Escola Politécnica da USP

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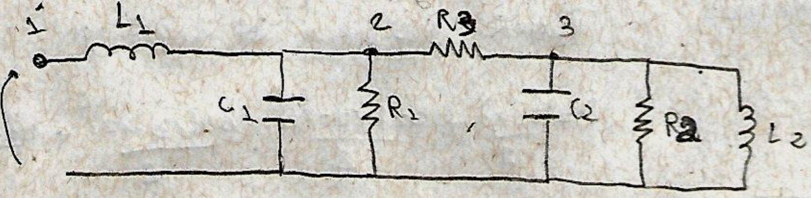
PME 3380 - Modelagem - Exs 3/09.

Ex 1) Equivalente elétrico do sistema rotativo:



* Aplicando a Lei dos nós em 1:

$$(V_1 - V_2)\cdot\frac{1}{L_1 D} = i(t) \xrightarrow{\text{Analogia}} \boxed{(\dot\theta_1 - \dot\theta_2)\cdot\frac{k_1}{D} = T(t)} \quad (I)$$

* Aplicando a Lei dos nós em 2:

$$V_2\left(C_1 D + \frac{1}{R_1}\right) + (V_2 - V_3)\frac{1}{R_3} - (V_1 - V_2)\cdot\frac{1}{L_1 D} = 0$$

$$-V_1\frac{1}{L_1 D} + V_2\left(C_1 D + \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{L_1 D}\right) - V_3\left(\frac{1}{R_3}\right) = 0$$

⇓ Analogia

$$-\frac{\dot\theta_1 k_1}{D} + \dot\theta_2\left(DJ_1 + b_1 + b_3 + \frac{k_1}{D}\right) - \dot\theta_3 b_3 = 0$$

$$J_1\ddot\theta_2 + \dot\theta_2(b_1 - b_3) - (\dot\theta_1 - \dot\theta_2)\frac{k_1}{D} - \dot\theta_3 b_3 = 0$$

$$\boxed{J_1\ddot\theta_2 + \dot\theta_2(b_1 - b_3) - \dot\theta_3 b_3 = T(t)}$$

(II)

* Aplicando a Lei dos nós em 3:

$$V_3\left(C_2 D + \frac{1}{R_2} + \frac{1}{L_2 D} - \frac{1}{R_3}\right) - V_2\frac{1}{R_3} = 0$$

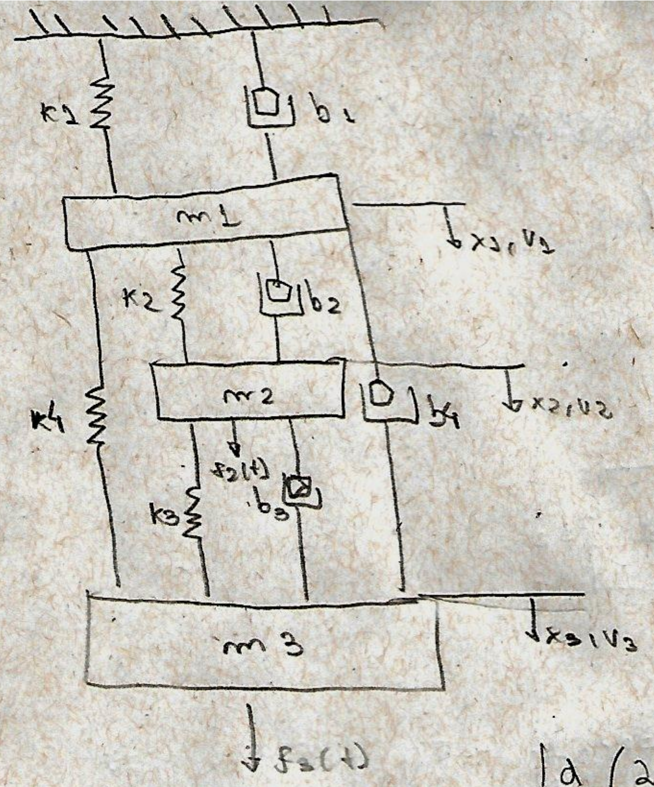
$$\dot\theta_3\left(J_2 D + b_2 + \frac{k_2}{D} - b_3\right) - \dot\theta_2 b_3 = 0$$

$$J_2\ddot\theta_3 + (b_2 - b_3)\dot\theta_3 + \dot\theta_3\frac{k_2}{D} - \dot\theta_2 b_3 = 0$$

$$\boxed{J_2\ddot\theta_3 + (b_2 - b_3)\dot\theta_3 + \theta_3 - b_3\dot\theta_2 = 0}$$

(III)

Ex2)



a) Energia cinética T

$$T = \frac{m_1}{2}\dot{x}_1^2 + \frac{m_2}{2}\dot{x}_2^2 + \frac{m_3}{2}\dot{x}_3^2$$

Energia potencial U:

$$U = \frac{k_1 x_1^2}{2} + \frac{k_2(x_2-x_1)^2}{2} + \frac{k_3(x_3-x_2)^2}{2} + \frac{k_4(x_3-x_1)^2}{2}$$

