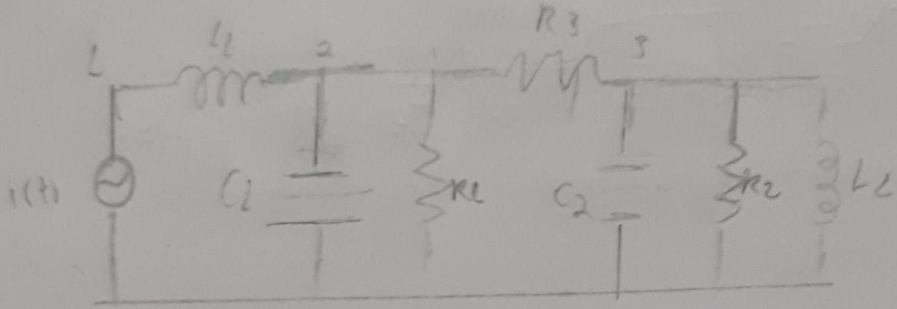
1-



Nó 1: $V_1\left(\frac{1}{L_1 D}\right) - V_2\left(\frac{1}{L_1 D}\right) = i(t)$

Nó 2: $V_2\left(C_1 D + \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{L_1 D}\right) - V_1\left(\frac{1}{L_1 D}\right) - V_3\left(\frac{1}{R_3}\right) = 0$

Nó 3: $V_3\left(C_2 D + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2 D}\right) - V_2\left(\frac{1}{R_3}\right) = 0$

Usando a analogia:

1- $K_1(\theta_1 - \theta_2) = T(t)$

2- $J_1\ddot{\theta}_2 + (B_1 + B_3)\dot{\theta}_2 + K_1\theta_2 - K_1\theta_1 - B_3\dot{\theta}_3 = 0$

3- $J_2\ddot{\theta}_3 + (B_2 + B_3)\dot{\theta}_3 + K_2\theta_3 - B_3\dot{\theta}_2 = 0$

2- Por Lagrange:

$T = \frac{1}{2}(m_1\dot{x}_1^2 + m_2\dot{x}_2^2 + m_3\dot{x}_3^2)$

$V = m_1 g x_1 + m_2 g x_2 + m_3 g x_3 + \frac{1}{2}(K_1 x_1^2 + K_2(x_2 - x_1)^2 + K_3(x_3 - x_2)^2 + K_4(x_3 - x_1)^2)$

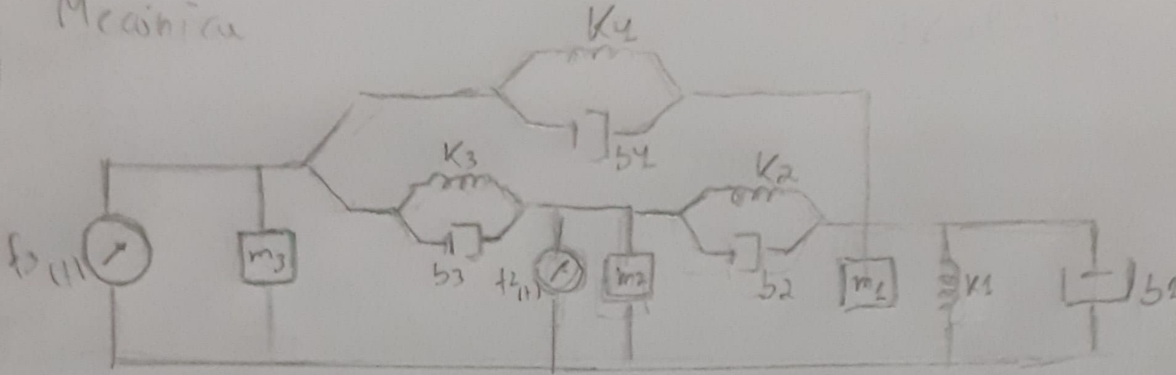
$R = \frac{1}{2}(b_1\dot{x}_1^2 + b_2(\dot{x}_2 - \dot{x}_1)^2 + b_3(\dot{x}_3 - \dot{x}_2)^2 + b_4(\dot{x}_3 - \dot{x}_1)^2)$

