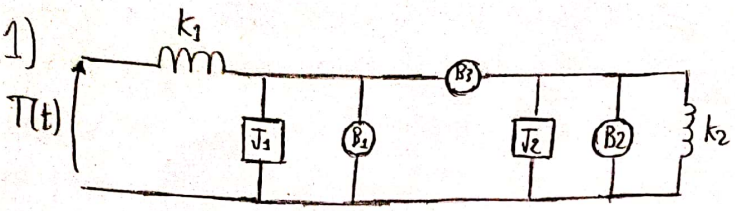
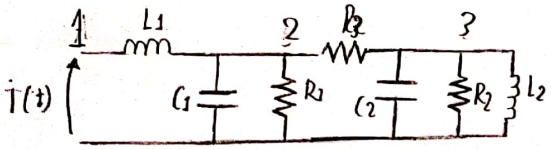
Exercícios p/ 17/09

1)



III p/ analogia elétrica:



Lei dos nós em A, B e C:

$$A: I(t) = \left(\frac{1}{L_1 D}\right)(V_1 - V_2)$$

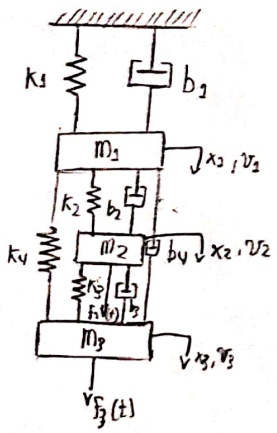
$$B: (V_2 - V_1)\left(\frac{1}{L_1 D}\right) + \frac{1}{R_3}(V_2 - V_3) + V_2\left(C_1 D + \frac{1}{R_1}\right) = 0$$

$$C: \frac{1}{R_3}(V_3 - V_2) + V_3\left(C_2 D + \frac{1}{R_2} + \frac{1}{L_2 D}\right) = 0$$

Que equivale a:

$$\begin{cases} T(t) = k_1\theta_1 - k_1\theta_2 \\ k_1(\theta_2 - \theta_1) + B_3(\dot{\theta}_2 - \dot{\theta}_3) + J_1\ddot{\theta}_2 + B_1\dot{\theta}_2 = 0 \\ B_3(\dot{\theta}_3 - \dot{\theta}_2) + J_2\ddot{\theta}_3 + B_2\dot{\theta}_3 + k_2\theta_3 = 0 \end{cases}$$

2)



a) Lagrange:

$$T = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{m_3 v_3^2}{2} \qquad V = \frac{k_1 x_1^2}{2} + \frac{k_2(x_2 - x_1)^2}{2} + \frac{k_3(x_3 - x_2)^2}{2} + \frac{k_4(x_3 - x_1)^2}{2}$$

$$R = \frac{b_1 v_1^2}{2} + \frac{b_2(v_2 - v_1)^2}{2} + \frac{b_3(v_3 - v_2)^2}{2} + \frac{b_4(v_3 - v_1)^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_1}\right) = m_1\dot{v}_1 = \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2(x_2 - x_1) + k_4(x_3 - x_1) \quad \frac{\partial R}{\partial v_1} = b_1 v_1 - b_2(v_2 - v_1) - b_4(v_3 - v_1)$$

$$\Rightarrow m_1\dot{v}_1 + k_1 x_1 - k_2(x_2 - x_1) - k_4(x_3 - x_1) + b_1 v_1 - b_2(v_2 - v_1) - b_4(v_3 - v_1)$$

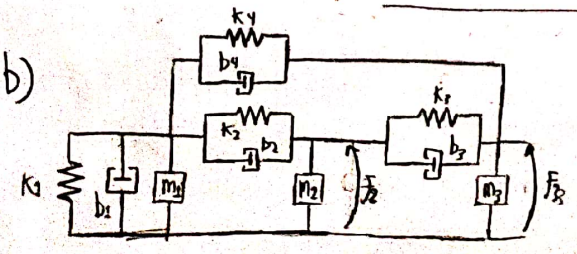
$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_2}\right) = m_2\dot{v}_2 \quad \frac{\partial L}{\partial x_2} = -k_2(x_2 - x_1) + k_3(x_3 - x_2) \quad \frac{\partial R}{\partial v_2} = b_2(v_2 - v_1) - b_3(v_3 - v_2)$$

$$\Rightarrow m_2\dot{v}_2 + k_2(x_2 - x_1) + k_3(x_2 - x_3) + b_2(v_2 - v_1) - b_3(v_3 - v_2)$$