Função dissipativa de Rayleigh

$$R = \frac{b_1\dot{x}_1^2}{2} + \frac{b_2(\dot{x}_2-\dot{x}_1)^2}{2} + \frac{b_3(\dot{x}_3-\dot{x}_2)^2}{2} + \frac{b_4(\dot{x}_3-\dot{x}_1)^2}{2}$$

Da equação de Lagrange:

$$L = T - U$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = 0}$$

$$L = T - V = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2} + \frac{m_3\dot{x}_3^2}{2} - \frac{k_1x_1^2}{2} - \frac{k_2(x_2-x_1)^2}{2} - \frac{k_3(x_3-x_2)^2}{2} - \frac{k_4(x_3-x_1)^2}{2}$$

→ Para a coordenada x_1:

$$\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1 \qquad \frac{\partial L}{\partial x_1} = -k_1x_1 + k_2(x_2-x_1) + k_4(x_3-x_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1\dot{x}_1 - b_2(\dot{x}_2-\dot{x}_1) - b_4(\dot{x}_3-\dot{x}_1)$$

$$\therefore m_1\ddot{x}_1 + k_1x_1 - k_2(x_2-x_1) - k_4(x_3-x_1) + b_1\dot{x}_1 - b_2(\dot{x}_2-\dot{x}_1) - b_4(\dot{x}_3-\dot{x}_1) = 0$$

$$m_1\ddot{x}_1 + k_1x_1 - k_2x_2 + k_2x_1 - k_4x_3 + k_4x_1 + b_1\dot{x}_1 - b_2\dot{x}_2 + b_2\dot{x}_1 - b_4\dot{x}_3 + b_4\dot{x}_1 = 0$$

$$m_1\ddot{x}_1 + k_1x_1 - k_2x_2 + k_2x_1$$

$$\boxed{m_1\ddot{x}_1 + (k_1+k_2+k_4)x_1 + (b_1+b_2+b_4)\dot{x}_1 = k_2x_2 + k_4x_3 + b_2\dot{x}_2 + b_4\dot{x}_3}$$ //

→ Para a coordenada x_2:

$$\frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2 \qquad \frac{\partial R}{\partial \dot{x}_2} = b_2(\dot{x}_2-\dot{x}_1) - b_3(\dot{x}_3-\dot{x}_2)$$

$$\frac{\partial L}{\partial x_2} = -k_2(x_2-x_1) + k_3(x_3-x_2)$$

Daí: $m_2\ddot{x}_2 + k_2x_2 - k_2x_1 - k_3x_3 + k_3x_2 + b_2\dot{x}_2 - b_2\dot{x}_1 - b_3\dot{x}_3 + b_3\dot{x}_2 = f_2(t)$

$$\boxed{m_2\ddot{x}_2 + (k_2+k_3)x_2 + (b_2+b_3)\dot{x}_2 = k_2x_1 + b_2\dot{x}_1 + k_3x_3 + b_3\dot{x}_3 + f_2(t)}$$ //

→ Para a coordenada x_3:

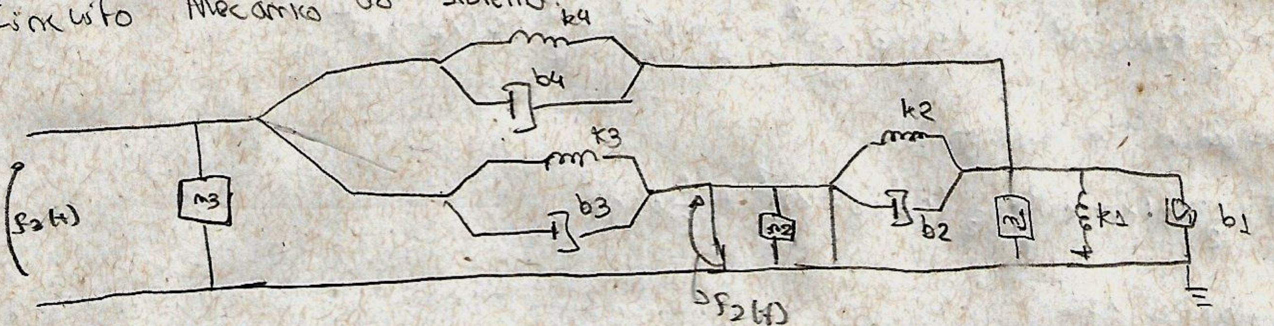
$$\frac{\partial L}{\partial \dot{x}_3} = m_3\dot{x}_3 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3\ddot{x}_3 \qquad \frac{\partial R}{\partial \dot{x}_3} = b_3(\dot{x}_3-\dot{x}_2)+b_4(\dot{x}_3-\dot{x}_1)$$

$$\frac{\partial L}{\partial x_3} = -k_3(x_3-x_2) - k_4(x_3-x_1)$$

Daí: $m_3\ddot{x}_3 + k_3x_3 - k_3x_2 + k_4x_3 - k_4x_1 + b_3\dot{x}_3 - b_3\dot{x}_2 + b_4\dot{x}_3 - b_4\dot{x}_1 = f_3(t)$

