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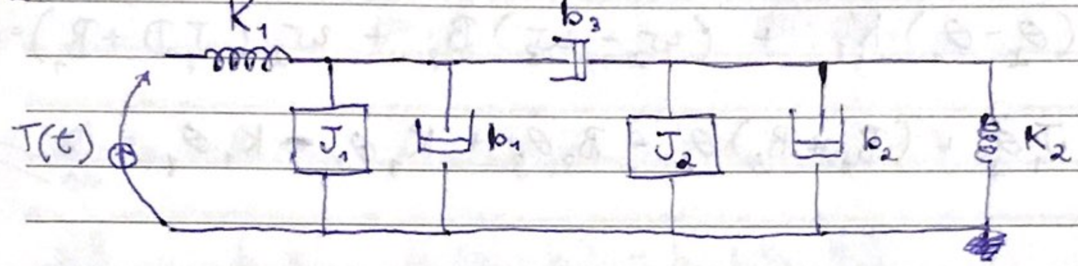
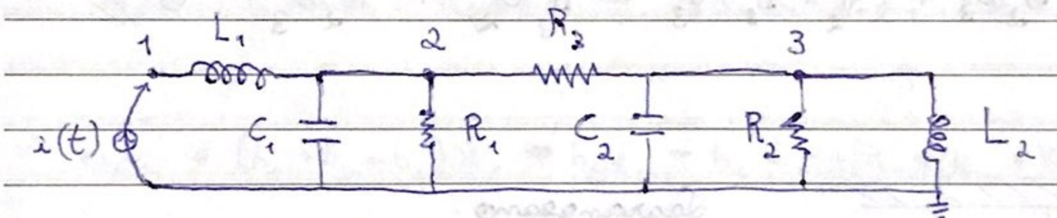
- Exercícios de modelagem -

1- Circuito mecânico:



Circuito elétrico:



Nó 1:

$$(V_1 - V_2) \cdot \frac{1}{L_1 \cdot D} = i(t)$$

Nó 2:

$$(V_2 - V_1) \cdot \frac{1}{L_1 \cdot D} + (V_2 - V_3) \cdot \frac{1}{R_3} + V_2 \left(C_1 \cdot D + \frac{1}{R_1} \right) = 0$$

Nó 3:

$$(V_3 - V_2) \cdot \frac{1}{R_3} + V_3 \left(C_2 \cdot D + \frac{1}{R_2} + \frac{1}{L_2 \cdot D} \right) = 0$$

Fazendo a analogia para o sistema mecânico:

Nó 1:

$$(\theta_1 - \theta_2) K_1 = T(t)$$

Nó 2:

$$(\theta_2 - \theta_1) K_1 + (\omega_2 - \omega_3) B_3 + \omega_2 (J_1 D + B_1) = 0$$

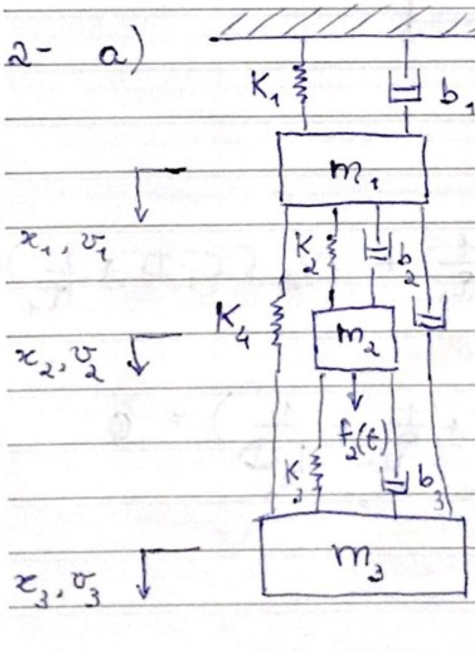
$$\Rightarrow J_1 \ddot{\theta}_2 + (B_1 + B_3)\dot{\theta}_2 - B_3 \dot{\theta}_3 + K_1 \theta_2 - K_1 \theta_1 = 0$$

Nó 3:

$$(\omega_3 - \omega_2) B_3 + \omega_3 \left(J_2 D + B_2 + \frac{K_2}{D}\right) = 0$$

$$\Rightarrow J_2 \ddot{\theta}_3 + (B_2 + B_3)\dot{\theta}_3 - B_3 \dot{\theta}_2 + K_2 \theta_3 = 0$$

2- a)



Lagrangeano:

$$L = T - V$$

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{m_3 \dot{x}_3^2}{2}$$

$$V = \frac{K_1 x_1^2}{2} + \frac{K_2 (x_2 - x_1)^2}{2} +$$

$$+ \frac{K_3 (x_3 - x_2)^2}{2} + \frac{K_4 (x_3 - x_1)^2}{2}$$

$$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b_3 (\dot{x}_3 - \dot{x}_2)^2}{2} + \frac{b_4 (\dot{x}_3 - \dot{x}_1)^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = f_{ext}$$

Coordenadas generalizadas: x_1, x_2 e x_3

Para x_1:

$$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (x_2 - x_1) + K_4 (x_3 - x_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_2 (\dot{x}_2 - \dot{x}_1) - b_4 (\dot{x}_3 - \dot{x}_1)$$

$$\therefore m_1 \ddot{x}_1 + K_1 x_1 - K_2 (x_2 - x_1) - K_4 (x_3 - x_1) + b_1 \dot{x}_1 - b_2 (\dot{x}_2 - \dot{x}_1) - b_4 (\dot{x}_3 - \dot{x}_1) = 0$$

$$\Rightarrow m_1 \ddot{x}_1 + (b_1 + b_2 + b_4) \dot{x}_1 - b_2 \dot{x}_2 - b_4 \dot{x}_3 + (K_1 + K_2 + K_4) x_1 - K_2 x_2 - K_4 x_3 = 0$$