P/x_1 → $m_1\ddot{x}_1 + (K_1 + K_2 + K_3 + K_4)x_1 + (b_1 + b_2 + b_4)\dot{x}_1 = K_2 x_2 + b_2\dot{x}_2 + K_4 x_3 + b_4\dot{x}_3$

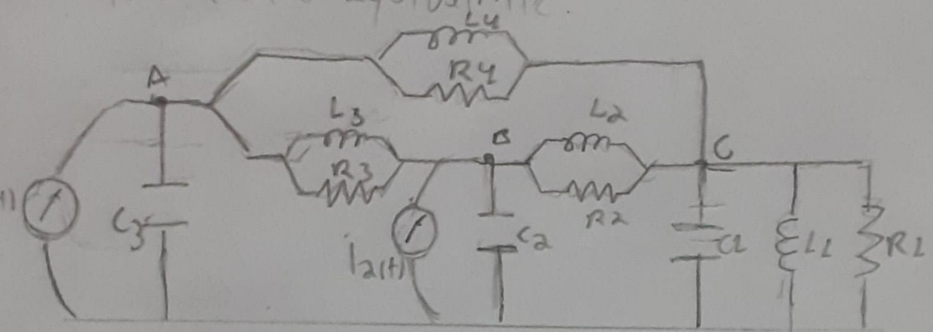
P/x_2 → $m_2\ddot{x}_2 + (K_2 + K_3)x_2 + (b_2 + b_3)\dot{x}_2 = K_2 x_1 + b_2\dot{x}_1 + K_3 x_3 + b_3\dot{x}_3 + f_2(t)$

P/x_3 → $m_3\ddot{x}_3 + (K_3 + K_4)x_3 + (b_3 + b_4)\dot{x}_3 = K_3 x_2 + b_3\dot{x}_2 + K_4 x_1 + b_4\dot{x}_1 + f_3(t)$

b) Mecanica



c) Circuito Elétrico Equivalente.



d) Nó A: $V_A\left(C_3D+\frac{1}{R_3}+\frac{1}{L_3D}+\frac{1}{R_4}+\frac{1}{L_4D}\right)-V_C\left(\frac{1}{R_4}+\frac{1}{L_4D}\right)-V_B\left(\frac{1}{R_3}+\frac{1}{L_3D}\right)=i_3(t)$

Nó B: $V_B\left(C_2D+\frac{1}{R_2}+\frac{1}{L_2D}+\frac{1}{R_3}+\frac{1}{L_3D}\right)-V_C\left(\frac{1}{R_2}+\frac{1}{L_2D}\right)-V_A\left(\frac{1}{R_3}+\frac{1}{L_3D}\right)=i_2(t)$

Nó C: $V_C\left(C_1D+\frac{1}{R_1}+\frac{1}{L_1D}+\frac{1}{R_2}+\frac{1}{L_2D}+\frac{1}{R_4}+\frac{1}{L_4D}\right)-V_B\left(\frac{1}{R_2}+\frac{1}{L_2D}\right)-V_A\left(\frac{1}{R_4}+\frac{1}{L_4D}\right)=0$

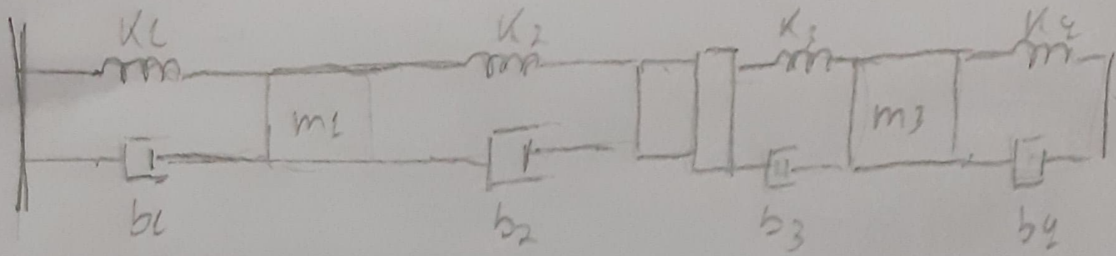
e) Equações

Nó A - $m_3\ddot{x}_3+(b_3+b_4)\dot{x}_3+(K_3+K_4)x_3=f_3(t)+b_4\dot{x}_1+K_4x_1+b_3\dot{x}_2+K_3x_2$

