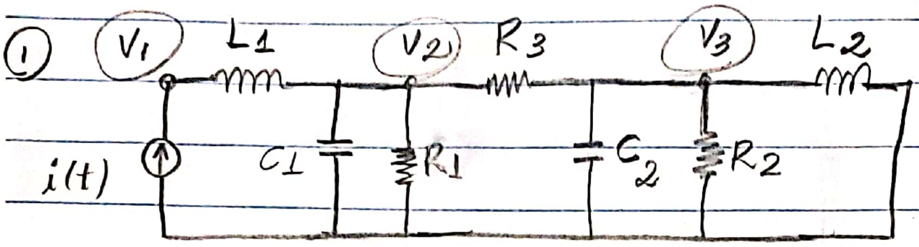
17 / 09 / 20

Bruno Nogueira Lucas - 10772668

1)



$$\frac{V_1}{L_1 D} + i(t) - \frac{V_2}{L_1 D} = 0 \qquad (I)$$

$$V_2 C_1 D + \frac{V_2}{R_1} - \frac{V_2}{R_3} - \frac{V_2}{L_1 D} + \frac{V_1}{L_1 D} + \frac{V_3}{R_3} = 0 \qquad (II)$$

$$V_3 C_2 D + \frac{V_3}{R_2} + \frac{V_3}{L_2 D} - \frac{V_3}{R_3} + \frac{V_2}{R_3} = 0 \qquad (III)$$

→ Retornando para o modelo mecânico:

$$\dot{\theta}_1 . K_1 + T - \dot{\theta}_2 K_1 = 0 \qquad (I)$$

$$\dot{\theta}_2 J_1 + \dot{\theta}_2 B_1 - \dot{\theta}_2 B_3 - \dot{\theta}_2 K_1 + \dot{\theta}_1 K_1 + \dot{\theta}_3 B_3 = 0 \qquad (II)$$

$$\dot{\theta}_3 J_2 + \dot{\theta}_3 B_2 + \dot{\theta}_3 K_2 - \dot{\theta}_3 B_3 + \dot{\theta}_2 B_3 = 0 \qquad (III)$$

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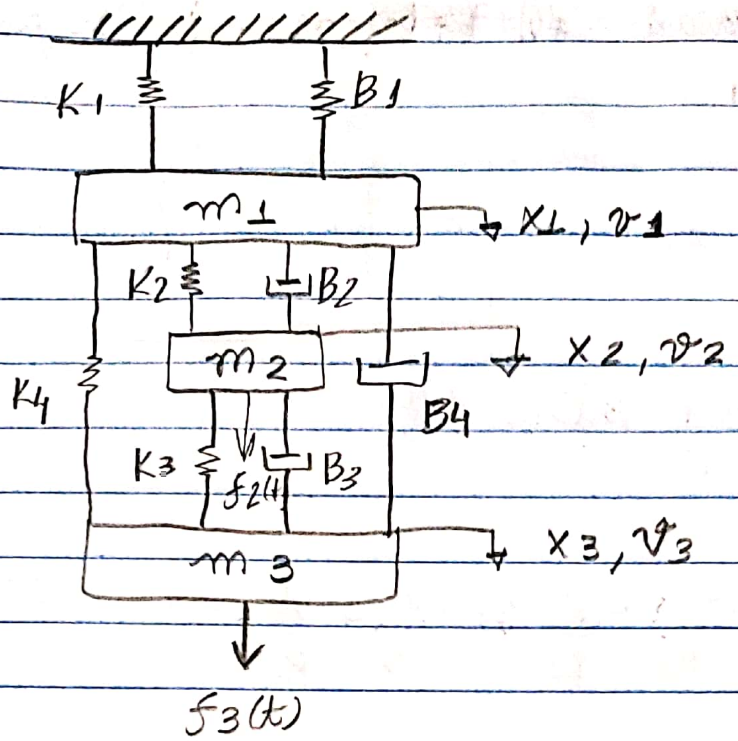
(2)



a) $$V = \frac{K_1 x_1^2}{2} + \frac{K_2 (x_1 - x_2)^2}{2} + \frac{K_3 (x_2 - x_3)^2}{2} + \frac{K_4 (x_1 - x_3)^2}{2}$$