Para x_2:

$$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = -K_2 (x_2 - x_1) + K_3 (x_3 - x_2) \quad \Big| \quad \frac{\partial R}{\partial \dot{x}_2} = b_2 (\dot{x}_2 - \dot{x}_1) - b_3 (\dot{x}_3 - \dot{x}_2)$$

$$\therefore m_2 \ddot{x}_2 + K_2 (x_2 - x_1) - K_3 (x_3 - x_2) + b_2 (\dot{x}_2 - \dot{x}_1) - b_3 (\dot{x}_3 - \dot{x}_2) = f_2(t)$$

$$\Rightarrow m_2 \ddot{x}_2 + (b_2 + b_3) \dot{x}_2 - b_2 \dot{x}_1 - b_3 \dot{x}_3 + (K_2 + K_3) x_2 - K_2 x_1 - K_3 x_3 = f_2(t)$$

Para x_3:

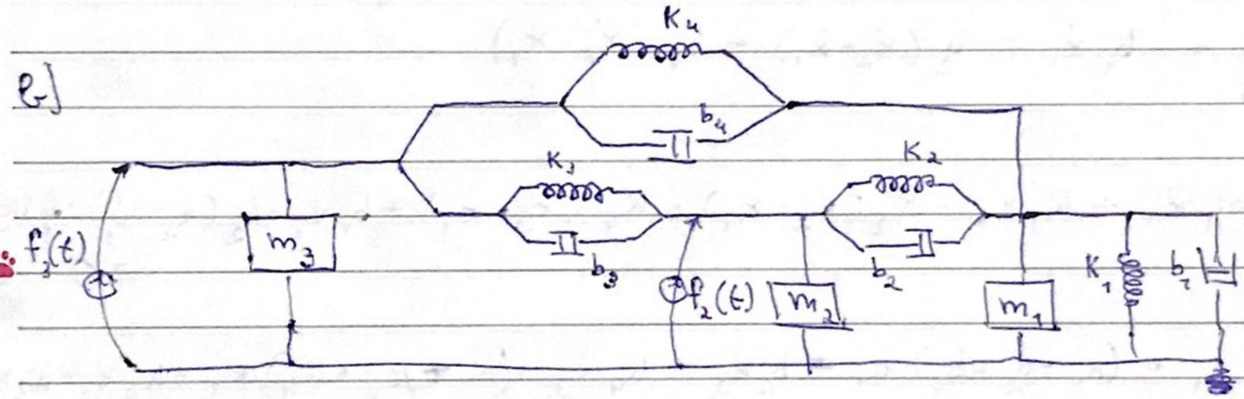
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3 \ddot{x}_3$$

$$\frac{\partial L}{\partial x_3} = -K_3(x_3 - x_2) - K_4(x_3 - x_1) \quad \Bigg| \quad \frac{\partial R}{\partial \dot{x}_3} = b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1)$$

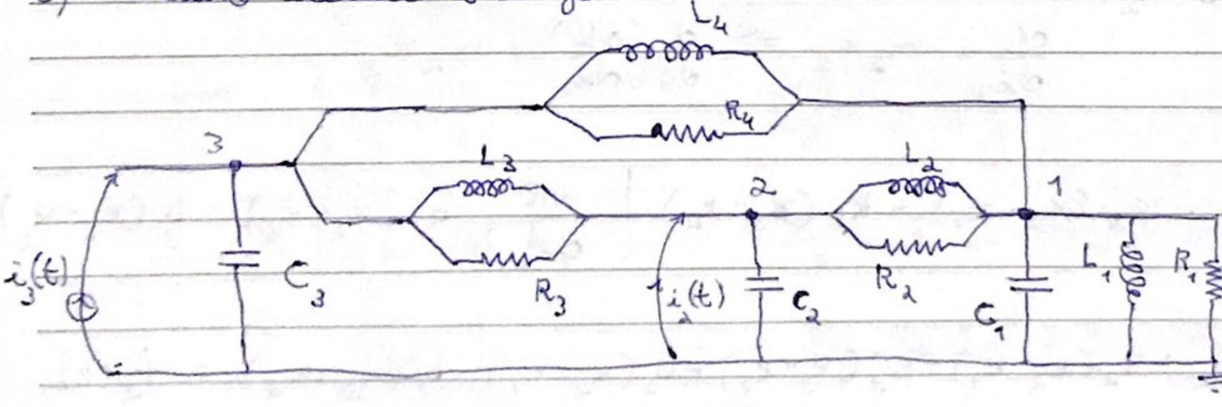
$$\therefore \; m_3\ddot{x}_3 + K_3(x_3 - x_2) + K_4(x_3 - x_1) + b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1) = f_3(t)$$

$$\Rightarrow m_3\ddot{x}_3 + (b_3 + b_4)\dot{x}_3 - b_3\dot{x}_2 - b_4\dot{x}_1 + (K_3 + K_4)x_3 - K_3x_2 - K_4x_1 = f_3(t)$$

b)



c) Circuito elétrico análogo:



d) Nó 1:

$$(V_1 - V_2)\left(\frac{1}{R_2} + \frac{1}{L_2 D}\right) + (V_1 - V_3)\left(\frac{1}{R_4} + \frac{1}{L_4 D}\right) + V_1\left(\frac{1}{R_1} + \frac{1}{L_1 D} + C_1 D\right) = 0$$

$$\Rightarrow V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{L_1 D} + \frac{1}{L_2 D} + \frac{1}{L_4 D} + C_1 D\right) - V_2\left(\frac{1}{R_2} + \frac{1}{L_2 D}\right) + - V_3\left(\frac{1}{R_4} + \frac{1}{L_4 D}\right) = 0$$

Nó 2:

$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2 D} + \frac{1}{L_3 D} + C_2 D\right) - V_1\left(\frac{1}{R_2} + \frac{1}{L_2 D}\right) - V_3\left(\frac{1}{R_3} + \frac{1}{L_3 D}\right) = i_2(t)$$

Nó 3:

$$V_3\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{L_3 D} + \frac{1}{L_4 D} + C_3 D\right) - V_1\left(\frac{1}{R_4} + \frac{1}{L_4 D}\right) - V_2\left(\frac{1}{R_3} + \frac{1}{L_3 D}\right) = i_3(t)$$

e) Nó 1:

$$v_1\left(b_1 + b_2 + b_4 + \frac{K_1}{D} + \frac{K_2}{D} + \frac{K_4}{D} + m_1 D\right) - v_2\left(b_2 + \frac{K_2}{D}\right) - v_3\left(b_4 + \frac{K_4}{D}\right) = 0$$

$$\Rightarrow m_1 \ddot{x}_1 + (b_1 + b_2 + b_4)\dot{x}_1 - b_2 \dot{x}_2 - b_4 \dot{x}_3 + (K_1 + K_2 + K_4)x_1 - K_2 x_2 - K_4 x_3 = 0$$