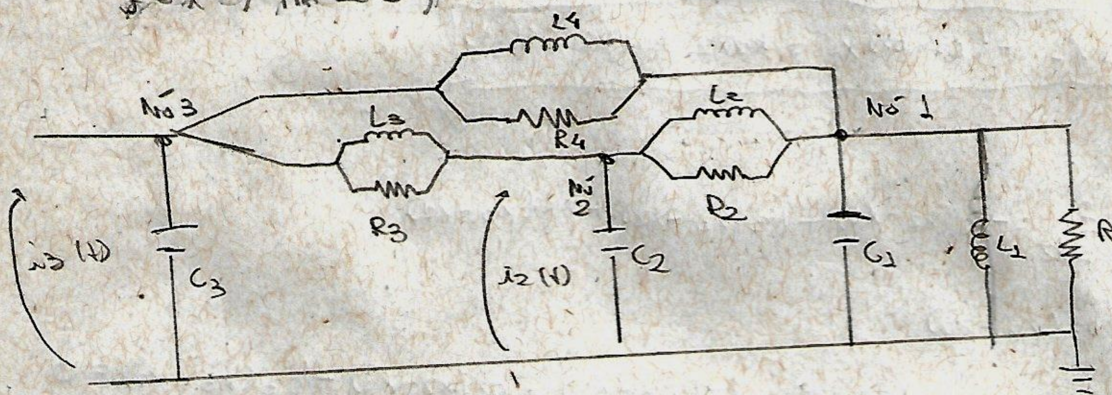
$$\boxed{m_3\ddot{x}_3 + (k_3+k_4)x_3 + (b_3+b_4)\dot{x}_3 = k_3x_2 + k_4x_1 + b_3\dot{x}_2 + b_4\dot{x}_1 + f_3(t)}$$ //

b) Circuito Mecânico do sistema:



c) Circuito elétrico equivalente

$f \Rightarrow i$, $m \Rightarrow C$, $b \Rightarrow R$, $k \Rightarrow L$



d) Aplicando as Leis de Kirchhoff para os nós 1, 2, 3 (Posições análogas a das massas)
↳ Método prático

* Nó 1:

$$V_1\left(C_1D + \frac{1}{R_1} + \frac{1}{L_1D} + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{R_4} + \frac{1}{L_4D}\right) - V_2\left(\frac{1}{R_2}+\frac{1}{L_2D}\right) - V_3\left(\frac{1}{R_4}+\frac{1}{L_4D}\right) = 0$$

* Nó 2:

$$V_2\left(C_2D + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{R_3} + \frac{1}{L_3D}\right) - V_1\left(\frac{1}{R_2}+\frac{1}{L_2D}\right) - V_3\left(\frac{1}{R_3}+\frac{1}{L_3D}\right) = i_2(t)$$

* Nó 3:

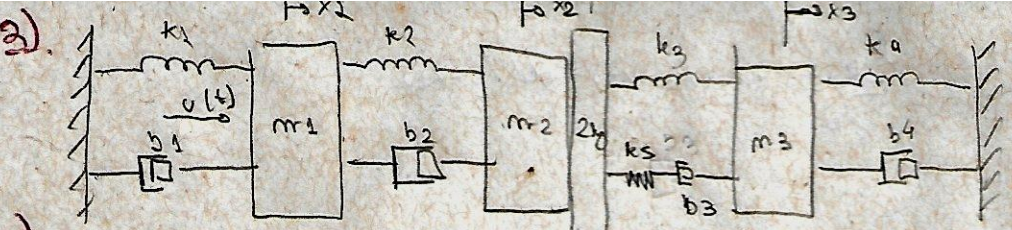
$$V_3\left(C_3D + \frac{1}{R_3} + \frac{1}{L_3D} + \frac{1}{R_4} + \frac{1}{L_4D}\right) - V_1\left(\frac{1}{R_4}+\frac{1}{L_4D}\right) - V_2\left(\frac{1}{R_3}+\frac{1}{L_3D}\right) = i_3(t)$$

e) Das analogias indicadas na letra c), se obtém as equações de movimento translacional: Nó 1 → massa 1 $\qquad i(t) \to f(t)$

Nó 2 → massa 2 $\qquad C \to m \qquad D \to \frac{d}{dt}$

Nó 3 → massa 3. $\qquad R \to b \qquad \frac{1}{D} \to \int dt$

$L \to k$

3) 

Dados: $m_1 = m_2 = m_3 = 1\ kg$

$b_1 = b_2 = b_3 = b_4 = 1\ Ns/m$

$k_1 = k_2 = k_3 = k_4 = k_5 = 1\ N/m$

a)

* Energia cinética: T:

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{(m_2+2)\dot{x}_2^2}{2} + \frac{m_3 \dot{x}_3^2}{2}$$