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{m_3 \dot{x}_3^2}{2}$$

$$R = \frac{B_1 \dot{x}_1^2}{2} + \frac{B_2 (\dot{x}_1 - \dot{x}_2)^2}{2} + \frac{B_3 (\dot{x}_2 - \dot{x}_3)^2}{2} + \frac{B_4 (\dot{x}_1 - \dot{x}_3)^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (y_1 - x_2) - K_4 (x_1 - x_3)$$

$$\frac{\partial R}{\partial \dot{x}_1} = B_1 \dot{x}_1 + B_2 (\dot{x}_1 - \dot{x}_2) + B_4 (\dot{x}_1 - \dot{x}_3)$$

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$$\rightarrow m_1 \ddot{x}_1 + K_1 x_1 + K_2(x_1 - x_2) + K_4(x_1 - x_3) + B_1 \dot{x}_1 + B_2(\dot{x}_1 - \dot{x}_2) + B_4(\dot{x}_1 - \dot{x}_3) = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = K_2(x_1 - x_2) - K_3(x_2 - x_3)$$

$$\frac{\partial R}{\partial \dot{x}_2} = -B_2(\dot{x}_1 - \dot{x}_2) + B_3(\dot{x}_2 - \dot{x}_3)$$

$$\rightarrow m_2 \ddot{x}_2 - K_2(x_1 - x_2) - K_3(x_2 - x_3) - B_2(\dot{x}_1 - \dot{x}_2) + B_3(\dot{x}_2 - \dot{x}_3) = f_2(t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_3 \ddot{x}_3$$

$$\frac{\partial L}{\partial x_3} = K_3(x_2 - x_3) + K_4(x_1 - x_3)$$

$$\frac{\partial R}{\partial \dot{x}_3} = -B_3(\dot{x}_2 - \dot{x}_3) - B_1(\dot{x}_1 - \dot{x}_3)$$

$$\rightarrow m_3 \ddot{x}_3 - K_3(x_2 - x_3) - K_4(x_1 - x_3) - B_3(\dot{x}_2 - \dot{x}_3) - B_1(\dot{x}_1 - \dot{x}_3) = f_3(t)$$