Nó B - $m_2\ddot{x}_2+(b_2+b_3)\dot{x}_2+(K_2+K_3)x_2=f_2(t)+b_2\dot{x}_1+K_2x_1+b_3\dot{x}_3+K_3x_3$

Nó C - $m_1\ddot{x}_1+(b_1+b_2+b_4)\dot{x}_1+(K_1+K_2+K_4)x_1=b_2\dot{x}_2+K_2x_2+b_4\dot{x}_3+K_4x_3$

3-



a) Por Lagrange:

$$T = \frac{1}{2}\left(m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2 + m_3 \dot{x}_3^2\right)$$

$$V = \frac{1}{2}\left(K_1 x_1^2 + K_2 (x_2 - x_1)^2 + K_3 (x_3 - x_2)^2 + K_4 x_3^2\right)$$

$$R = \frac{1}{2}\left(b_1 \dot{x}_1^2 + b_2 (\dot{x}_2 - \dot{x}_1)^2 + b_3 (\dot{x}_3 - \dot{x}_2) + K_4 \dot{x}_3^2\right)$$

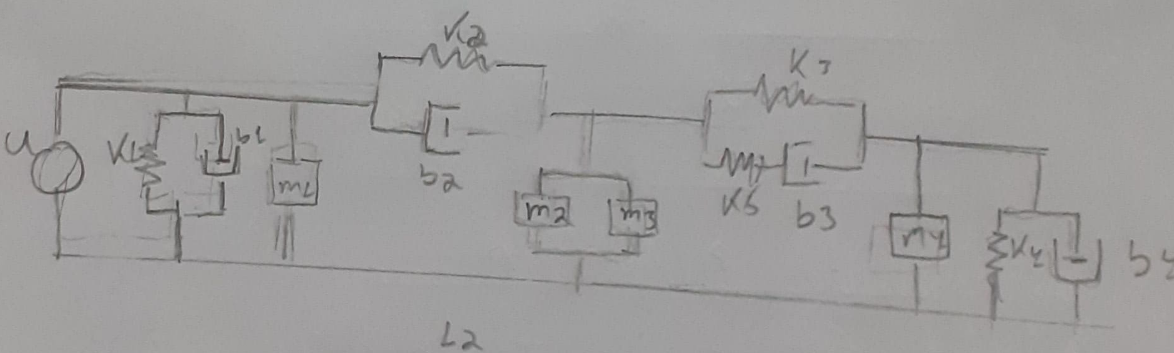
Equações:

$$m_1 \ddot{x}_1 + (b_1 + b_2)\dot{x}_1 - b_2 \dot{x}_2 + (K_1 + K_2)x_1 - K_2 x_2 = u$$

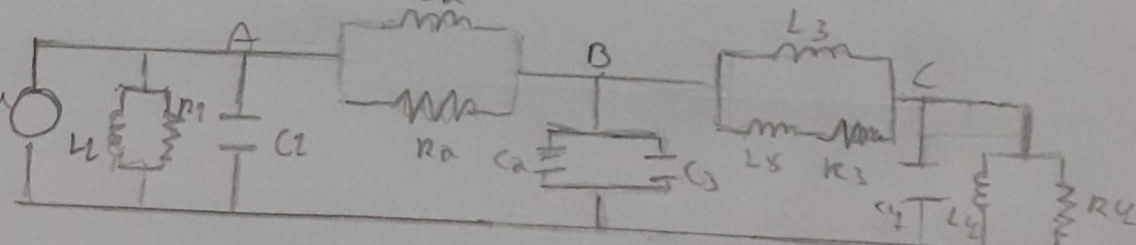
$$m_2 \ddot{x}_2 - b_2 \dot{x}_1 + (b_2 + b_3)\dot{x}_2 - b_3 \dot{x}_3 - K_2 x_1 + (K_2 + K_3)x_2 - K_3 x_3 = 0$$

$$m_3 \ddot{x}_3 - b_3 \dot{x}_2 + (b_3 + b_4)\dot{x}_3 - K_3 x_2 + (K_3 + K_4)x_3 = 0$$

b)