* Energia potencial U:

$$U = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{1}{2} k_3 (x_3 - x_2)^2 + \frac{1}{2} k_4 x_3^2 + \frac{1}{2} k_5 (x_3 - x_2)^2$$

* Função Dissipativa de Rayleigh

$$R = \frac{1}{2} b_1 \dot{x}_1^2 + \frac{1}{2} b_2 (\dot{x}_2 - \dot{x}_1)^2 + \frac{1}{2} (b_3 \dot{x}_3 - \dot{x}_2)^2 + \frac{1}{2} b_4 \dot{x}_3^2$$

* Na coordenada x_1:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \quad , \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2 (x_2 - x_1) \quad \frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_2 (\dot{x}_2 - \dot{x}_1)$$

$$\therefore \boxed{m_1 \ddot{x}_1 + (k_1 + k_2) x_1 + (b_1 + b_2) \dot{x}_1 = k_2 x_2 + b_2 \dot{x}_2 + u(t)}$$

//

* Na coordenada x_2:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = (m_2 + 2) \ddot{x}_2 \qquad \frac{\partial L}{\partial x_2} = -k_2 (x_2 - x_1) + k_3 (x_3 - x_2) + k_5 (x_3 - x_2)$$

$$\frac{\partial R}{\partial \dot{x}_2} = b_2 (\dot{x}_2 - \dot{x}_1) - b_3 (\dot{x}_3 - \dot{x}_2)$$

$$\therefore (m_2 + 2) \ddot{x}_2 + k_2 (x_2 - x_1) - k_3 (x_3 - x_2) - k_5 (x_3 - x_2) + b_2 (\dot{x}_2 - \dot{x}_1) - b_3 (\dot{x}_3 - \dot{x}_2) = 0$$

$$\boxed{(m_2 + 2) \ddot{x}_2 + (k_2 + k_3 + k_5) x_2 + (b_2 + b_3) \dot{x}_2 = k_2 x_1 + k_3 x_3 + k_5 x_3 + b_2 \dot{x}_1 + b_3 \dot{x}_3}$$

//

* Na coordenada x_3:

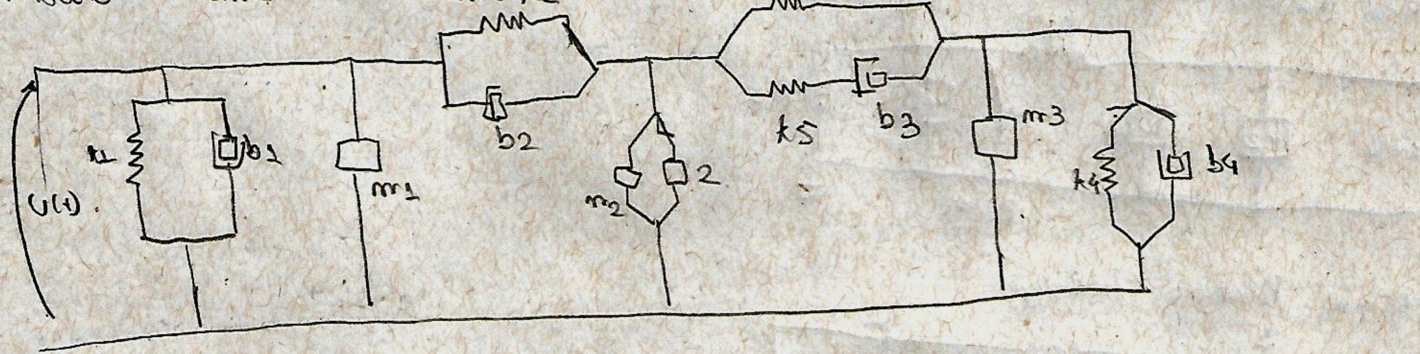
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3 \ddot{x}_3 \qquad \frac{\partial L}{\partial x_3} = +k_3 (x_3 - x_2) + k_4 x_3 + k_5 (x_3 - x_2) \qquad \frac{\partial R}{\partial \dot{x}_3} = b_3 (\dot{x}_3 - \dot{x}_2) + b_4 \dot{x}_3$$

$$\therefore m_3 \ddot{x}_3 + k_3 (x_3 - x_2) + k_4 x_3 + k_5 (x_3 - x_2) + b_3 (\dot{x}_3 - \dot{x}_2) + b_4 \dot{x}_3 = 0$$