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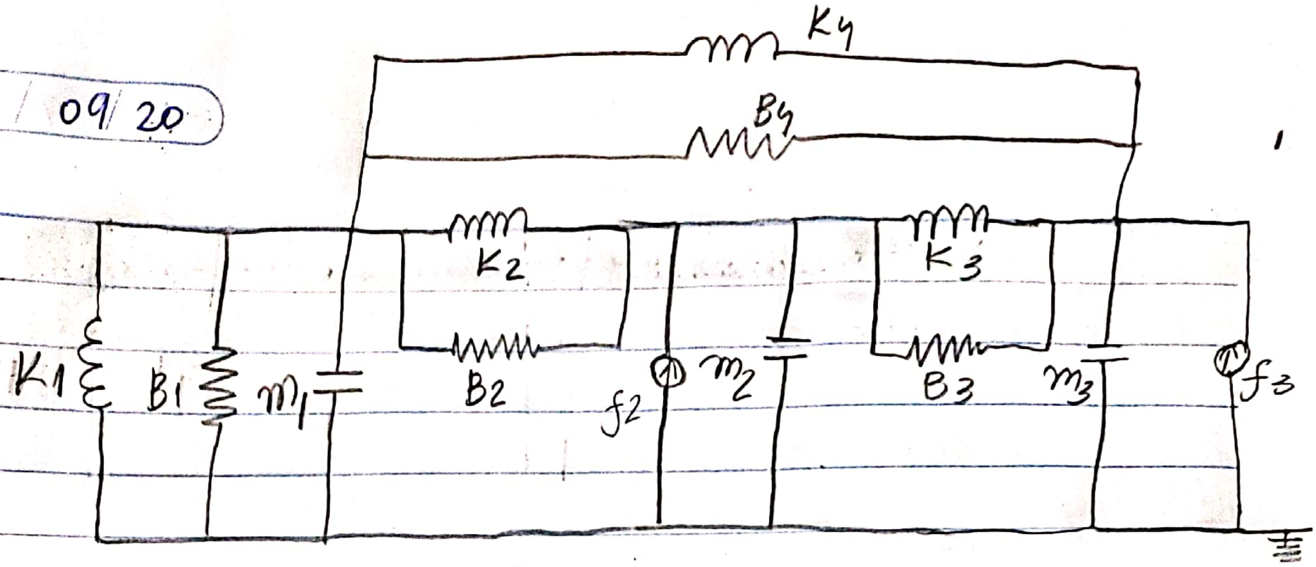
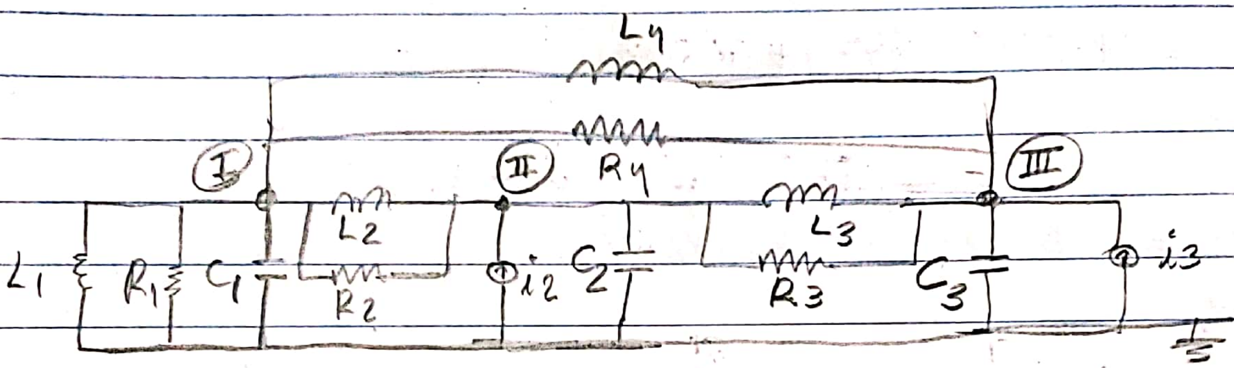
b)



c)



d) I : $V_I\left(C_1D+\frac{1}{L_1D}+\frac{1}{R_1}\right)=(V_2-V_1)\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)+$

$+\ (V_3-V_1)\left(\frac{1}{L_4D}+\frac{1}{R_4}\right)$

II : $i_2=-V_1\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)+V_2\left(\frac{1}{L_2D}+\frac{1}{R_2}+\frac{1}{L_3D}+\frac{1}{R_3}+C_2D\right)$

III : $i_3=-V_1\left(\frac{1}{L_4D}+\frac{1}{R_4}\right)-V_B\left(\frac{1}{L_3D}+\frac{1}{R_3}\right)+V_C\left(\frac{1}{L_3D}+\frac{1}{R_3}+\frac{1}{L_4D}+\frac{1}{R_4}\right)$

e) I : $m_1\ddot{x}_1+K_1x_1+K_2(x_1-x_2)+K_4(x_1-x_3)+B_1\dot{x}_1+B_2(\dot{x}_1-\dot{x}_2)+$
$+\ B_4(\dot{x}_1-\dot{x}_3)=0$

II : $m_2\ddot{x}_2-K_2(x_1-x_2)-K_3(x_2-x_3)-B_2(\dot{x}_1-\dot{x}_2)+B_3(\dot{x}_2-\dot{x}_3)=f_2(t)$

III : $m_3\ddot{x}_3-K_3(x_2-x_3)-K_4(x_1-x_3)-B_3(\dot{x}_2-\dot{x}_3)-B_4(\dot{x}_1-\dot{x}_3)=f_3(t)$

