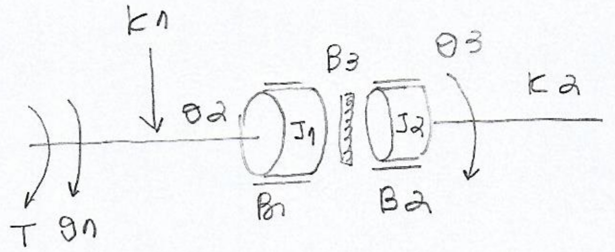
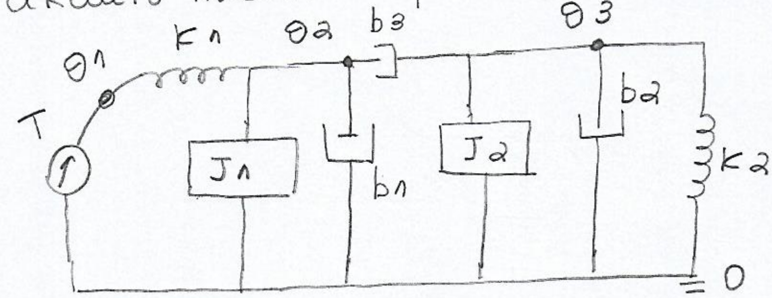
Lista Modelagem – 9348644

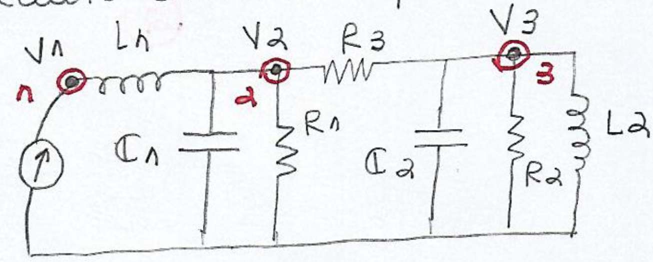
Ex. 1

Modelo matemático por analogia para o mod físico do sistema



circuito mecânico equivalente ⟶ circuito elétrico equivalente





- Resolução circuito

Nó 2:

$$V_2\left(1/L_1D + 1/R_1 + C_1D + 1/R_3\right) - V_3\left(1/R_3\right) = \vec{i}$$

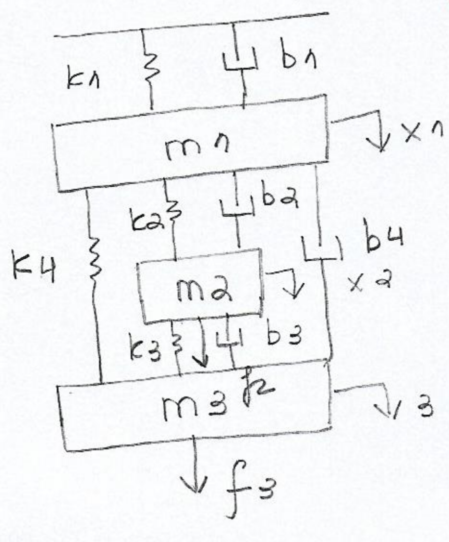
Nó 3

$$V_3\left(1/R_3 + 1/R_2 + C_2D + 1/L_2D\right) - V_2\left(1/R_3\right) = 0$$

- Equações matemáticas utilizando analogia:

$$\begin{cases} \theta_2 k_1 + \dot{\theta}_2 b_1 + J_1 \ddot{\theta}_2 + \dot{\theta}_2 b_3 - \dot{\theta}_3 b_3 = T(t) \\ \dot{\theta}_3 b_2 + \dot{\theta}_3 R_2 + J_2 \ddot{\theta}_3 + \theta_3 k_2 - \dot{\theta}_2 b_3 = 0 \end{cases}$$

ex 2.