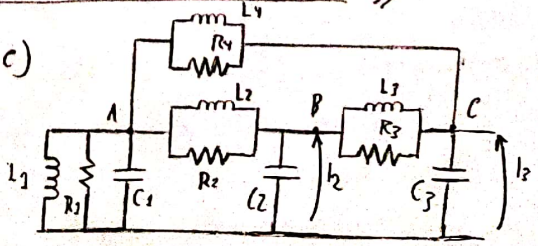
$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_3}\right) = m_3\dot{v}_3 \quad \frac{\partial L}{\partial x_3} = -k_3(x_3 - x_2) + k_4(x_3 - x_1) \quad \frac{\partial R}{\partial v_3} = b_3(v_3 - v_2) + b_4(v_3 - v_1)$$

$$\Rightarrow m_3\dot{v}_3 + k_3(x_3 - x_2) - k_4(x_3 - x_1) + b_3(v_3 - v_2) + b_4(v_3 - v_1)$$

b)



c)



d) Lei dos nós em A, B e C:

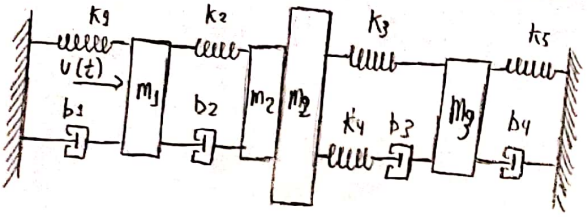
A: $V_A\left(C_3D + \frac{1}{L_4D} + \frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{L_3D}\right) - V_B\left(\frac{1}{C_3D} + \frac{1}{R_3}\right) - V_C\left(\frac{1}{L_4D} + \frac{1}{R_4}\right) = I_3$

B: $V_B\left(C_2D + \frac{1}{L_3D} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2D}\right) - V_A\left(\frac{1}{L_3D} + \frac{1}{R_3}\right) - V_C\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) = I_2$

C: $V_C\left(\frac{1}{L_4D} + C_1D + \frac{1}{L_2D} + \frac{1}{L_1D} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - V_A\left(\frac{1}{R_4} + \frac{1}{L_4D}\right) - V_B\left(\frac{1}{R_2} + \frac{1}{L_2D}\right) = 0$

e) Convertendo as eq de volta p/ o modelo físico chegamos às mesmas eq obtidas por lagrange

3)



$k_1 : k_5 = 1\ N/m$

$b_1 : b_4 = 1\ Ns/m$

$m_1, m_3 = 1$

$m_2 = 3$

a) Lagrange:

$$T = \frac{m_1\dot{x}_1^2}{2} + \frac{(m_2+m_3)\dot{x}_2^2}{2} + \frac{m_3\dot{x}_3^2}{2} = \frac{3\dot{x}_2^2 + \dot{x}_1^2 + \dot{x}_3^2}{2}$$

$$V = \frac{k_1x_1^2}{2} + \frac{k_2(x_2-x_1)^2}{2} + \frac{k_3(x_3-x_2)^2}{2} + \frac{k_4(x_3-x_2)^2}{2} + \frac{k_5(x_3)^2}{2} = \frac{x_1^2 + (x_2-x_1)^2 + 2(x_3-x_2)^2 + x_3^2}{2}$$