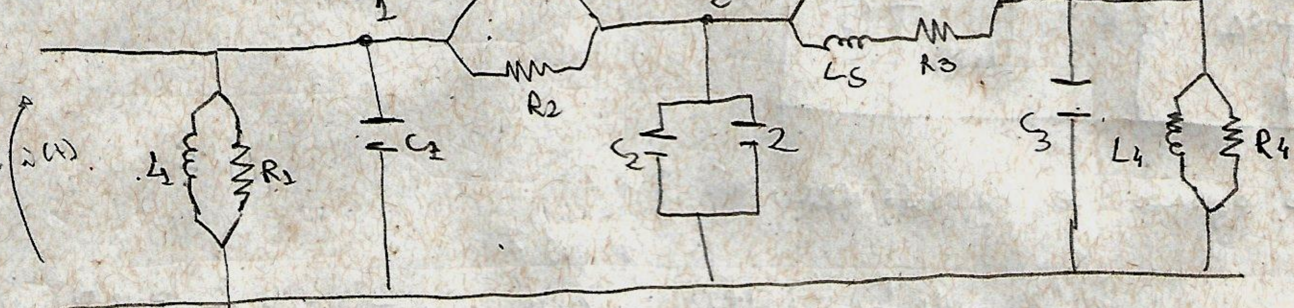
$$\boxed{m_3 \ddot{x}_3 + (k_3 + k_4 + k_5) x_3 + (b_3 + b_4) \dot{x}_3 = + k_3 x_2 + k_5 x_2 + b_3 \dot{x}_2}$$

//

b) Modelo Mecanico do Sistema



c) Analogo eletrico:



D) Aplicando as Leis de Kirchoff nos nós 1, 2 e 3:

- Nó 1: $\left(\frac{1}{L_1D} + \frac{1}{R_1} + C_1D + \frac{1}{L_2D} + \frac{1}{R_2}\right)V_1 - \left(\frac{1}{L_2D} + \frac{1}{R_2}\right)V_2 = i(t)$

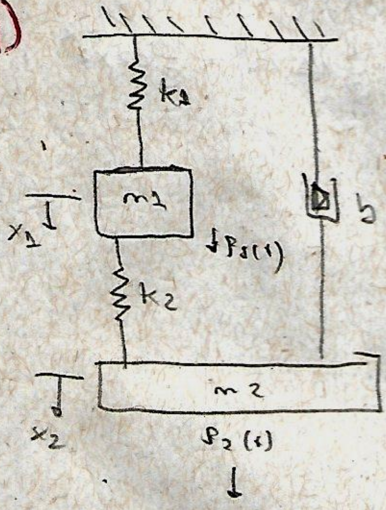
- Nó 2: $\left(\frac{1}{L_2D} + \frac{1}{R_2} + C_2D + 2D + \frac{1}{L_3D} + \frac{1}{L_5D} + \frac{1}{R_3}\right)V_2 - \left(\frac{1}{L_2D} + \frac{1}{R_2}\right)V_1 - \left(\frac{1}{L_3D} + \frac{1}{L_5D} + \frac{1}{R_3}\right)V_3 = 0.$

- Nó 3: $\left(\frac{1}{L_3D} + \frac{1}{L_5D} + \frac{1}{R_3} + C_3D + \frac{1}{L_4D} + \frac{1}{R_4}\right)V_3 - \left(\frac{1}{L_3D} + \frac{1}{L_5D} + \frac{1}{R_3}\right)V_2 = 0$

E) Analogia conseguida substituindo

$i(t) \to v(t)$; $R \to b$, $C \to m$; $L \to k$

4)



a) Energia Cinetica: $T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$

Energia Potencial: $U = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2$

Função Dissipativa de Rayleigh: $R = \frac{1}{2}b\dot{x}_2^2$

- Para x_1:

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1 \quad \frac{\partial L}{\partial x_1} = -k_1x_1 + k_2(x_2 - x_1) \quad \frac{\partial R}{\partial \dot{x}_1} = 0$

$\therefore \boxed{m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = f_1(t)}$

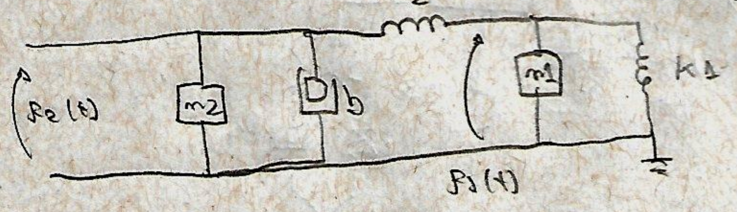
- Para x_2:

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2 \quad \frac{\partial L}{\partial x_2} = -k_2(x_2 - x_1) \quad \frac{\partial R}{\partial \dot{x}_2} = b\dot{x}_2$

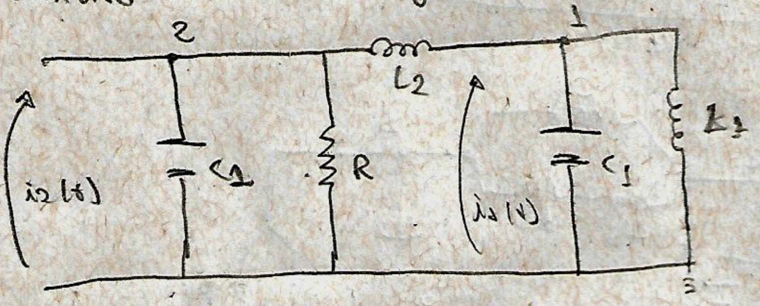
$\therefore \boxed{m_2\ddot{x}_2 + k_2x_2 - k_2x_1 + b\dot{x}_2 = f_2(t)}$