1 Lagrange

$$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2} + \frac{m_3\dot{x}_3^2}{2}$$

$$V = \frac{k_1x_1^2}{2} + \frac{k_2(x_2-x_1)^2}{2} + \frac{k_3(x_3-x_2)^2}{2} + \frac{k_4(x_3-x_1)^2}{2}$$

$$R = \frac{b_1\dot{x}_1^2}{2} + \frac{b_2(\dot{x}_2-\dot{x}_1)^2}{2} + \frac{b_3(\dot{x}_3-\dot{x}_2)^2}{2} + \frac{b_4(\dot{x}_3-\dot{x}_1)^2}{2}$$

Equacionamento

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = 0$$

$$\begin{cases} R * i = V \rightarrow i = V/R \\ i = V/LD \\ i = V \times CD \end{cases}$$

para $q_i = x_1$:

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) + k_4(x_1 - x_3) + b_1\dot{x}_1 + b_2(\dot{x}_1 - \dot{x}_2) + b_4(\dot{x}_1 - \dot{x}_3) = 0$$

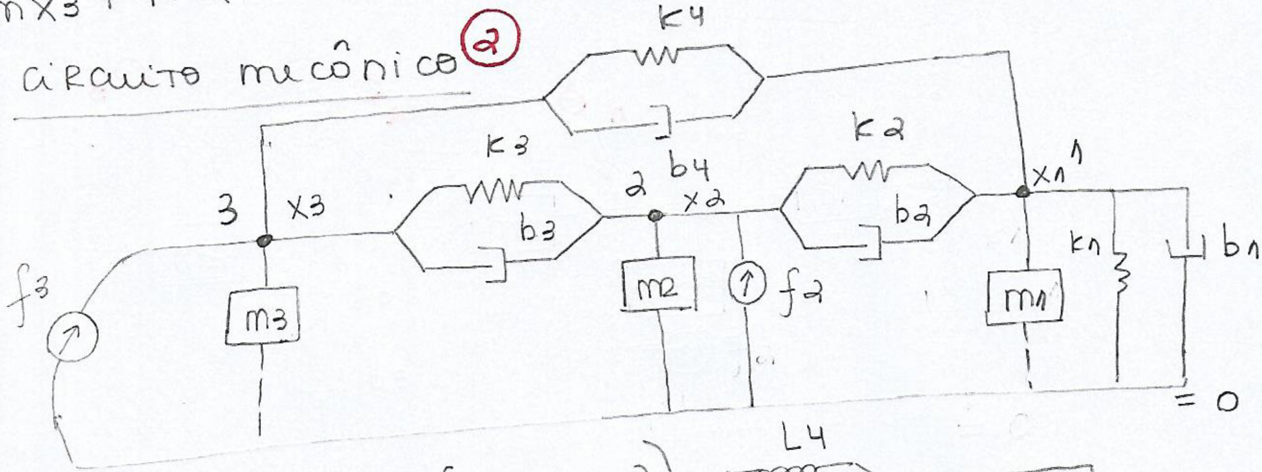
para $q_i = x_2$:

$$m\ddot{x}_2 + k_2(x_2 - x_1) + k_3(x_2 - x_3) + b_2(\dot{x}_2 - \dot{x}_1) + b_3(\dot{x}_2 - \dot{x}_3) = f_2$$