$$R = \frac{b_1\dot{x}_1^2}{2} + \frac{b_2(\dot{x}_2-\dot{x}_1)^2}{2} + \frac{b_3(\dot{x}_3-\dot{x}_2)^2}{2} + \frac{b_4\dot{x}_3^2}{2} = \frac{\dot{x}_1^2 + (\dot{x}_2-\dot{x}_1)^2 + (\dot{x}_3-\dot{x}_2)^2 + \dot{x}_3^2}{2}$$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \ddot{x}_1$ $\quad -\frac{\partial L}{\partial x_1} = x_1 + (x_2-x_1) = x_2 - 2x_1$ $\quad \frac{\partial R}{\partial \dot{x}_1} = \dot{x}_1 - (\dot{x}_2-\dot{x}_1) = 2\dot{x}_1 - \dot{x}_2$

$$\ddot{x}_1 - 2x_1 + x_2 + 2\dot{x}_1 - \dot{x}_2 = v(t)$$

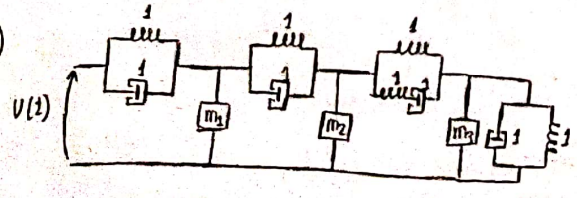
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = 3\ddot{x}_2$ $\quad -\frac{\partial L}{\partial x_2} = -(x_2-x_1) + 2(x_3-x_2) = -3x_2 + x_1 + 2x_3$ $\quad \frac{\partial R}{\partial \dot{x}_2} = (\dot{x}_2-\dot{x}_1) - (\dot{x}_3-\dot{x}_2) = 2\dot{x}_2 - \dot{x}_1 - \dot{x}_3$

$$3\ddot{x}_2 - 3x_2 + x_1 + 2x_3 + 2\dot{x}_2 - \dot{x}_1 - \dot{x}_3 = 0$$

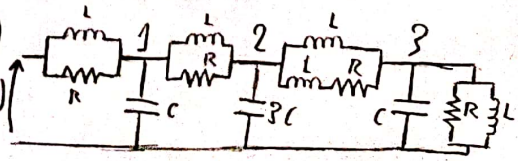
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = \ddot{x}_3$ $\quad -\frac{\partial L}{\partial x_3} = -x_3 - 2(x_3-x_2)$ $\quad \frac{\partial R}{\partial \dot{x}_3} = (\dot{x}_3-\dot{x}_2) + \dot{x}_3$

$$\ddot{x}_3 - 3x_3 + 2x_2 + 2\dot{x}_3 - \dot{x}_2 = 0$$

b)



c)



d) Lei dos nós em 1, 2 e 3:

1: $V_1\left(\frac{1}{LD}+\frac{1}{R}+CD+\frac{1}{LD}+\frac{1}{R}\right)-V_2\left(\frac{1}{LD}+\frac{1}{R}\right)=I(t)$

2: $V_2\left(\frac{1}{LD}+\frac{1}{R}+3CD+\frac{1}{LD}+\frac{1}{R}+\frac{1}{LD}\right)-V_1\left(\frac{1}{LD}+\frac{1}{R}\right)-V_3\left(\frac{1}{LD}+\frac{1}{LD}+\frac{1}{R}\right)=0$

e) Equivalente ao ex 2 //

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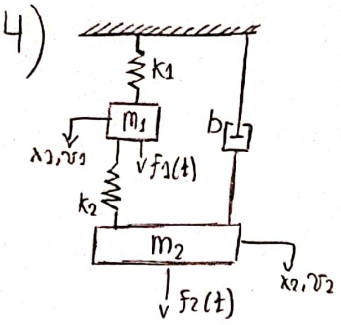
4)



a) Lagrange:

$T=\frac{m_1 v_1^2}{2}+\frac{m_2 v_2^2}{2}$ $\quad V=\frac{k_1 x_1^2}{2}+\frac{k_2(x_2-x_1)^2}{2}$ $\quad R=\frac{b v_2^2}{2}$