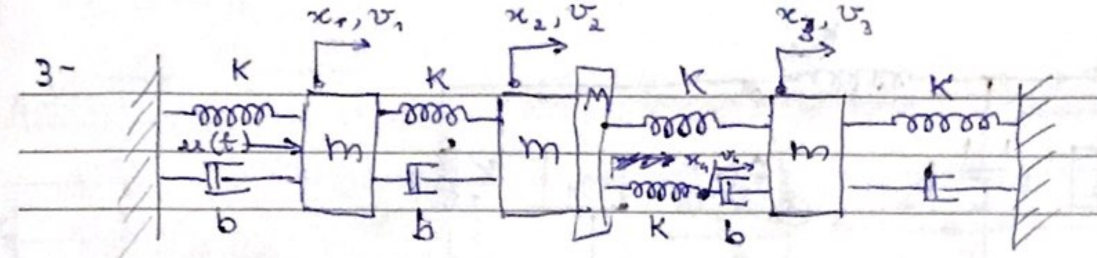
Nó 2:

$$v_2\left(b_2 + b_3 + \frac{K_2}{D} + \frac{K_3}{D} + m_2 D\right) - v_1\left(b_2 + \frac{K_2}{D}\right) - v_3\left(b_3 + \frac{K_3}{D}\right) = f_2(t)$$

$$\Rightarrow m_2 \ddot{x}_2 + (b_2 + b_3)\dot{x}_2 - b_2 \dot{x}_1 - b_3 \dot{x}_3 + (K_2 + K_3)x_2 - K_2 x_1 - K_3 x_3 = f_2(t)$$

Nó 3: $m_3 \ddot{x}_3 + (b_3 + b_4)\dot{x}_3 - b_4 \dot{x}_1 - b_3 \dot{x}_2 + (K_3 + K_4)x_3 - K_4 x_1 - K_3 x_2 = f_3(t)$

3 -



$K = 1\ N/m$
$b = 1\ N \cdot s/m$
$m = 1\ kg$
$M = 2m = 2\ kg$

a) Lagrangeano: $L = T - V$

$$T = \frac{m \cdot \dot{x}_1^2}{2} + \frac{(m+M)\dot{x}_2^2}{2} + \frac{m\dot{x}_3^2}{2}$$

$$V = \frac{Kx_1^2}{2} + \frac{K(x_2 - x_1)^2}{2} + \frac{K(x_3 - x_2)^2}{2} + \frac{K(x_4 - x_2)^2}{2} + \frac{Kx_3^2}{2}$$

$$R = \frac{b\dot{x}_1^2}{2} + \frac{b(\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b(\dot{x}_3 - \dot{x}_4)^2}{2} + \frac{b\dot{x}_3^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = f_{ext}$$

Coordenadas generalizadas: x_1, x_2, x_3 e x_4

Para x_1:

$$\frac{\partial L}{\partial \dot{x}_1} = m\dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m\ddot{x}_1$$

$$\frac{\partial L}{\partial x_1} = -Kx_1 + K(x_2 - x_1) \qquad \frac{\partial R}{\partial \dot{x}_1} = b\dot{x}_1 - b(\dot{x}_2 - \dot{x}_1)$$

$$\boxed{\therefore m\ddot{x}_1 + 2b\dot{x}_1 - b\dot{x}_2 + 2kx_1 - kx_2 = u(t)}$$

Para x_2:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = (m+M)\ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = -K(x_2 - x_1) + K(x_3 - x_2) + K(x_4 - x_2) \qquad \frac{\partial R}{\partial \dot{x}_2} = b(\dot{x}_2 - \dot{x}_1)$$

$$\therefore\ (m+M)\ddot{x}_2 + b\dot{x}_2 - b\dot{x}_1 + 3Kx_2 - Kx_1 - Kx_3 - Kx_4 = 0$$

Para x_3:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m\ddot{x}_3 \quad \Big| \quad \frac{\partial L}{\partial x_3} = -K(x_3 - x_2) - Kx_3$$

$$\frac{\partial R}{\partial \dot{x}_3} = b(\dot{x}_3 - \dot{x}_4) + b\dot{x}_3$$

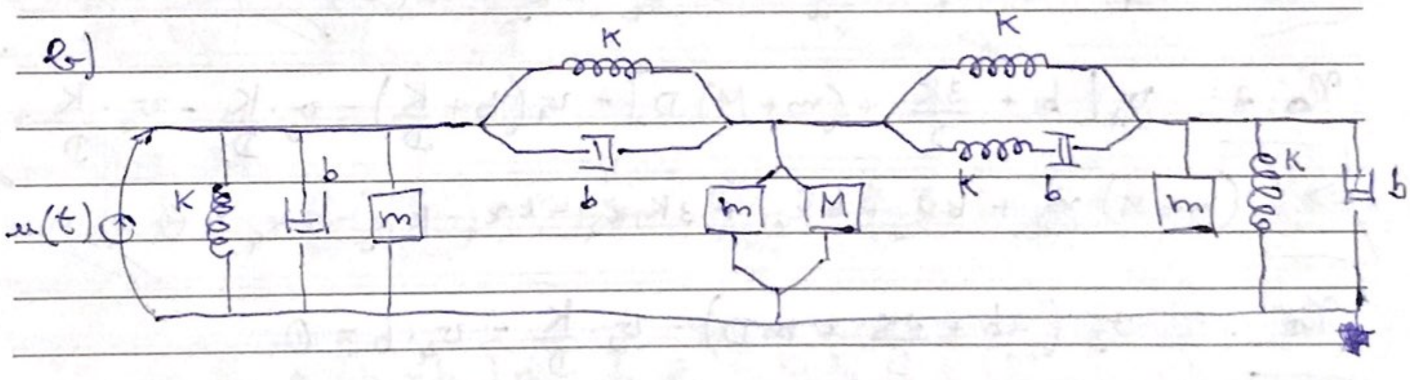
$$\therefore\ m\ddot{x}_3 + 2b\dot{x}_3 - b\dot{x}_4 + 2Kx_3 - Kx_2 = 0$$