(5)

b) Circuito Mecanico Equivalente



c) Circuito Elétrico Análogo.



d) Aplicando as equações de Kirchhoff nos nós 1 e 2:

* Nó 1:

$$\boxed{V_1\left(C_1D + \frac{1}{L_1D} + \frac{1}{L_2D}\right) - V_2\frac{1}{L_2D} = i_1(t)}$$

* Nó 2:

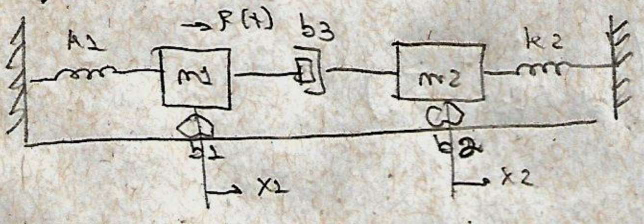
$$\boxed{V_2\left(C_2D + \frac{1}{R} + \frac{1}{L_2D}\right) - V_1\frac{1}{L_2D} = i_2(t)}$$

e) Usando a analogia: $C \to m$, $L \to k$, $R \to b$, $i \to f$, $v \to \dot{x}$

$\Rightarrow$ 1: $m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = f_1(t)$

2: $m_2\ddot{x}_2 + b\dot{x}_2 + k_2x_2 - k_2x_1 = f_2(t)$

5)



d) Energia cinética T:

$$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2}$$

Energia Potencial U:

$$U = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$$

Função de Rayleigh:

$$R = \frac{1}{2}b_3(\dot{x}_2 - \dot{x}_1)^2 + \frac{1}{2}b_1\dot{x}_1^2 + \frac{1}{2}b_2\dot{x}_2^2$$

• Para x_1:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1 \qquad \frac{\partial R}{\partial \dot{x}_1} = b_1\dot{x}_1 - b_3(\dot{x}_2 - \dot{x}_1)$$

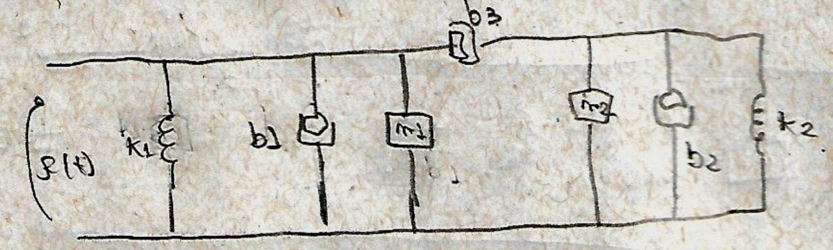
$$\frac{\partial L}{\partial x_1} = -k_1x_1 \qquad \therefore \boxed{m_1\ddot{x}_1 + k_1x_1 + (b_1+b_3)\dot{x}_1 = b_3\dot{x}_2 + f(t)} //$$

• Para x_2:

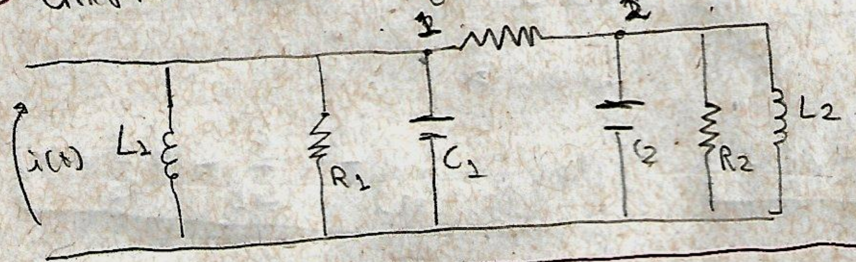
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2 \qquad \frac{\partial R}{\partial \dot{x}_2} = b_2\dot{x}_2 + b_3(\dot{x}_2 - \dot{x}_1)$$

$$\frac{\partial L}{\partial x_2} = -k_2x_2 \qquad \therefore \boxed{m_2\ddot{x}_2 + k_2x_2 + (b_2+b_3)\dot{x}_2 = b_3\dot{x}_1} //$$

b) Circuito Mecânico do Sistema



c) Circuito elétrico Análogo:



d) Aplicando as Leis de Kirchhoff:

1:

$$\boxed{\left(\frac{1}{L_1 D} + \frac{1}{R_1} + C_1 D + \frac{1}{R_3}\right)V_1 - \left(\frac{1}{R_3}\right)V_2 = i(t)}$$

