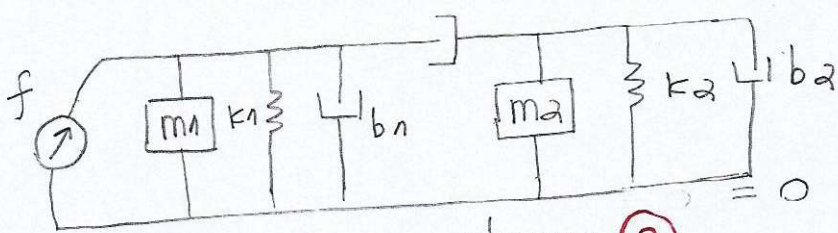
$$L = T - V = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} - \frac{k_1 x_1^2}{2} - \frac{k_a x_a^2}{2}; R = b_3 \frac{(\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 \dot{x}_2^2}{2}$$



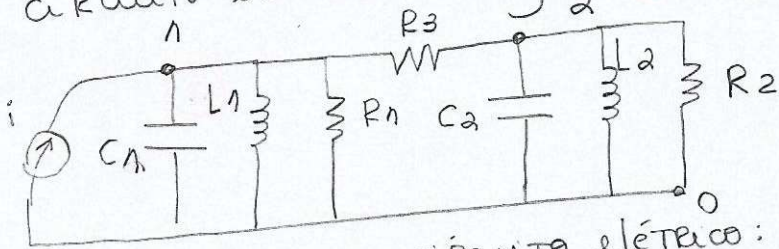
$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = \Sigma Q$$

$$\text{Equacionamento: } q = x_1 \rightarrow m_1 \ddot{x}_1 + k_1 x_1 + b_3 (\dot{x}_1 - \dot{x}_2) + b_1 \dot{x}_1 = f \quad 1$$

$$q = x_2 \rightarrow m_2 \ddot{x}_2 + k_2 x_2 + b_3 (\dot{x}_2 - \dot{x}_1) + b_2 \dot{x}_2 = 0$$



Circuito elétrico análogo: 3



Equacionamento circuito elétrico: analogia

Nó 1:

$$V_1 \left(C_1 D + \frac{1}{L_1 D} + \frac{1}{R_1} + \frac{1}{R_3} \right) - V_2 \left(\frac{1}{R_3} \right) = i \quad (4)$$

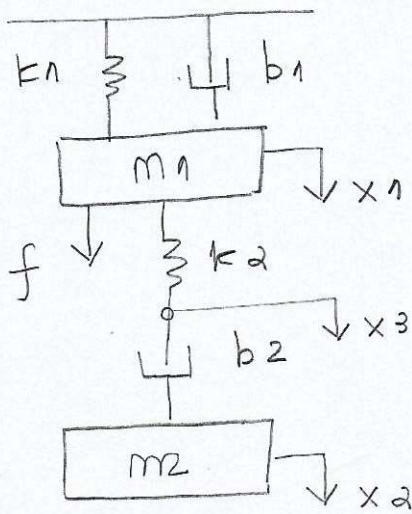
Nó 2:

$$V_2 \left(C_2 D + \frac{1}{L_2 D} + \frac{1}{R_2} + \frac{1}{R_3} \right) - V_1 \left(\frac{1}{R_3} \right) = 0$$

$$V_1 (m_1 D + k_1 / D + b_1 + b_3) - v_2 b_3 = f \quad (5)$$

$$v_2 (m_2 D + k_2 / D + b_2 + b_3) - v_1 b_3 = 0$$

ex. 6



Lagrange

$$L = T - V = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} \quad (1)$$

$$- \frac{k_1 x_1^2}{2} - \frac{k_2 (x_1 - x_3)^2}{2}$$

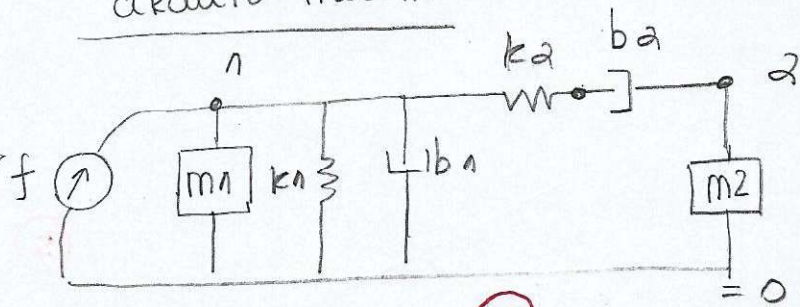
$$R = \frac{b_2 (x_2 - x_3)^2}{2}$$

Equacionamento:

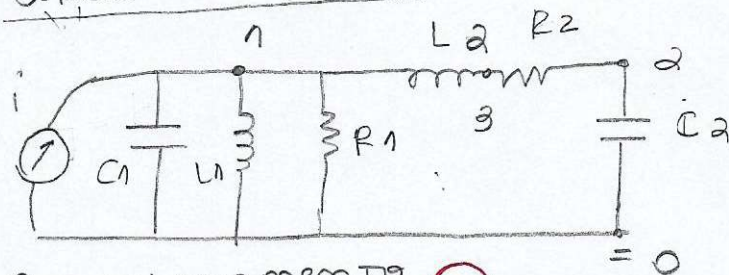
$$m_1 \ddot{x}_1 + k_1 x_1 - k_2 (x_1 - x_3) = f$$

$$\ddot{x}_2 + b_2 (x_2 - x_3) = 0$$

Circuito mecânico: 2



Circuito elétrico 3



Equacionamento circuito: 4

$$V_1 (C_1 D + 1/L_1 D + 1/R_1 + 1/L_2 D) - V_3 (1/L_2 D) = i$$

$$V_2 (C_2 D + 1/R_2) - V_3 (1/R_2) = 0$$

Circuito mecânico:

$$v_1 (m_1 D + k_1 / D + b_1 + k_2 / D) - v_3 (k_2 / D) = f$$

$$v_2 (m_2 D + b_2) - v_3 b_2 = 0 \quad (5)$$