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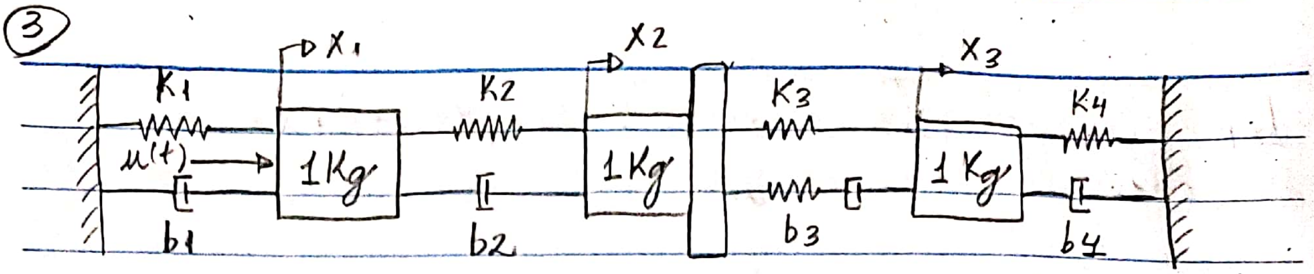

a) $V = \frac{K_1 x_1^2}{2} + \frac{K_2 (x_1 - x_2)^2}{2} + \frac{K_3 (x_2 - x_3)^2}{2} + \frac{K_4 x_3^2}{2}$

$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{m_3 \dot{x}_3^2}{2}$

$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_1 - \dot{x}_2)^2}{2} + \frac{b_3 (\dot{x}_2 - \dot{x}_3)^2}{2} + \frac{b_4 \dot{x}_3^2}{2}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$ $\qquad \frac{\partial L}{\partial x_1} = -K_1 x_1 - K_2 (x_1 - x_2)$

$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 + b_2 (\dot{x}_1 - \dot{x}_2)$

$\rightarrow m_1 \ddot{x}_1 + K_1 x_1 + K_2 (x_1 - x_2) + b_1 \dot{x}_1 + b_2 (\dot{x}_1 - \dot{x}_2) = u(t)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$ $\qquad \frac{\partial L}{\partial x_2} = K_2 (x_1 - x_2) - K_3 (x_2 - x_3)$

$\frac{\partial R}{\partial \dot{x}_2} = -b_2 (\dot{x}_1 - \dot{x}_2) + b_3 (\dot{x}_2 - \dot{x}_3)$

$\rightarrow m_2 \ddot{x}_2 - K_2 (x_1 - x_2) + K_3 (x_2 - x_3) - b_2 (\dot{x}_1 - \dot{x}_2) + b_3 (\dot{x}_2 - \dot{x}_3) = 0$

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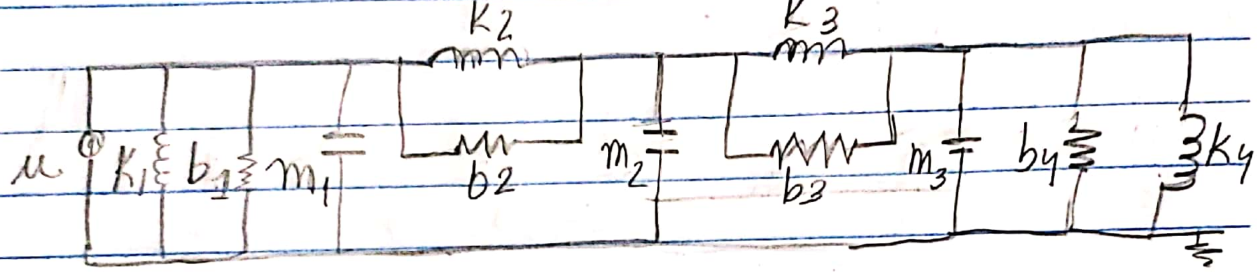
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3 \ddot{x}_3 \qquad \frac{\partial L}{\partial x_3} = k_3(x_2 - x_3) - k_4 x_3$$