para $q_i = x_3$

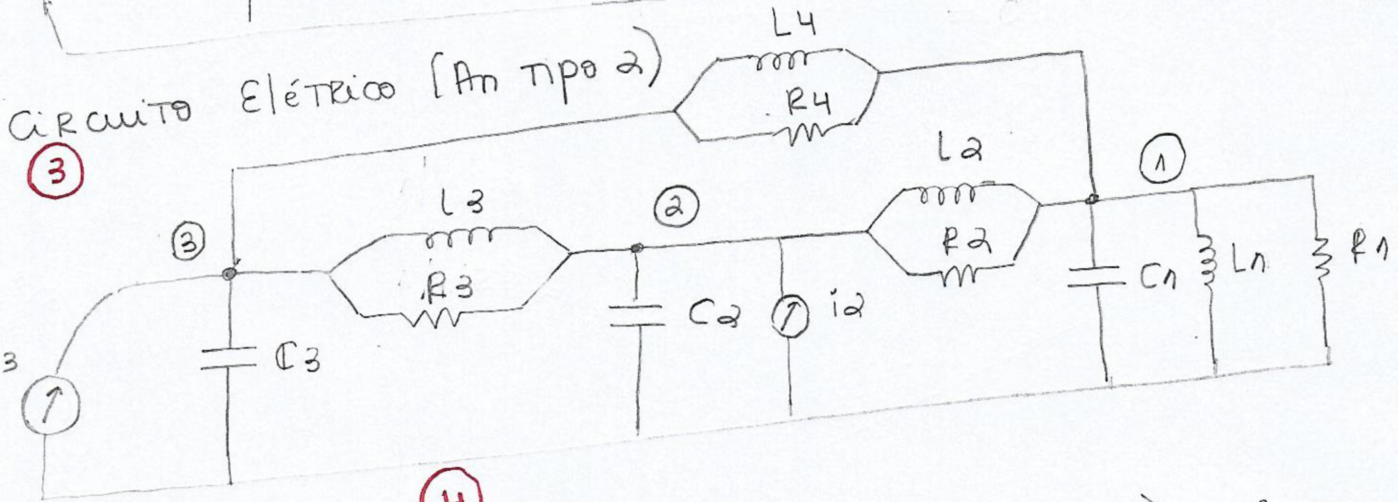
$$m\ddot{x}_3 + k_4(x_3 - x_1) + k_3(x_3 - x_2) + b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1) = f_3$$

circuito mecânico (2)



circuito Elétrico (An tipo 2)

(3)



Equacionamento (4)

Nó 3

$$V_3\left(C_3D + \frac{1}{R_3} + \frac{1}{L_3D} + 1/R_4 + 1/L_4D\right) - V_2\left(\frac{1}{R_3} + \frac{1}{L_3D}\right) - V_1\left(\frac{1}{L_4D} + \frac{1}{R_4}\right) = i_3$$

Nó 2

$$V_2\left(C_2D + \frac{1}{R_3} + \frac{1}{L_3D} + \frac{1}{L_2D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{R_3} + \frac{1}{L_3DR}\right) - V_1\left(\frac{1}{L_2D} + \frac{1}{R_2}\right) = i_2$$

Nó 1

$$V_1\left(C_1D + \frac{1}{R_1} + \frac{1}{L_1D} + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{R_4} + \frac{1}{L_4D}\right) - V_3\left(\frac{1}{R_4} + \frac{1}{L_4D}\right) - V_2\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) = 0$$

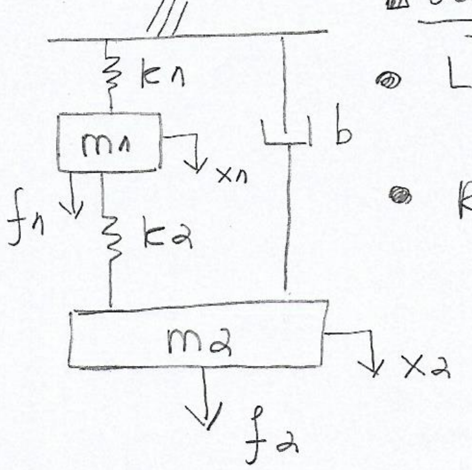
equazionalmente

$$v_3\left(m_3 D + b_3 + k_3/D + b_4 + k_4/D\right) - v_2\left(b_3 + k_3/D\right) - v_1\left(b_4 + k_4/D\right) = f_3$$

$$v_2\left(m_2 D + b_3 + k_3/D + k_2/D + b_2\right) - v_3\left(b_3 + k_3/D\right) - v_1\left(b_2 + k_2/D\right) = f_2$$

$$v_1\left(m_3 D + b_1 + k_1/D + b_2 + k_2/D + b_4 + k_4/D\right) - v_3\left(b_4 + k_4/D\right) - v_2\left(b_2 + \frac{k_2}{D}\right) = 0$$

ex4.