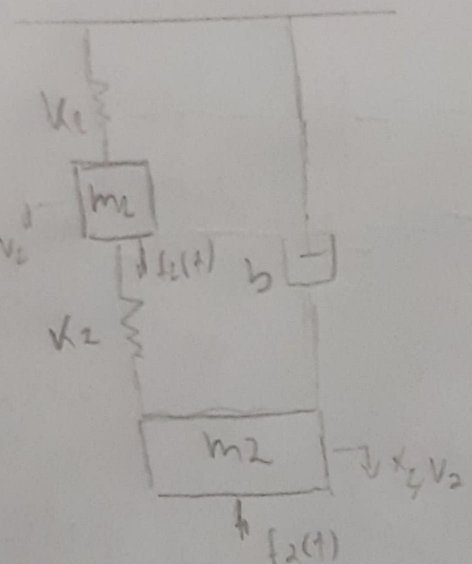
c)



Nó A: $\left(\frac{1}{L_1 D} + \frac{1}{R_1} + C_1 D + \frac{1}{L_2 D} + \frac{1}{R_2}\right) V_A - \left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) V_B = u(t)$

Nó B: $\left(\frac{1}{L_2 D} + \frac{1}{R_2} + C_2 D + C_3 D + \frac{1}{L_3 D} + \frac{1}{L_5 D} + \frac{1}{R_3}\right) V_B - \left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) V_A - \left(\frac{1}{L_3 D} + \frac{1}{L_5 D} + \frac{1}{R_3}\right) V_C = 0$

Nó C: $\left(\frac{1}{L_3 D} + \frac{1}{L_5 D} + \frac{1}{R_3} + \frac{1}{L_4 D} + \frac{1}{R_4} + C_4 D\right) V_C - \left(\frac{1}{L_3 D} + \frac{1}{L_5 D} + \frac{1}{R_3}\right) V_B = 0$



$$T = \frac{1}{2}\left(m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2\right)$$

$$V = \frac{1}{2}\left(k_1 x_1^2 + k_2 (x_2 - x_1)^2\right)$$

$$R = \frac{1}{2} b \dot{x}_2^2$$

$P/x_1 \to$ $\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1$ $\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$

$\frac{\partial L}{\partial x_1} = -k_1 x_1 - k_2(x_2 - x_1)$ $\quad \frac{\partial R}{\partial \dot{x}_1} = 0$

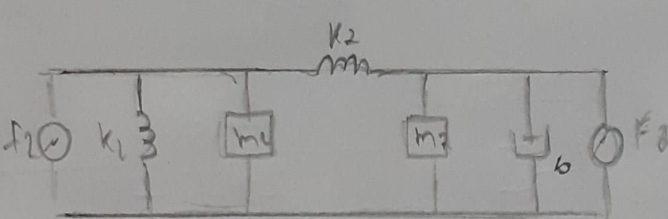
$L = T - V$

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_2 - x_1) = f_1(t)$$

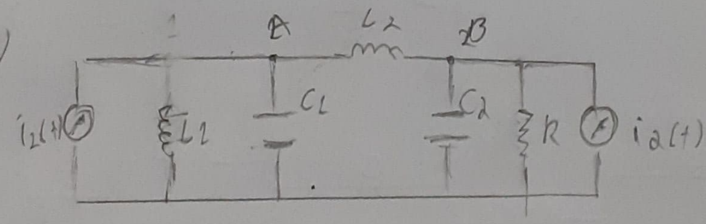
$P/x_2 \to$ $\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2$ $\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$ $\quad \frac{\partial L}{\partial x_2} = -k_2(x_2 - x_1)$ $\quad \frac{\partial R}{\partial \dot{x}_2} = b\dot{x}_2$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) + b\dot{x}_2 = 0$$

b)



c)