Para x_4:

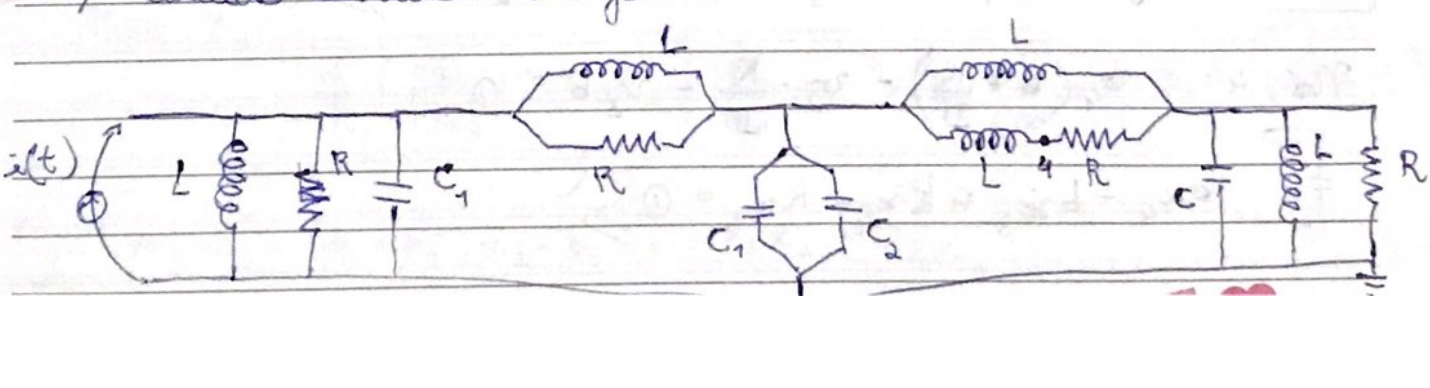
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_4}\right) = 0 \quad \Big| \quad \frac{\partial L}{\partial x_4} = -K(x_4 - x_2) \quad \Big| \quad \frac{\partial R}{\partial \dot{x}_4} = -b(\dot{x}_3 - \dot{x}_4)$$

$$\therefore\ b\dot{x}_4 - b\dot{x}_3 + Kx_4 - Kx_2 = 0$$

b)



c) Circuito elétrico análogo



d) Nó 1:

$$V_1\left(\frac{2}{R}+\frac{2}{LD}+CD\right)-V_2\left(\frac{1}{R}+\frac{1}{LD}\right)=i(t)$$

Nó 2:

$$V_2\left(\frac{1}{R}+\frac{3}{LD}+C_1D+C_2D\right)-V_1\left(\frac{1}{R}+\frac{1}{LD}\right)-V_3\cdot\frac{1}{LD}-V_4\cdot\frac{1}{LD}=0$$

Nó 3:

$$V_3\left(\frac{2}{R}+\frac{2}{LD}+CD\right)-V_2\cdot\frac{1}{LD}-V_4\cdot\frac{1}{R}=0$$

Nó 4:

$$V_4\left(\frac{1}{R}+\frac{1}{LD}\right)-V_2\cdot\frac{1}{LD}-V_3\cdot\frac{1}{R}=0$$

e) Nó 1:

$$v_1\left(2b+\frac{2K}{D}+mD\right)-v_2\left(b+\frac{K}{D}\right)=u(t)$$

$$\Rightarrow\quad m\ddot{x}_1+2b\dot{x}_1-b\dot{x}_2+2Kx_1-Kx_2=u(t)$$

Nó 2: $v_2\left[b+\frac{3K}{D}+(m+M)D\right]-v_1\left(b+\frac{K}{D}\right)-v_3\cdot\frac{K}{D}-v_4\cdot\frac{K}{D}=0$

$$\Rightarrow\quad (m+M)\ddot{x}_2+b\dot{x}_2+b\dot{x}_1+3Kx_2-Kx_1-Kx_3-Kx_4=0$$

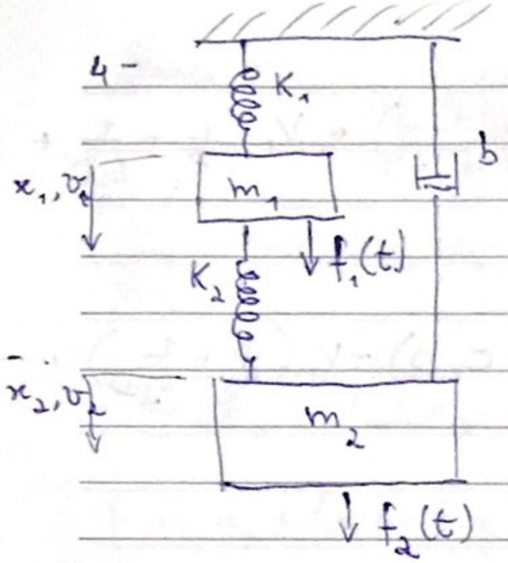
Nó 3: $v_3\left(2b+\frac{2K}{D}+mD\right)-v_2\cdot\frac{K}{D}-v_4\cdot b=0$

$$\Rightarrow\quad m\ddot{x}_3-b\dot{x}_4+2b\dot{x}_3+2Kx_3-Kx_2=0$$

Nó 4: $v_4\left(b+\frac{K}{D}\right)-v_2\cdot\frac{K}{D}-v_3 b=0$

$$\Rightarrow\quad b\dot{x}_4-b\dot{x}_3+Kx_4-Kx_2=0$$

4 -



a) Lagrangeano: $L = T - V$

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$$

$$V = \frac{K_1 x_1^2}{2} + \frac{K_2 (x_2 - x_1)^2}{2}$$

$$R = \frac{b \dot{x}_2^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = f_{ext}$$