Lagrange (1)

$$L = T - V = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} - \frac{k_1 x_1^2}{2} - \frac{k_2 (x_2 - x_1)^2}{2}$$

$$R = b \dot{x}_2^2 / 2$$

Equacionamento:

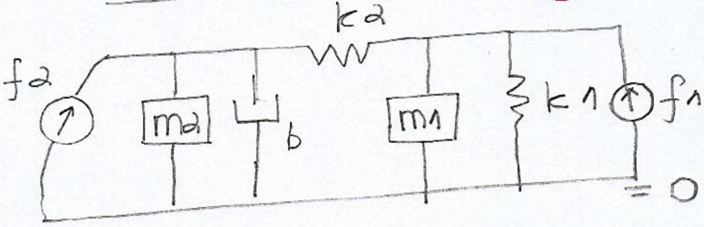
$q = x_1$

$$m \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = f_1$$

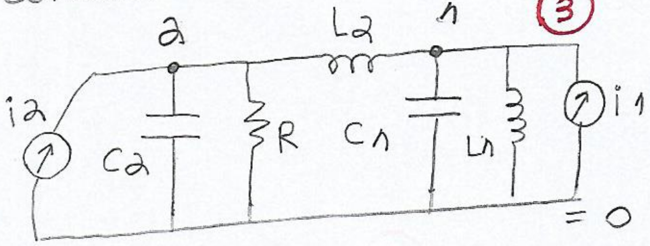
$q = x_2$

$$m \ddot{x}_2 + k_2 (x_2 - x_1) + b \dot{x}_2 = f_2$$

Circuito mecânico: (2)



Circuito elétrico Equivalente: (3)



Equacionamento: (4)

Nó 1:

$$V_2 \left(C_2 D + \frac{1}{R} + \frac{1}{L_2 D} \right) - V_1 \left(\frac{1}{L_2 / D} \right) = i_2$$

Nó 2:

$$V_1 \left(C_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D} \right) - V_2 \left(\frac{1}{L_1 D} \right) = i_1$$

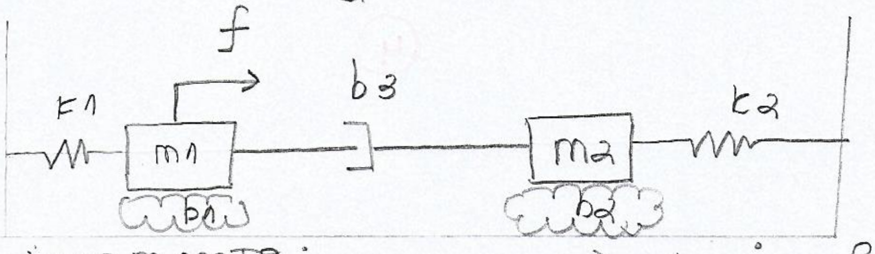
Equacionamento por analogia: (5)

$$v_1 (m_1 D + k_1 / D + k_2 / D) - v_1 (k_1 / D) = f_1$$

$$v_2 (m_2 D + b + k_2 / D) - v_1 (k_2 / D) = f_2$$

ex5.

$$L = T - V = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} - \frac{k_1 x_1^2}{2} - \frac{k_2 x_2^2}{2} ; \quad R = b_3 \frac{(\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 \dot{x}_2^2}{2}$$



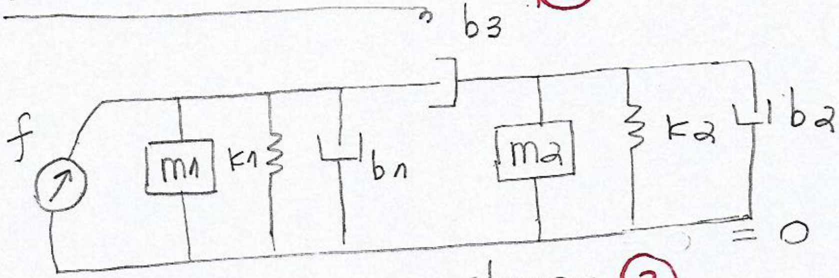
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = \Sigma Q$$

Equacionamento: (1)