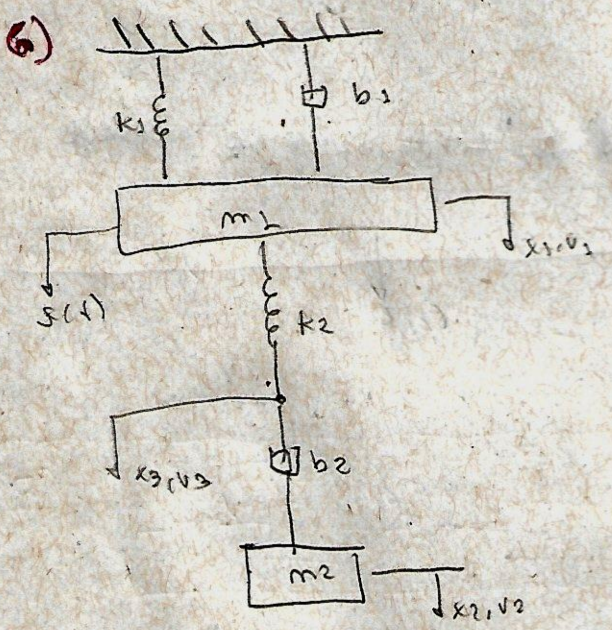
2:

$$\boxed{\left(\frac{1}{L_2 D} + \frac{1}{R_2} + C_2 D + \frac{1}{R_3}\right)V_2 - \frac{1}{R_3}V_1 = 0}$$

e) Fazendo a Analogia

$m \to C$, $F \to i$, $L \to k$, $R \to b$, $V \to \dot{x}$

6)



a) Energia cinética

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$$

Energia potencial

$$U = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_3 - x_1)^2}{2}$$

Função Dissipativa de Rayleigh

$$R = \frac{b_2(\dot{x}_3 - \dot{x}_2)^2}{2} + \frac{b_1 \dot{x}_1^2}{2}$$

- Para x_1:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \qquad \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2(x_3 - x_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1$$

$$\therefore \boxed{m_1 \ddot{x}_1 + (k_1 + k_2)x_1 + b_1 \dot{x}_1 = k_2 x_3 + f(t)} //$$

- Para x_2:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \qquad \frac{\partial L}{\partial x_2} = 0, \quad \frac{\partial R}{\partial \dot{x}_2} = -b_2(\dot{x}_3 - \dot{x}_2) \quad \therefore \boxed{m_2 \ddot{x}_2 + b_2 \dot{x}_2 = b_2 \dot{x}_3} //$$

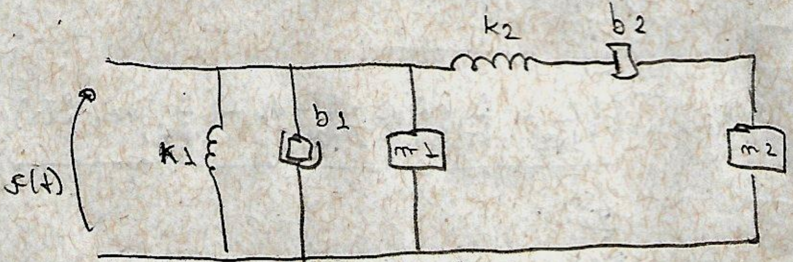
- Para x_3:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = 0 \qquad \frac{\partial L}{\partial x_3} = -k_2(x_3 - x_1) \qquad \frac{\partial R}{\partial \dot{x}_3} = b_2(\dot{x}_3 - \dot{x}_2)$$

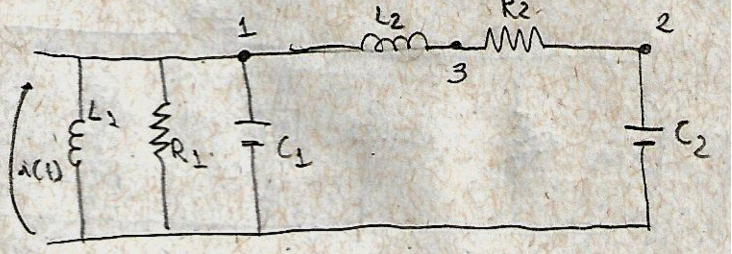
$$\therefore k_2 x_3 - k_2 x_1 + b_2 \dot{x}_3 - b_2 \dot{x}_2 = 0$$

$$\boxed{k_2 x_3 + b_2 \dot{x}_3 = k_2 x_1 + b_2 \dot{x}_2} //$$

b) Circuito Mecânico do Sistema



c) Circuito Elétrico Análogo



d) Aplicando as Leis de Kirchhoff nos nós 1, 2 e 3:

→ Nó 1:

$$\boxed{V_1\left(C_1D + \frac{1}{R_1} + \frac{1}{L_1D} + \frac{1}{L_2D}\right) - V_3\,\frac{1}{LD} = i(t)}$$

→ Nó 3

$$\boxed{V_3\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) - V_1\,\frac{1}{L_2D} - V_2\,\frac{1}{R_2} = 0}$$

→ Nó 2:

$$\boxed{V_2\left(C_2D + \frac{1}{R_2}\right) - V_3\,\frac{1}{R_2} = 0}$$

e) Aplicando a analogia $m \to C$, $V \to \dot{x}$, $i \to f$, $R \to b$, $L \to k$, obtém-se novamente as equações para o sistema mecânico.