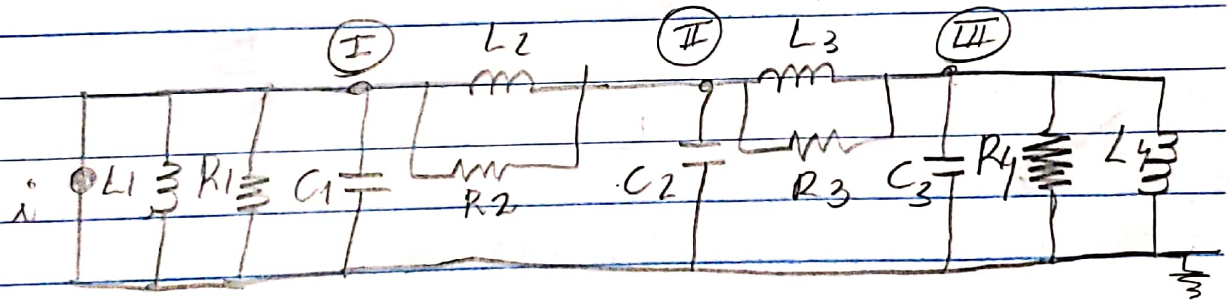
$$\frac{\partial R}{\partial \dot{x}_3} = -b_3(\dot{x}_2 - \dot{x}_3) + b_4 \dot{x}_3$$

$$\rightarrow m_3\ddot{x}_3 - k_3(x_2 - x_3) + k_4 x_3 - b_3(\dot{x}_2 - \dot{x}_3) + b_4 \dot{x}_3 = 0$$

b)



c)



d) I: $V_1\left(\frac{1}{L_1 D} + \frac{1}{R_1} + C_1 D + \frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) = i$

II: $V_2\left(\frac{1}{L_2 D} + \frac{1}{R_2} + C_2 D + \frac{1}{L_3 D} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{L_3 D} + \frac{1}{R_3}\right) = 0$

III: $V_3\left(\frac{1}{R_4} + \frac{1}{L_4 D} + C_3 D + \frac{1}{L_3 D} + \frac{1}{R_3 D}\right) - V_2\left(\frac{1}{L_3 D} + \frac{1}{R_3 D}\right) = 0$

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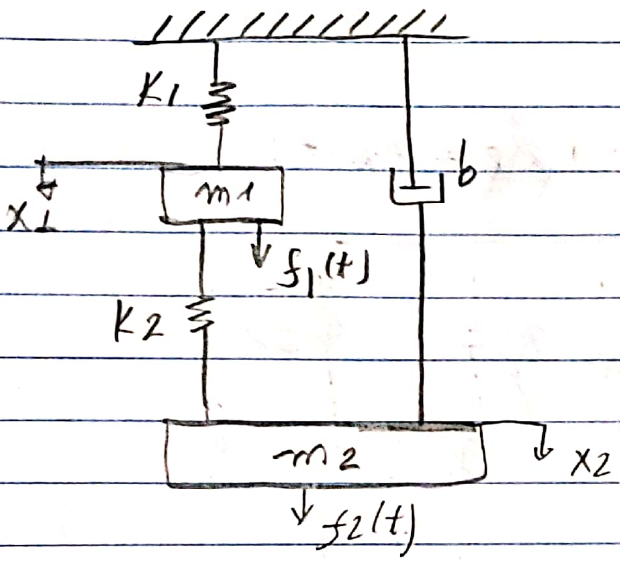
c) I: $m_1\ddot{x}_1 + k_1x_1 + k_2(x_1-x_2) + b_1\dot{x}_1 + b_2(\dot{x}_1-\dot{x}_2) = u(t)$

II: $m_2\ddot{x}_2 - k_2(x_1-x_2) + k_3(x_2-x_3) - b_2(\dot{x}_1-\dot{x}_2) + b_3(\dot{x}_2-\dot{x}_3) = 0$

III: $m_3\ddot{x}_3 - k_3(x_2-x_3) + k_4x_3 - b_3(\dot{x}_2-\dot{x}_3) + b_4\dot{x}_3 = 0$