Coordenadas generalizadas: x_1 e x_2

Para x_1:

$$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (x_2 - x_1) \quad \Bigg| \quad \frac{\partial R}{\partial \dot{x}_1} = 0$$

$$\therefore \; m \ddot{x}_1 + K_1 x_1 - K_2 (x_2 - x_1) = f_1(t)$$

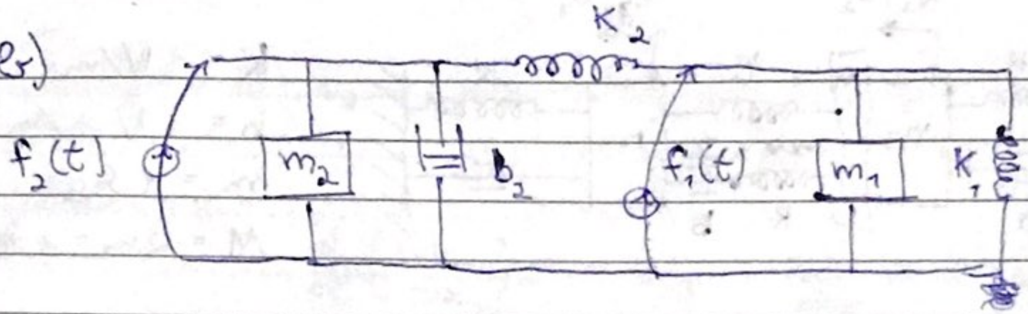
$$\boxed{\Rightarrow \; m \ddot{x}_1 + (K_1 + K_2) x_1 - K_2 x_2 = f_1(t)}$$

Para x_2:

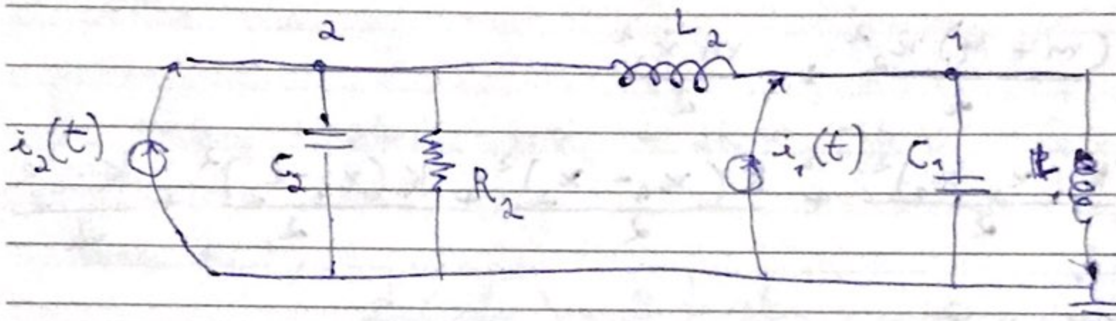
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \quad \Bigg| \quad \frac{\partial L}{\partial x_2} = -K_2 (x_2 - x_1) \quad \Bigg| \quad \frac{\partial R}{\partial \dot{x}_2} = b \dot{x}_2$$

$$\boxed{\therefore \; m_2 \ddot{x}_2 + b \dot{x}_2 + K_2 x_2 - K_2 x_1 = f_2(t)}$$

b)



c) Circuito elétrico análogo:



d) Nó 1:

$$V_1\left(\frac{1}{L_1D}+\frac{1}{L_2D}+C_1D\right) - V_2\cdot\frac{1}{L_2D} = i_1(t)$$

Nó 2:

$$V_2\left(\frac{1}{R_2}+\frac{1}{L_2D}+C_2D\right) - V_1\cdot\frac{1}{L_2D} = i_2(t)$$

e) Nó 1:

$$v_1\left(\frac{K_1}{D}+\frac{K_2}{D}+m_1D\right) - v_2\cdot\frac{K_2}{D} = f_1(t)$$

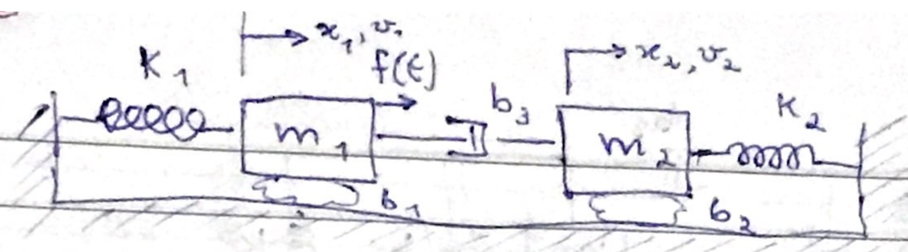
$$\Rightarrow m_1\ddot{x}_1 + (K_1+K_2)x_1 - K_2x_2 = f_1(t)$$

Nó 2:

$$v_2\left(b_2+\frac{K_2}{D}+m_2D\right) - v_1\cdot\frac{K_2}{D} = f_2(t)$$

$$\Rightarrow m_2\ddot{x}_2 + b_2\dot{x}_2 + k_2x_2 - k_2x_1 = f_2(t)$$

5-



a) Lagrangeano : $L = T - V$

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$$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 \dot{x}_2^2}{2} + \frac{b_3 (\dot{x}_2 - \dot{x}_1)^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = f_{ext}$$ | Coordenadas generalizadas: x_1 e x_2

Para x_1:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \quad \Big| \quad \frac{\partial L}{\partial x_1} = -K_1 x_1 \quad \Big| \quad \frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_3(\dot{x}_2 - \dot{x}_1)$$

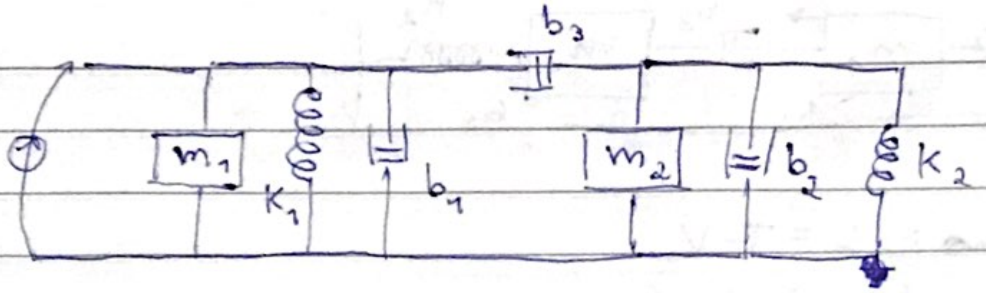
$$\therefore \; m_1 \ddot{x}_1 + (b_1 + b_3)\dot{x}_1 - b_3 \dot{x}_2 + K_1 x_1 = f(t)$$