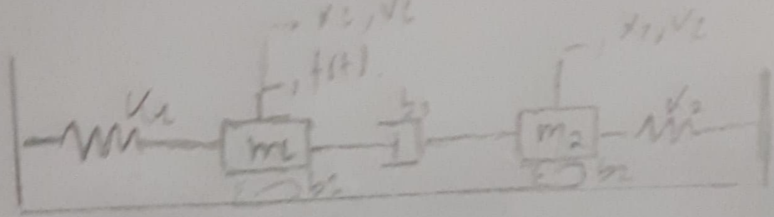
d) Nó A: $V_1\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D}\right) - V_2 \frac{1}{L_2 D} = i_1(t)$

Nó B: $V_2\left(C_2 D + \frac{1}{R} + \frac{1}{L_2 D}\right) - V_1 \frac{1}{L_2 D} = i_2(t)$

e) Nó A: $m_1 \ddot{x}_1 + (k_1 + k_2) x_1 = f_1(t) + k_2 x_2$

Nó B: $m_2 \ddot{x}_2 + b\dot{x}_2 + k_2 x_2 = f_2(t) + k_2 x_1$

5.

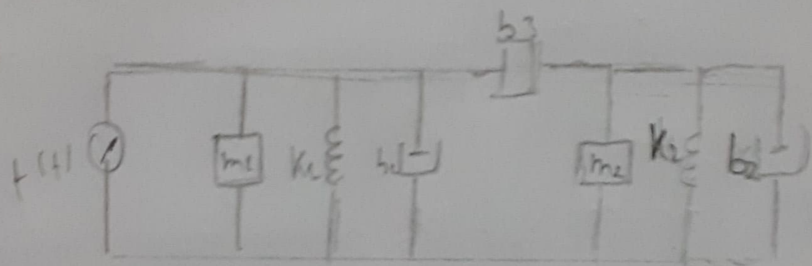


$T = \frac{1}{2}(m_1\dot{x}_1^2 + m_2\dot{x}_2^2)$ $V = \frac{1}{2}(k_1 x_1^2 + k_2 x_2^2)$ $R = \frac{1}{2}(b_1\dot{x}_1^2 + b_2\dot{x}_2^2 + b_3(\dot{x}_2 - \dot{x}_1)^2)$

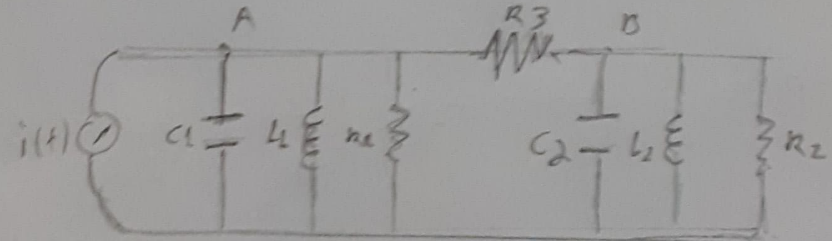
p/ x_1: $m_1\ddot{x}_1 + k_1 x_1 + (b_1+b_3)\dot{x}_1 = b_3\dot{x}_2 + f(t)$

p/ x_2 → $m_2\ddot{x}_2 + k_2 x_2 + (b_2+b_3)\dot{x}_2 = b_3\dot{x}_1$

b)



c)



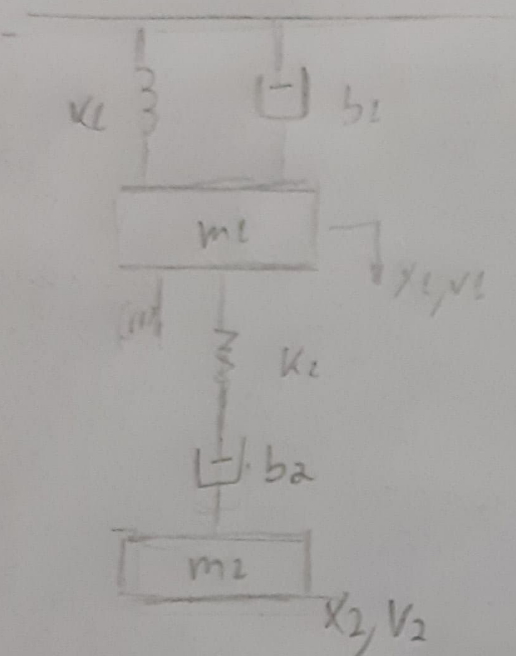
d) Nó A: $V_1\left(C_1 D + \frac{1}{R_1} + \frac{1}{L_1 D} + \frac{1}{R_3}\right) - V_2\frac{1}{R_3} = i(t)$