4)



a) $V = \frac{k_1x_1^2}{2} + \frac{k_2(x_1-x_2)^2}{2}$

$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2}$

$R = \frac{b\dot{x}_2^2}{2}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1 \qquad \frac{\partial L}{\partial x_1} = -k_1x_1 - k_2(x_1-x_2)$

$\frac{\partial R}{\partial x_1} = 0$

$\rightarrow m_1\ddot{x}_1 + k_1x_1 + k_2(x_1-x_2) = f_1(t)$

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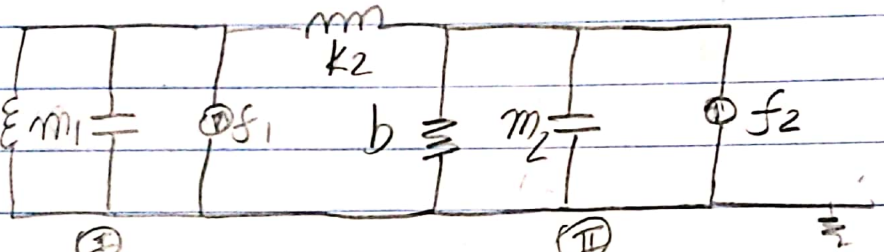
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2 \qquad \frac{\partial L}{\partial x_2} = k_2(x_1 - x_2)$$

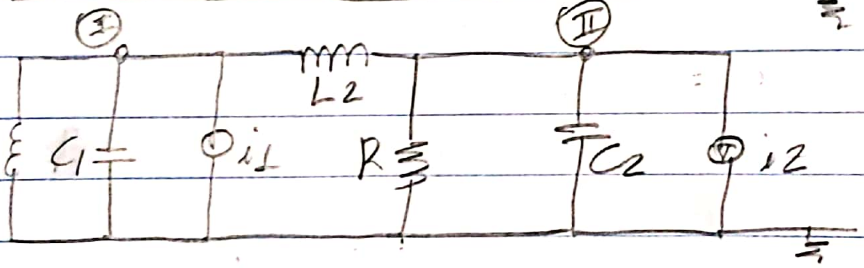
$$\frac{\partial R}{\partial \dot{x}_2} = b\dot{x}_2$$

$$\rightarrow m_2\ddot{x}_2 - k_2(x_1 - x_2) + b\dot{x}_2 = f_2(t)$$

b)



c)



d) I : $V_1\left(C_1D + \frac{1}{L_1D} + \frac{1}{L_2D}\right) - V_2\left(\frac{1}{L_2D}\right) = i_1$

II : $V_2\left(C_2D + \frac{1}{R} + \frac{1}{L_2D}\right) - V_1\left(\frac{1}{L_2D}\right) = i_2$

e) I : $m_1\ddot{x}_1 + K_1x_1 + K_2(x_1 - x_2) = f_1(t)$

II : $m_2\ddot{x}_2 - K_2(x_1 - x_2) + b\dot{x}_2 = f_2(t)$

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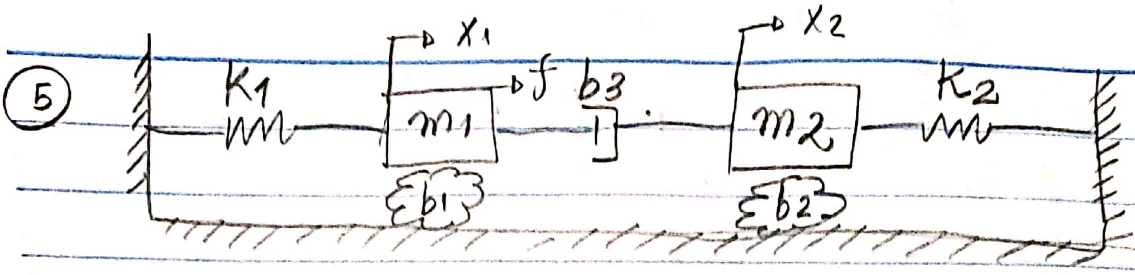


$$V = \frac{K_1 x_1^2}{2} + \frac{K_2 x_2^2}{2} \qquad T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$$

$$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 \dot{x}_2^2}{2} + \frac{b_3 (\dot{x}_1 - \dot{x}_2)^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \qquad \frac{\partial L}{\partial x_1} = -K_1 x_1$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 + b_3 (\dot{x}_1 - \dot{x}_2)$$