Para x_2:

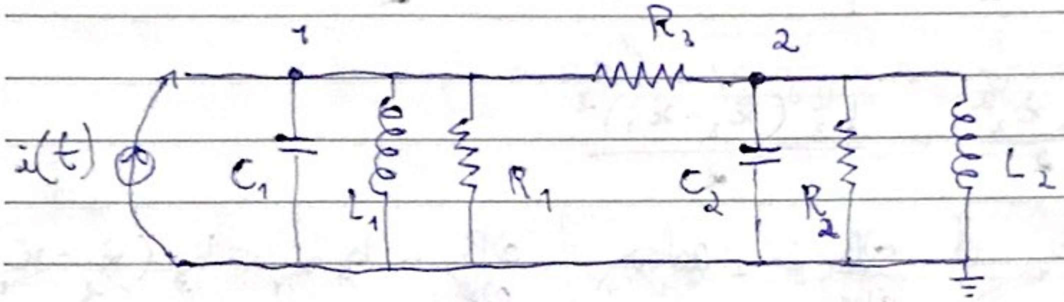
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \quad \Big| \quad \frac{\partial L}{\partial x_2} = -K_2 x_2 \quad \Big| \quad \frac{\partial R}{\partial \dot{x}_2} = b_2 \dot{x}_2 + b_3(\dot{x}_2 - \dot{x}_1)$$

$$\therefore \; m_2 \ddot{x}_2 + (b_2 + b_3)\dot{x}_2 - b_3 \dot{x}_1 + K_2 x_2 = 0$$

b)



c) Circuito elétrico análogo:



d) Nó 1:

$$\boxed{V_1\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{L_1 D}+C_1 D\right) - V_2\cdot\frac{1}{R_3} = i(t)}$$

Nó 2:

$$\boxed{V_2\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{L_2 D}+C_2 D\right) - V_1\cdot\frac{1}{R_3} = 0}$$

e) Nó 1:

$$v_1\left(b_1+b_1+\frac{K_1}{D}+m_1 D\right) - v_2 b_3 = f(t)$$

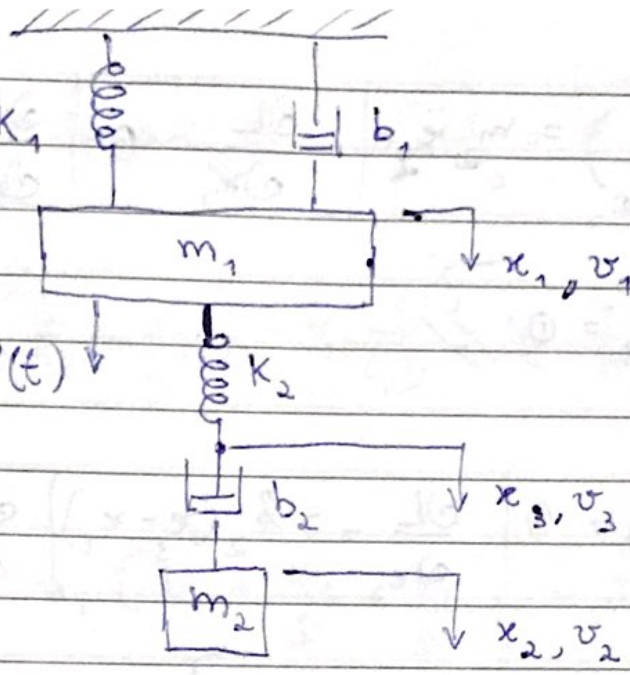
$$\boxed{\Rightarrow m_1\ddot{x}_1 + (b_1+b_3)\dot{x}_1 - b_3\dot{x}_2 + K_1 x_1 = f(t)}$$

Nó 2:

$$v_2\left(b_2+b_3+\frac{K_2}{D}+m_2 D\right) - v_1 b_3 = 0$$

$$\boxed{\Rightarrow m_2\ddot{x}_2 + (b_2+b_3)\dot{x}_2 - b_3\dot{x}_1 + k_2 x_2 = 0}$$

6-



a) Lagrangeano: $L = T - V$

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} \quad \Big| \quad V = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_3 - x_1)^2}{2}$$

$$R = \frac{b_1 \dot{x}_1}{2} + \frac{b_2 (\dot{x}_2 - \dot{x}_3)^2}{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = f_{ext}$$

Coordenadas generalizadas: x_1, x_2 e x_3

Para x_1:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \quad \Big| \quad \frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (x_3 - x_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1$$

$$\boxed{\therefore m_1 \ddot{x}_1 + b_1 \dot{x}_1 + (K_1 + K_2) x_1 - K_2 x_3 = f(t)}$$

Para x_2:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \quad \Big| \quad \frac{\partial L}{\partial x_2} = 0 \quad \Big| \quad \frac{\partial R}{\partial \dot{x}_2} = b_2(\dot{x}_2 - \dot{x}_3)$$

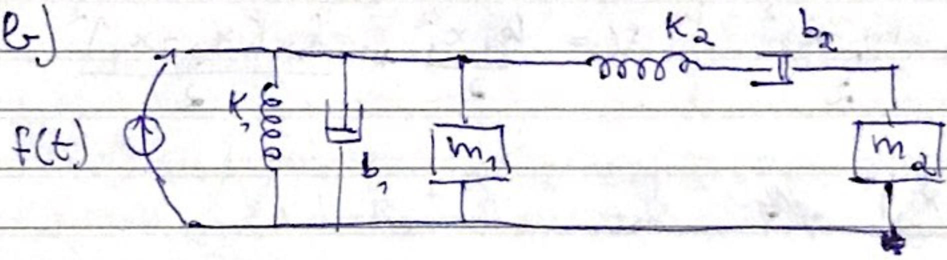
$$\boxed{\therefore m\ddot{x}_2 + b_2 \dot{x}_2 - b_2 \dot{x}_3 = 0}$$

Para x_3:

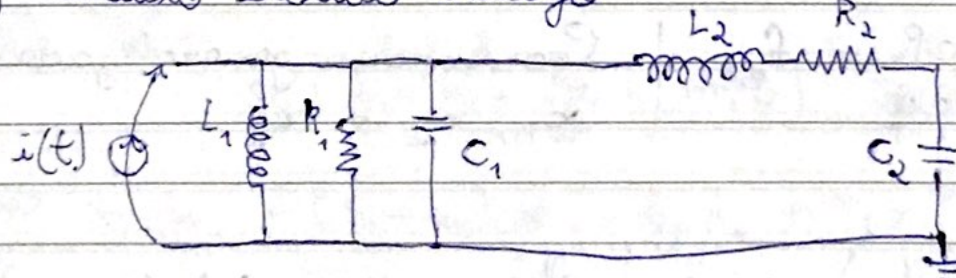
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = 0 \quad \Big| \quad \frac{\partial L}{\partial x_3} = -k_2(x_3 - x_1) \quad \Big| \quad \frac{\partial R}{\partial \dot{x}_3} = -b_2(\dot{x}_2 - \dot{x}_3)$$

$$\boxed{\therefore b_2 \dot{x}_3 - b_2 \dot{x}_2 + k_2 x_3 - k_2 x_1 = 0}$$

b)



c) Circuito elétrico análogo



d) Nó 1:

$$\boxed{V_1\left(\frac{1}{R_1} + \frac{1}{L_1 D} + \frac{1}{L_2 D} + C_1 D\right) - V_3 \cdot \frac{1}{L_2 D} = i(t)}$$

Nó 2:

$$V_2\left(\frac{1}{R_2} + C_2 D\right) - V_3 \cdot \frac{1}{R_2} = 0$$

Nó 3:

$$V_3\left(\frac{1}{R_2} + \frac{1}{L_2 D}\right) - V_1 \cdot \frac{1}{L_2 D} - V_2 \cdot \frac{1}{R_2} = 0$$

e) Nó 1:

$$v_1\left(b_1 + \frac{K_1}{D} + \frac{K_2}{D} + m_1 D\right) - v_3 \cdot \frac{K_2}{D} = f(t)$$

$$\Rightarrow m_1 \ddot{x}_1 + b_1 \dot{x}_1 + (K_1 + K_2)x_1 - K_2 x_3 = f(t)$$

Nó 2:

$$v_2(b_2 + m_2 D) - v_3 b_2 = 0$$

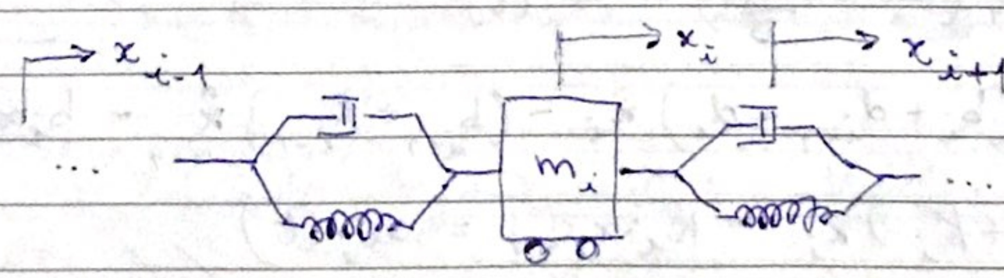
$$\Rightarrow m_2 \ddot{x}_2 + b_2 \dot{x}_2 - b_2 \dot{x}_3 = 0$$

Nó 3:

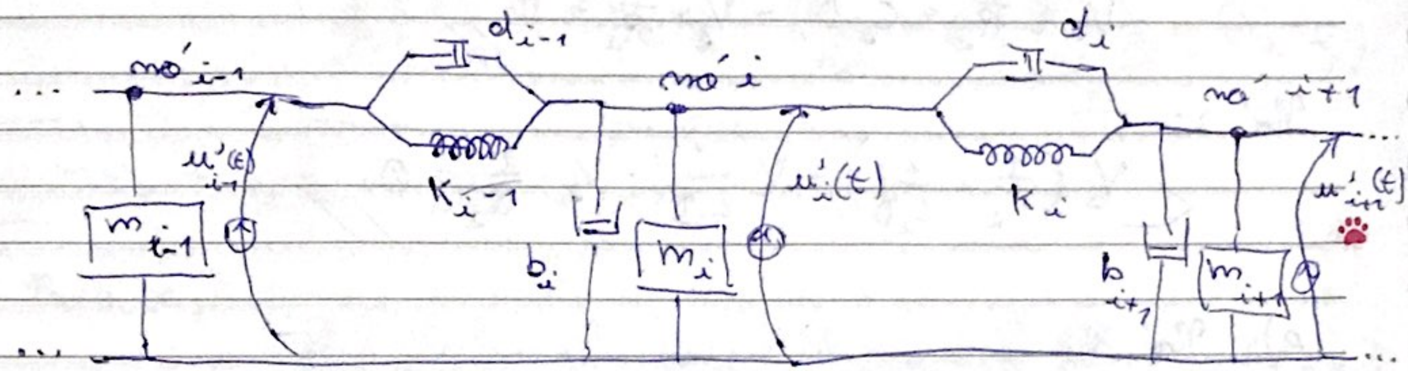
$$v_3\left(b_2 + \frac{K_2}{D}\right) - v_1 \cdot \frac{K_2}{D} - v_2 b_2 = 0$$

$$\Rightarrow b_2 \dot{x}_3 - b_2 \dot{x}_2 + K_2 x_3 - K_2 x_1 = 0$$

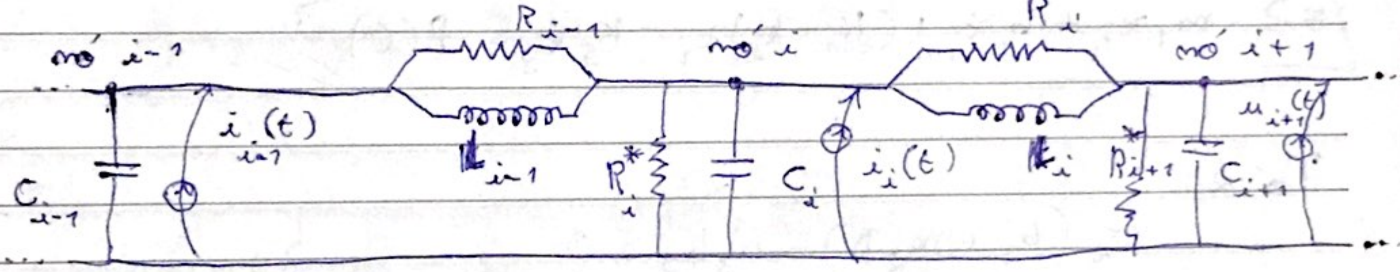
7-



7 - Dos vagões $i-1$ a $i+1$:



Circuito elétrico análogo



Nó i:

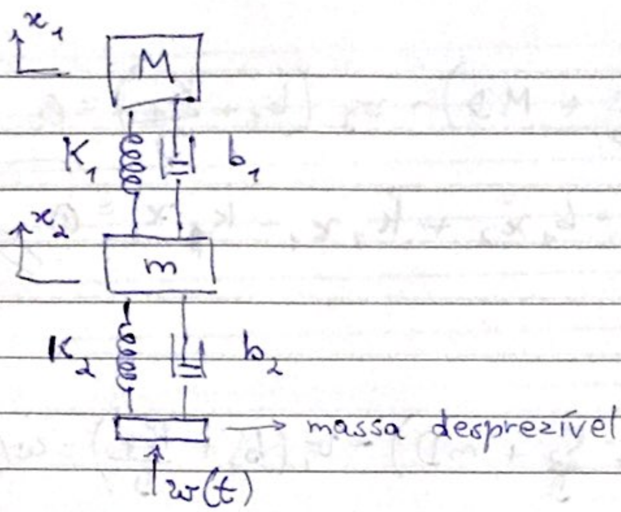
$$\begin{cases} V_i\left(\frac{1}{R_{i-1}}+\frac{1}{R_i}+\frac{1}{R_i^*}+\frac{1}{L_{i-1}D}+\frac{1}{L_i}+C_iD\right)-V_{i-1}\left(\frac{1}{R_{i-1}}+\frac{1}{L_{i-1}D}\right)+ \\ -V_{i+1}\left(\frac{1}{R_1}+\frac{1}{L_iD}\right)=i_i(t) \end{cases}$$

$$\Rightarrow v_1\left(d_{i-1}+d_i+b_1+\frac{K_{i-1}}{D}+\frac{K_i}{D}\right)-V_{i-1}\left(d_{i-1}+\frac{K_{i-1}}{D}\right)+$$

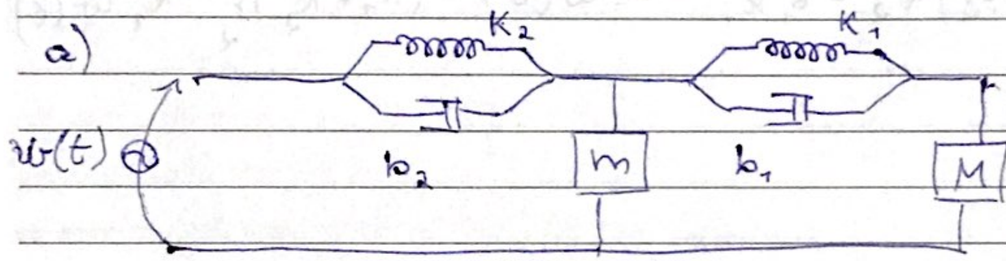
$$-v_{i+1}\left(d_i+\frac{K_i}{D}\right)=u_i(t)$$

$$\Rightarrow \boxed{\begin{aligned} & m_i\ddot{x}_i+(b_i+d_{i-1}+d_i)\dot{x}_i-(b_{i-1}+K_{i-1})\dot{x}_{i-1}-b_ix_{i+1}+ \\ & +(K_{i-1}+K_i)x_i-K_ix_{i+1}=u_i(t) \end{aligned}}$$

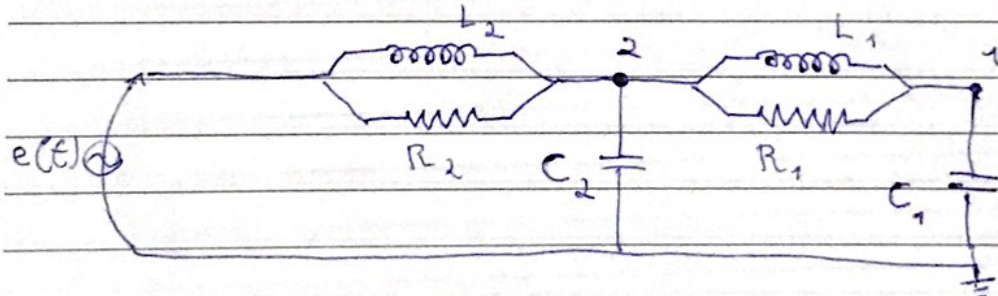
8-



a)



Circuito elétrico análogo:



Nó 1:

$$V_1\left(\frac{1}{R_1} + \frac{1}{L_1 D} + C_1 D\right) - V_2\left(\frac{1}{R_1} + \frac{1}{L_1 D}\right) = 0$$

Nó 2:

$$V_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{L_1 D} + \frac{1}{L_2 D} + C_2 D\right) - V_1\left(\frac{1}{R_1} + \frac{1}{L_1 D}\right) - e(t)\left(\frac{1}{R_1} + \frac{1}{L_1 D}\right) = 0$$

$\Rightarrow$ Nó 1:

$$v_1\left(b_1+\frac{K_1}{D}+MD\right)-v_2\left(b_1+\frac{K_1}{D}\right)=0$$

$$\boxed{\Rightarrow \quad M\ddot{x}_1+b_1\dot{x}_1-b_1\dot{x}_2+K_1x_1-K_1x_2=0}$$

Nó 2:

$$v_2\left(b_1+b_2+\frac{K_1}{D}+\frac{K_2}{D}+mD\right)-v_1\left(b_1+\frac{K_1}{D}\right)-\dot{w}(t)\left(b_1+\frac{K_1}{D}\right)=0$$

$$\Rightarrow \quad m\ddot{x}_2+(b_1+b_2)\dot{x}_2-b_1\dot{x}_1-b_1\dot{w}(t)+(K_1+K_2)x_2-K_1w(t)$$

$$-K_1x_1=0$$