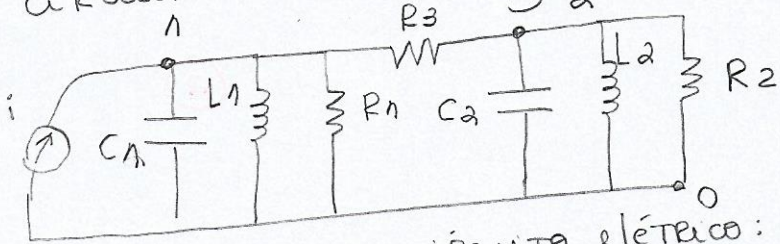
$q = x_1 \rightarrow m_1 \ddot{x}_1 + k_1 x_1 + b_3 (\dot{x}_1 - \dot{x}_2) + b_1 \dot{x}_1 = f$

$q = x_2 \rightarrow m_2 \ddot{x}_2 + k_2 x_2 + b_3 (\dot{x}_2 - \dot{x}_1) + b_2 \dot{x}_2 = 0$

circuito mecânico (2)



circuito elétrico análogo: (3)



Equacionamento circuito elétrico: analogia

Nó 1:

$$V_1\left(C_1D+\frac{1}{L_1D}+\frac{1}{R_1}+\frac{1}{R_3}\right)-V_2\left(\frac{1}{R_3}\right)=\vec{i}$$ (4)

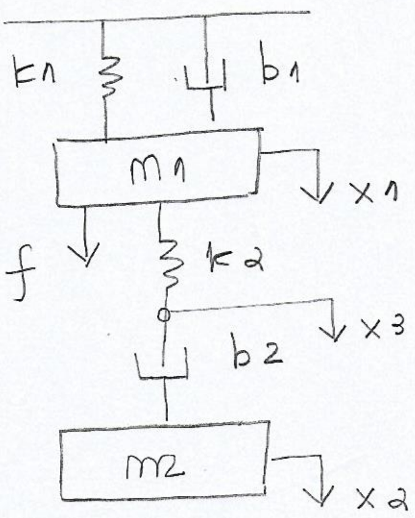
Nó 2:

$$V_2\left(C_2D+\frac{1}{L_2D}+\frac{1}{R_2}+\frac{1}{R_3}\right)-V_1\left(\frac{1}{R_3}\right)=0$$

$$v_1(m_1D+k_1/D+b_1+b_3)-v_2b_3=f$$ (5)

$$v_2(m_2D+k_2/D+b_2+b_3)-v_1b_3=0$$

ex. 6



Lagrange (1)

$$L=T-V=\frac{m_1\dot{x}_1^2}{2}+\frac{m_2\dot{x}_2^2}{2}-\frac{k_1x_1^2}{2}-\frac{k_2(x_1-x_3)^2}{2}$$

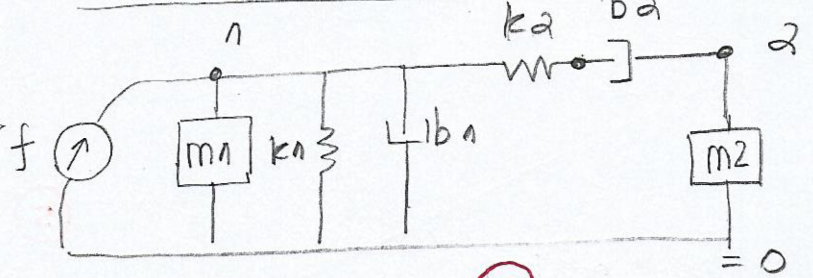
$$R=\frac{b_2(\dot{x}_2-\dot{x}_3)^2}{2}$$

Equacionamento:

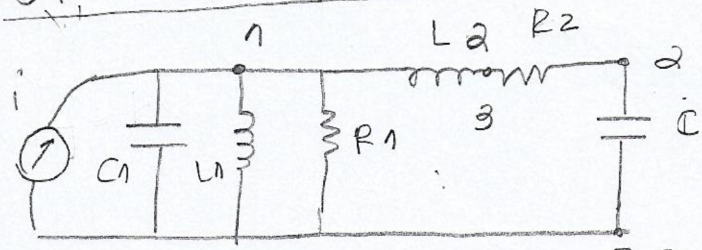
$$m\ddot{x}_1+k_1x_1-k_2(x_1-x_3)=f$$

$$m\ddot{x}_2+b_2(\dot{x}_2-\dot{x}_3)=0$$

circuito mecânico: (2)