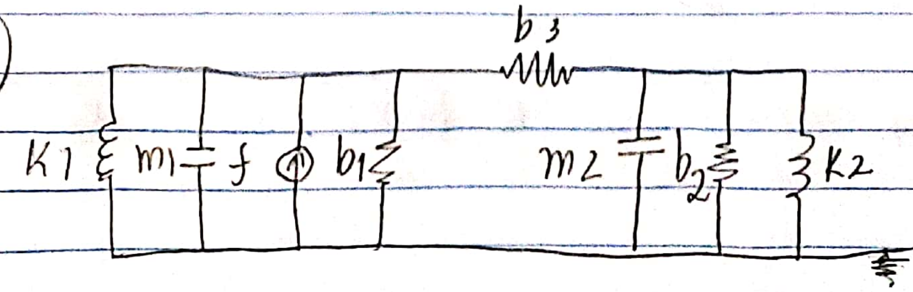
$$\rightarrow m_1 \ddot{x}_1 + K_1 x_1 + b_1 \dot{x}_1 + b_3 (\dot{x}_1 - \dot{x}_2) = f(t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \qquad \frac{\partial L}{\partial x_2} = -K_2 x_2$$

$$\frac{\partial R}{\partial \dot{x}_2} = b_2 \dot{x}_2 - b_3 (\dot{x}_1 - \dot{x}_2)$$

$$\rightarrow m_2 \ddot{x}_2 + K_2 x_2 + b_2 \dot{x}_2 - b_3 (\dot{x}_1 - \dot{x}_2) = 0$$

b)



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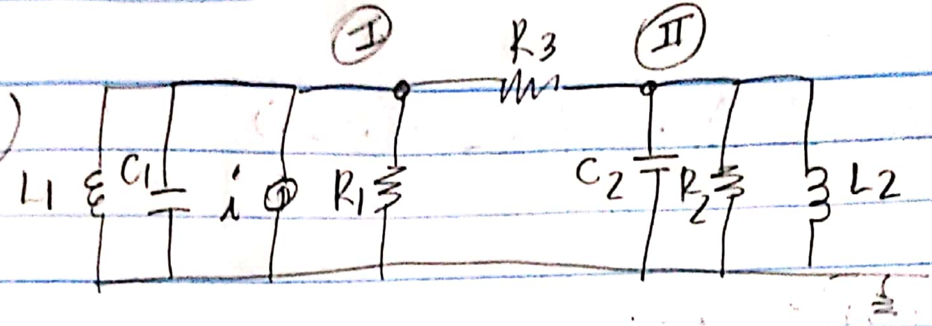
c)



d) I: $i = V_1\left(\frac{1}{L_1D} + C_1D + \frac{1}{R_1} + \frac{1}{R_3}\right) - V_2\left(\frac{1}{R_3}\right)$

II: $V_2\left(C_2D + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_3}\right) = 0$

e) I: $m_1\ddot{x}_1 + K_1x_1 + b_1\dot{x}_1 + b_3(\dot{x}_1 - x_2) = f(x)$

II: $m_2\ddot{x}_2 + K_2x_2 + b_2\dot{x}_2 - b_3(\dot{x}_1 - \dot{x}_2) = 0$

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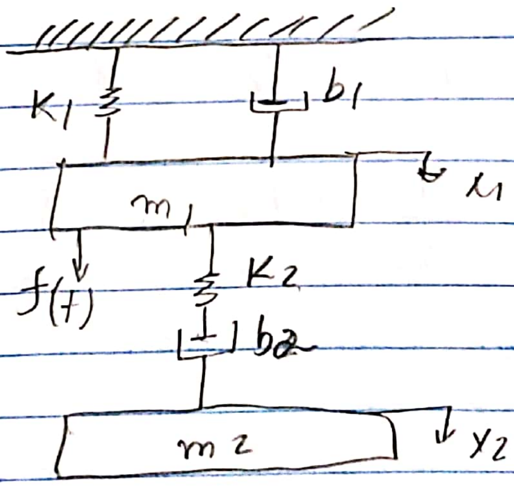