Nó B: $V_2\left(C_2 D + \frac{1}{R_2} + \frac{1}{L_2 D} + \frac{1}{R_3}\right) - V_1\frac{1}{R_3} = 0$

e) A: $m_1\ddot{x}_1 + (b_1+b_3)\dot{x}_1 + k_1 x_1 = f(t) + b_3\dot{x}_2$

B: $m_2\ddot{x}_2 + (b_2+b_3)\dot{x}_2 + k_2 x_2 = b_3\dot{x}_1$

6.



$$T = \frac{1}{2}\left(m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2\right)$$

$$V = \frac{1}{2}\left(k_1 x_1^2 + k_2 (x_3 - x_1)^2\right)$$

$$R = \frac{1}{2}\left(b_1 \dot{x}_1^2 + b_2 (\dot{x}_2 - \dot{x}_3)^2\right)$$

$P/x_1 =$

$$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \quad \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2 (x_3 - x_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 \rightarrow m_1 \ddot{x}_1 + (k_1 + k_2) x_1 + b_1 \dot{x}_1 = f(t) + k_2 x_3$$

P/x_2 :

$$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \quad \frac{\partial L}{\partial x_2} = 0 \quad \frac{\partial R}{\partial \dot{x}_2} = b_2 (\dot{x}_2 - \dot{x}_3)$$

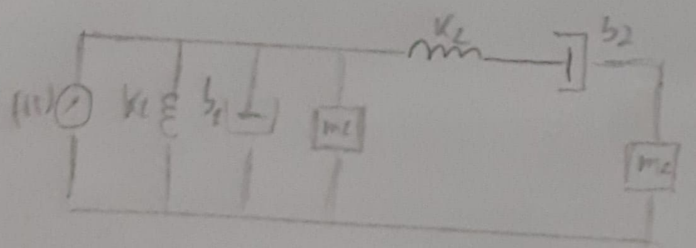
$$m \ddot{x}_2 + b_2 \dot{x}_2 = b_2 \dot{x}_3$$

P/x_3

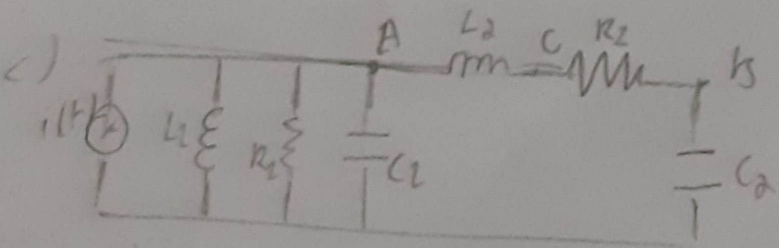
$$\frac{\partial L}{\partial \dot{x}_3} = 0 \quad \frac{\partial L}{\partial x_3} = -k_2 (x_3 - x_1) \quad \frac{\partial R}{\partial \dot{x}_3} = -b_2 (\dot{x}_2 - \dot{x}_3)$$

$$k_2 x_3 + b_2 \dot{x}_3 = k_2 x_1 + b_2 \dot{x}_2$$

b)



c)



d) Nó A: $V_1\left(C_1D+\frac{1}{R_1}+\frac{1}{L_1D}+\frac{1}{L_2D}\right)-V_3\frac{1}{L_2D}=i(t)$

Nó B: $V_2\left(C_2D+\frac{1}{R_2}\right)-V_3\frac{1}{R_2}=0$

Nó C: $V_3\left(\frac{1}{R_2}+\frac{1}{L_2D}\right)-V_1\frac{1}{L_2D}-V_2\frac{1}{R_2}=0$