Circuito Elétrico (3)



Equacionamento circuito: (4)

$$V_1(C_1D+1/L_1D+1/R_1+1/L_2D)-V_3(1/L_2D)=\vec{i}$$

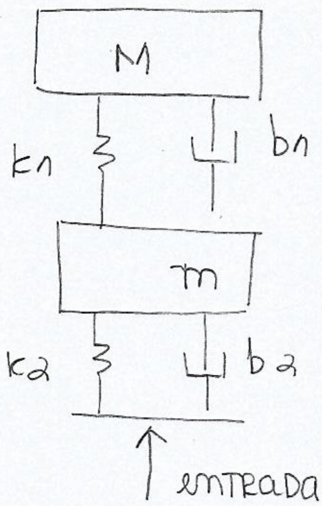
$$V_2(C_2D+1/R_2)-V_3(1/R_2)=0$$

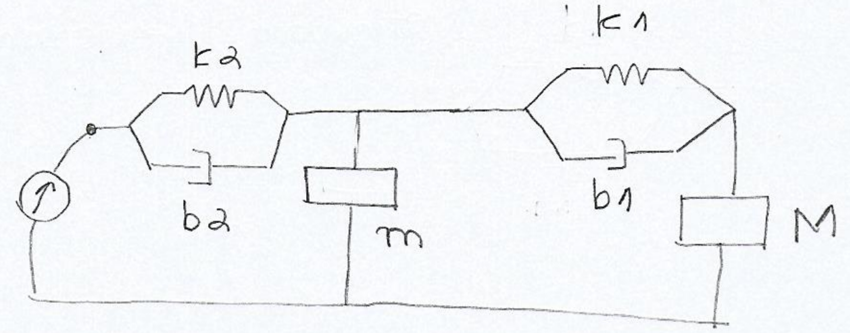
circuito mecânico: (5)

$$v_1(m_1D+k_1/D+b_1+k_2/D)-v_3(k_2/D)=\vec{f}$$

$$v_2(m_2D+b_2)-v_3b_2=0$$

ex 8.



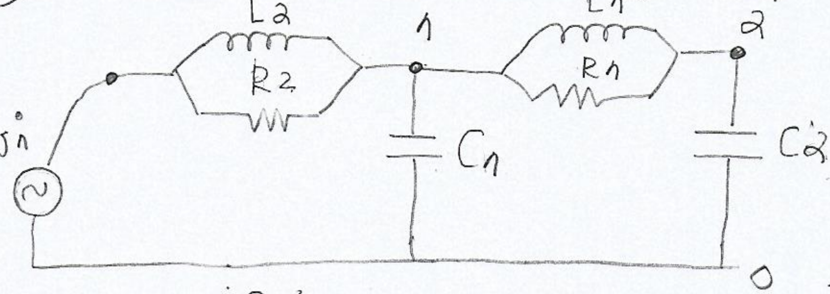


Eq. Elétrico:

$$V_1\left(C_1D+\frac{1}{L_2D}+\frac{1}{R_2}+\frac{1}{L_1D}+\frac{1}{R_1}\right)-v_r\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)-V_2\left(\frac{1}{L_1D}+\frac{1}{R_1}\right)=0$$

$$V_2\left(\frac{1}{L_1D}+\frac{1}{R_1}+C_2D\right)-V_1\left(\frac{1}{L_1D}+\frac{1}{R_1}\right)=0$$

(1) entrada = deslocamento (AN. 2)



Por analogia:

$$v_1(m_1D+k_2/D+b_2+k_1/D+b_1)-v_r(k_2/D+b_2)-v_2(k_1/D+b_1)=0$$

$$m_1\ddot{x}_1+k_2x_1+b_2\dot{x}_1+k_1x+b_1\dot{x}_1-k_1x_2-\dot{x}_2b=u(t)k_2+\dot{u}(t)b_2$$

ainda

$$m\ddot{x}_2+x_2k_1+\dot{x}_2b_1-\dot{x}_1b-x_1k_1=0$$