a) $V = \frac{K_1x_1^2}{2} + \frac{K_2(x_1-x_2)^2}{2}$

$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2}$ $R = \frac{b_1\dot{x}_1^2}{2} + \frac{b_2(\dot{x}_1-\dot{x}_2)^2}{2}$

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$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \qquad \frac{\partial L}{\partial x_1} = -k_1 x_1^2 - k_2(x_1 - x_2)$$

$$\frac{\partial R}{\partial x_1} = b_1 \dot{x}_1 + b_2(\dot{x}_1 - \dot{x}_2)$$

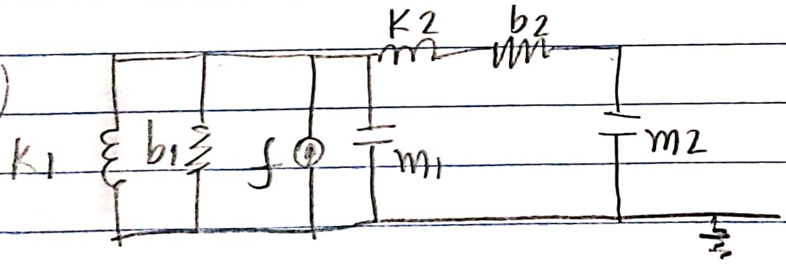
$$\rightarrow m_1 \ddot{x}_1 + k_1 x_1^2 + k_2(x_1 - x_2) + b_1 \dot{x}_1 + b_2(\dot{x}_1 - \dot{x}_2) = f(t)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \qquad \frac{\partial L}{\partial x_2} = k_2(x_1 - x_2)$$

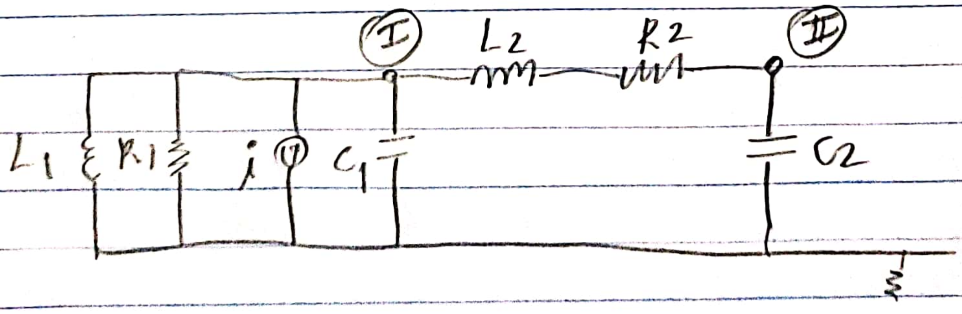
$$\frac{\partial R}{\partial \dot{x}_2} = -b_2(\dot{x}_1 - \dot{x}_2)$$

$$\rightarrow m_2 \ddot{x}_2 - k_2(x_1 - x_2) - b_2(\dot{x}_1 - \dot{x}_2) = 0$$

b)



c)



d)

$$\text{I}: V_1\left(\frac{1}{L_1 D} + \frac{1}{R_1} + C_1 D + \frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) = i$$

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II : $V_2\left(C_2D + \frac{1}{L_2D} + \frac{1}{R_2}\right) - V_1\left(\frac{1}{L_2D} + \frac{1}{R_2}\right) = 0$

e) I : $m_1\ddot{x}_1 + K_1x_1^2 + K_2(x_1 - x_2) + b_2(\dot{x}_1 - \dot{x}_2) + b_1\dot{x}_1 = f(t)$

II : $m_2\ddot{x}_2 - K_2(x_1 - x_2) - b_2(\dot{x}_1 - \dot{x}_2) = 0$