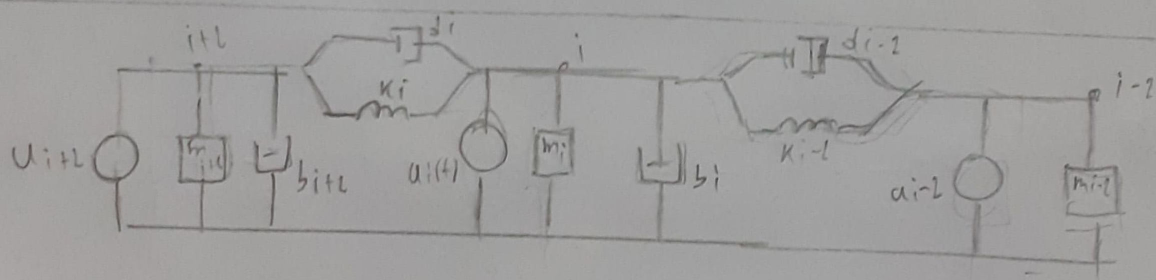
e)

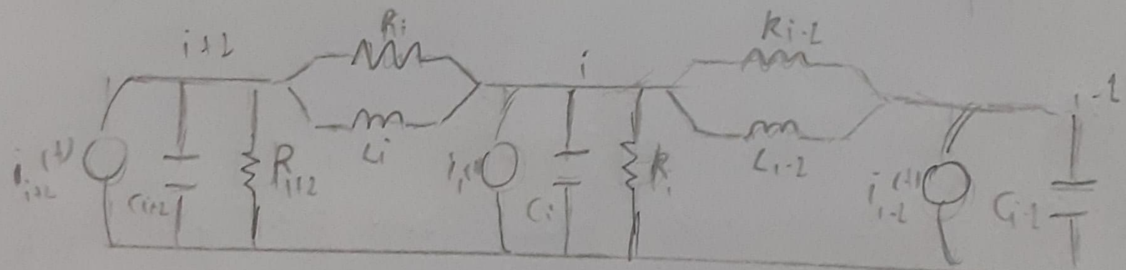
$$m_1\ddot{x}_1+b_1\dot{x}_1+(K_1+K_2)x_1=f(t)+K_2x_3$$

$$m_2\ddot{x}_2+b_2\dot{x}_2=b_2\dot{x}_3$$

$$b_2\dot{x}_3+K_2x_3=K_2x_1+b_2\dot{x}_2$$

7.





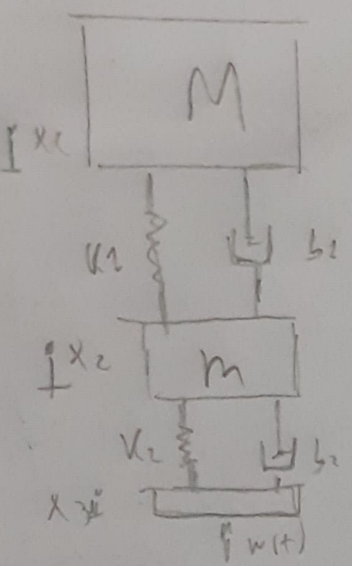
Eq: $V_i\left(C_iD+\frac{1}{r_i}+\frac{1}{L_iD}+\frac{1}{L_{i-1}D}+\frac{1}{R_i}+\frac{1}{R_{i-1}}\right)-V_{i-1}\left(\frac{1}{R_{i-1}}+\frac{1}{L_{i-1}D}\right)$

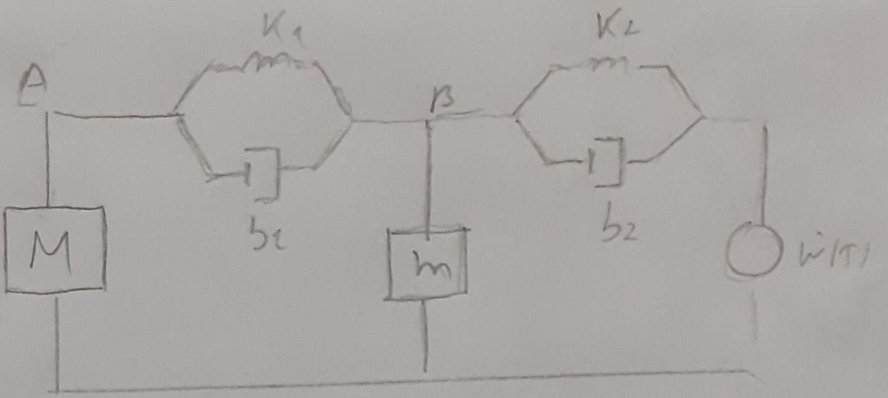
$-V_{i+1}\left(\frac{1}{R_i}+\frac{1}{L_iD}\right)=i_i(t)$