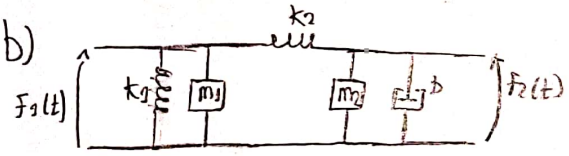
$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial v_1}\right)=m_1\dot{v}_1$ $\quad -\frac{\partial L}{\partial x_1}=-k_1x_1+k_2(x_2-x_1)$ $\quad \frac{\partial R}{\partial v_1}=0$

$m_1\dot{v}_1+k_1x_1-k_2(x_2-x_1)=f_1(t)$ //

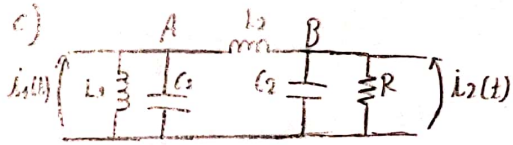
$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial v_2}\right)=m_2\dot{v}_2$ $\quad \frac{\partial L}{\partial x_2}=-k_2(x_2-x_1)$ $\quad \frac{\partial R}{\partial v_2}=bv_2$

$m_2\dot{v}_2+k_2(x_2-x_1)+bv_2=f_2(t)$ //

b)



c)



e) Equivalente aos ex 2 e 3.

d) Lei dos nós em A e B:

$\begin{cases} A: V_1\left(\frac{1}{L_1D}+C_1D+\frac{1}{L_2D}\right)-\left(\frac{1}{L_2D}\right)V_2=I_1(t) \\ B: V_2\left(\frac{1}{L_2D}+C_2D+\frac{1}{R}\right)-V_1\left(\frac{1}{L_2D}\right)=I_2(t) \end{cases}$

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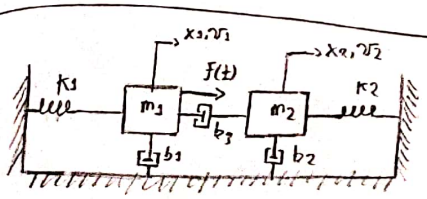
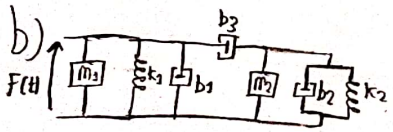
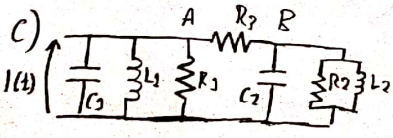
5)



b)



c)



a) Lagrange: $T=\frac{m_1 v_1^2}{2}+\frac{m_2 v_2^2}{2}$ $\quad V=\frac{k_1x_1^2}{2}+\frac{k_2x_2^2}{2}$

$R=\frac{b_1 v_1^2}{2}+\frac{b_2 v_2^2}{2}+\frac{b_3(v_2-v_1)^2}{2}$

$\frac{d}{dt}\left(\frac{dL}{dv_1}\right)=m_1\dot{v}_1$ $\quad \frac{dL}{dx_1}=-k_1x_1$ $\quad \frac{dR}{dv_1}=b_1v_1-b_3(v_2-v_1)$

$\frac{d}{dt}\left(\frac{dL}{dv_2}\right)=m_2\dot{v}_2$ $\quad \frac{dL}{dx_2}=-k_2x_2$ $\quad \frac{dR}{dv_2}=b_2v_2+b_3(v_2-v_1)$

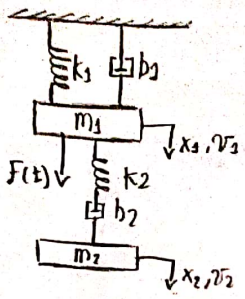
$\begin{cases} m_1\dot{v}_1+k_1x_1+b_1v_1-b_3(v_2-v_1)=f(t) \\ m_2\dot{v}_2+k_2x_2+b_2v_2+b_3(v_2-v_1)=0 \end{cases}$

d) Lei dos nós em A e B:

A: $V_1\left(C_1D+\frac{1}{L_1D}+\frac{1}{R_1}+\frac{1}{R_3}\right)-\frac{V_2}{R_3}=F(t)$ $\quad$ B: $V_2\left(C_2D+\frac{1}{R_3}+\frac{1}{L_2D}+\frac{1}{R_2}\right)-\frac{V_1}{R_3}=0$

e) Ver 2, 3 e 4

6)



a) Lagrange:

$$T = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \qquad V = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_2 - x_1)^2}{2} \qquad R = \frac{b_1 v_1^2}{2} + \frac{b_2 (v_3 - v_2)^2}{2}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial v_1}\right) = m_1 \dot{v}_1 \qquad \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2 (x_2 - x_1) \qquad \frac{\partial R}{\partial v_1} = b_1 v_1$$

$$m_1 \dot{v}_1 + k_1 x_1 - k_2 (x_2 - x_1) + b_1 v_1 = F(t)$$

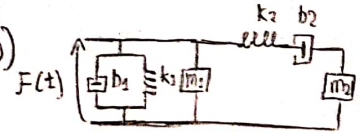
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial v_2}\right) = 0 \qquad \frac{\partial L}{\partial x_2} = -k_2 (x_2 - x_1) \qquad \frac{\partial R}{\partial v_2} = -b_2 (v_3 - v_2)$$

$$k_2 (x_2 - x_1) - b_2 (v_3 - v_2) = 0$$

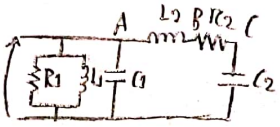
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial v_3}\right) = m_2 \dot{v}_3 \qquad \frac{\partial L}{\partial x_3} = 0 \qquad \frac{\partial R}{\partial v_3} = b_2 (v_3 - v_2)$$

$$m_2 \dot{v}_3 + b_2 (v_3 - v_2) = 0$$

b)



c)



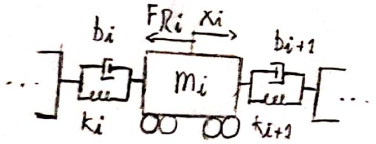
d) Lei dos nós em A, B e C:

$$A: V_1\left(\frac{1}{L_1 D} + \frac{1}{R_1} + C_1 D + \frac{1}{R_2}\right) - \frac{V_2}{L_2 D} = F(t)$$

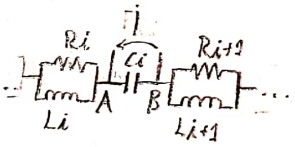
$$B: V_2\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) - \frac{V_2}{R_2} - \frac{V_1}{L_2 D} = 0$$

$$C: V_3\left(\frac{1}{R_2} + C_2 D\right) - \frac{V_2}{R_2} = 0$$

7)



analogia elétrica:

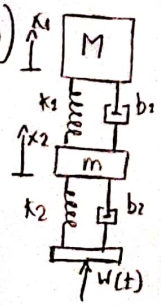


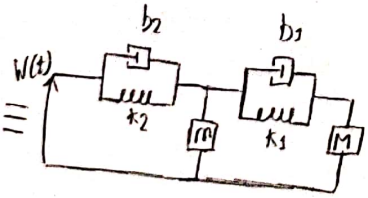
Lei dos nós:

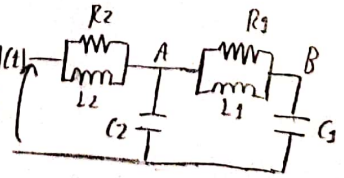
$$A: V_i\left(\frac{1}{R_i} + \frac{1}{L_i D} + C_i D\right)$$

$$- V_{i-1}\left(\frac{1}{R_i} + \frac{1}{L_i D}\right) - C_i D V_{i+1} = -I$$

8)







Lei dos nós:

$$A: V_3\left(\frac{1}{L_2 D} + \frac{1}{R_2} + C_2 D + \frac{1}{L_1 D} + \frac{1}{R_1}\right)$$

$$-\left(\frac{1}{L_1 D} + \frac{1}{R_1}\right) V_2 = I(t)$$

$$B: V_2\left(\frac{1}{L_1 D} + \frac{1}{R_1} + C_1 D\right) - V_1\left(\frac{1}{L_1 D} + \frac{1}{R_1}\right) = 0$$