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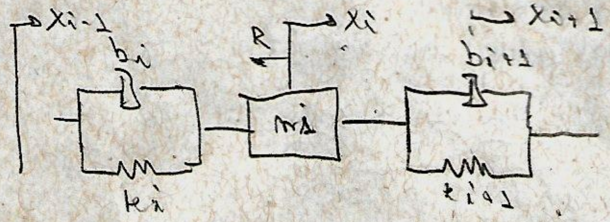
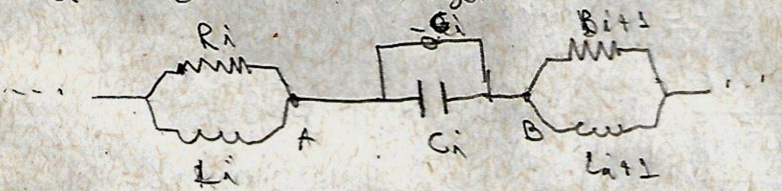
7)



• Sistema Elétrico análogo



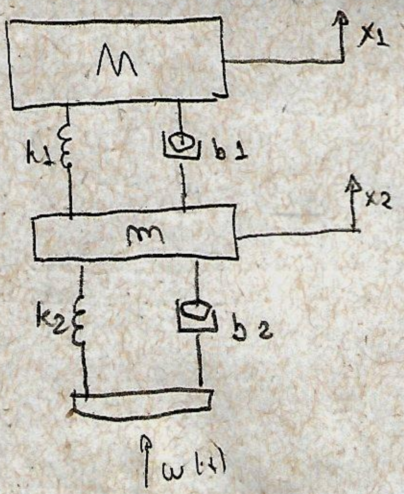
→ Usando as Leis de Kirchhoff para o nó A e B:

$$*A)\ \left(\frac{1}{R_i} + \frac{1}{L_iD} + C_iD\right)V_i - \left(\frac{1}{R_i} + \frac{1}{L_iD}\right)V_{i-1} - C_iD\,V_{i+1} = -G_i$$

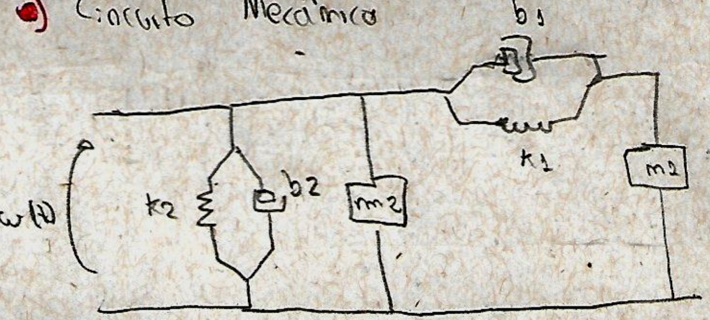
$$*B)\ \left(\frac{1}{R_{i+1}} + \frac{1}{L_{i+1}D} + C_iD\right)V_{i+1} - \left(\frac{1}{R_{i+1}} + \frac{1}{L_{i+1}D}\right)V_{i+1} - C_iD\,V_i = -G_i$$

→ Usando o princípio da indução, pode-se generalizar para os de novo $(i+m)$ vagões. Depois, usa-se a analogia tipo II para retornar o sistema de equações ao problema mecânico original.

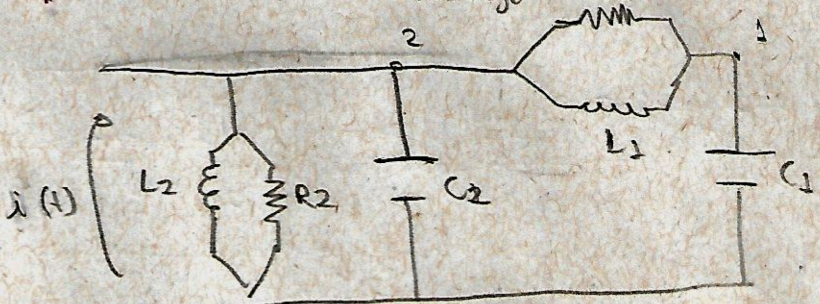
8)



Circuito Mecânico



Circuito Eletro Análogo



Usando as Leis de Kirchhoff nos nós 1 e 2:

$$V_1\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{R_1}\right) - V_2\left(\frac{1}{R_1} + \frac{1}{L_1 D}\right) = 0 \quad (I)$$

$$V_2\left(C_2 D + \frac{1}{L_1 D} + \frac{1}{R_1} + \frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_1\left(\frac{1}{R_1} + \frac{1}{L_1 D}\right) = i(t)$$

Colocando na analogia mecânica novamente

de I) $\rightarrow \boxed{m_1\ddot{x}_1 + b_1\dot{x}_1 - b_1\dot{x}_2 + k_1x_1 - k_1x_2 = 0}$ //

II) $\rightarrow \boxed{m_2\ddot{x}_1 - b_1\dot{x}_1 + (b_1+b_2)\dot{x}_2 - k_1x_1 + (k_1+k_2)x_2 = w(t)}$ //