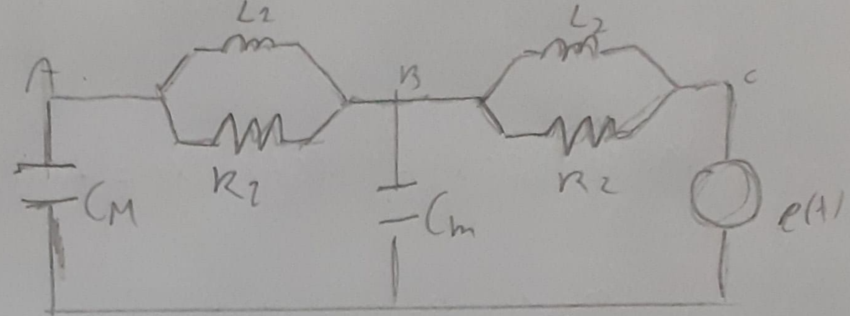
Por analogia

$m_i\ddot{x}_i+(b_i+d_i+d_{i-1})\dot{x}_i+(K_i+K_{i-1})x_i=u_i(t)-m_ig\sin\theta_i+b_{i-1}\dot{x}_{i-1}+b_i\dot{x}_{i+1}+K_{i-1}x_{i-1}+K_ix_{i+1}$

1,







Nó A: $V_1\left(C_M D + \frac{1}{L_1 D} + \frac{1}{R_1}\right) - V_2\left(\frac{1}{L_1 D} + \frac{1}{R_1}\right) = 0$

Nó B. $V_2\left(C_m D + \frac{1}{L_1 D} + \frac{1}{L_2 D} + \frac{1}{R_1} + \frac{1}{R_2}\right) - V_1\left(\frac{1}{L_1 D} + \frac{1}{R_1}\right) - e(t)\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) = 0$

$M\ddot{x}_1 + K_2 x_1 - K_1 x_2 + b_2 \dot{x}_1 - b_1 \dot{x}_2 = 0$

$m\ddot{x}_2 + K_1 x_2 + K_2 x_2 - K_1 x_1 - K_2 w(t) + b_1 \dot{x}_2 + b_2 \dot{x}_2 - b_1 \dot{x}_1 - b_2 \dot{w}(t) = 0$

b) Mesmo circuito, mas: $e(t) \to i(t)$ $\dot{w}(t) \to w(t)$

Nó A: $V_1\left(C_M D + \frac{1}{L_1 D} + \frac{1}{R_1}\right) - V_2\left(\frac{1}{L_1 D} + \frac{1}{R_1}\right) = 0$

Nó B: $V_2\left(C_m D + \frac{1}{L_1 D} + \frac{1}{L_2 D} + \frac{1}{R_1} + \frac{1}{R_2}\right) - V_1\left(\frac{1}{L_1 D} + \frac{1}{R_1}\right) - V_3\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) = 0$

Nó C: $V_3\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) = i(t)$

$m_1\ddot{x}_1 + K_1 x_1 - K_2 x_2 + b_1 \dot{x}_1 - b_1 \dot{x}_2 = 0$

$m_2\ddot{x}_2 + K_1 x_2 + K_2 x_2 - K_1 x_1 - K_2 x_3 + b_1 \dot{x}_2 + b_2 \dot{x}_2 - b_1 \dot{x}_1 - b_2 \dot{x}_3 = 0$

$K_2 x_3 - K_2 x_2 + b_2 \dot{x}_3 - b_2 \dot{x}_2 = w(t)$