Exercícios - Aula 03/09

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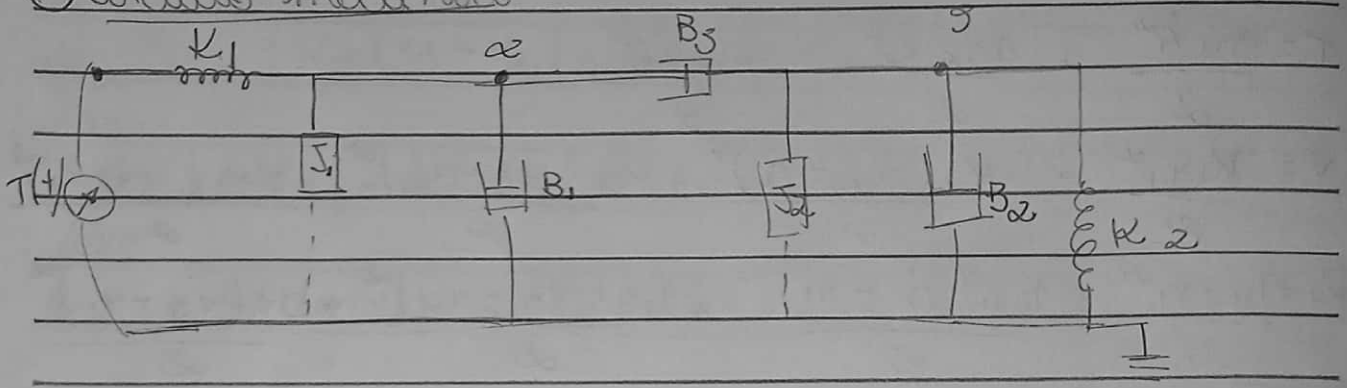
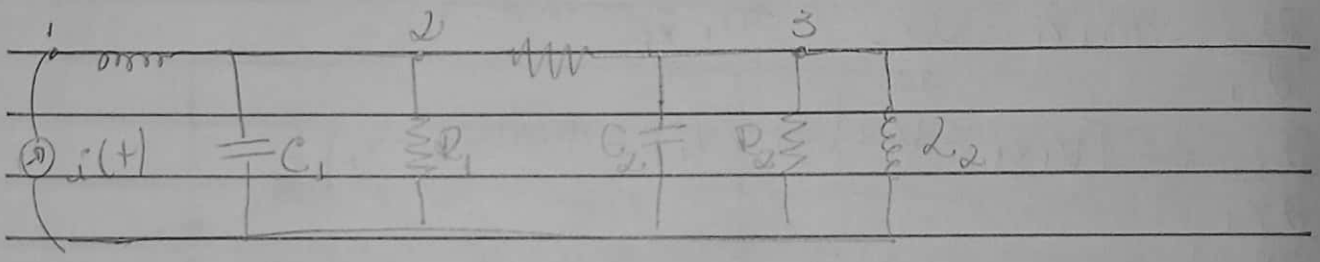
① Circuito mecânico



Circuito elétrico



Nó 1: $\frac{V_1}{L_1 D} - \frac{V_2}{L_1 D} = i(t)$

Nó 2: $V_2\left(C_1 D + \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{L_1 D}\right) - \frac{V_1}{L_1 D} - \frac{V_3}{R_3} = 0$

Nó 3: $V_3\left(C_2 D + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2 D}\right) - \frac{V_2}{R_3} = 0$

f. → T → i $\begin{cases} \text{Nó 1: } K_1(x_1 - x_2) = T(t) \\ \text{Nó 2: } \omega_2\left(J_1 D + B_1 + B_3 + \frac{K_1}{D}\right) - \omega_1 \frac{K_1}{D} - \omega_3 B_3 \end{cases}$

v → ω → T

$$\boxed{\text{Nó 2: } J_1 \ddot{\theta}_2 + (B_1 + B_3)\dot{\theta}_2 + K_1\theta_2 = K_1\theta_1 + B_3\dot{\theta}_3}$$

Nó 3: $\omega_3\left(J_2 D + B_2 + B_3 + K_2/D\right) = B_3 \omega_2$

Nó 3: $J_2\ddot{\theta}_3 + (B_2 + B_3)\dot{\theta}_3 + K_2\theta_3 = B_3\dot{\theta}_2$

② $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_{ext}$

$L = T - V$

$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{m_3 \dot{x}_3^2}{2}$

$V = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_2 - x_1)^2}{2} + \frac{k_3 (x_3 - x_2)^2}{2} + \frac{k_4 (x_3 - x_1)^2}{2}$

$R = \frac{b_1 \cdot \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b_3 (\dot{x}_3 - \dot{x}_2)^2}{2} + \frac{b_4 (\dot{x}_3 - \dot{x}_1)^2}{2}$

x_1 | $F_1(t) = 0$

$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 = \frac{d}{dt} = m_1 \ddot{x}_1$

$\frac{\partial L}{\partial x_1} = -[k_1 x_1 - k_2 (x_2 - x_1) - k_4 (x_3 - x_1)]$

$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_2 (\dot{x}_2 - \dot{x}_1) - b_4 (\dot{x}_3 - \dot{x}_1)$

$m_1 \ddot{x}_1 + b_1 \dot{x}_1 - b_2 (\dot{x}_2 - \dot{x}_1) - b_4 (\dot{x}_3 - \dot{x}_1) + k_1 x_1 - k_2 (x_2 - x_1) - k_4 (x_3 - x_1) = 0$

x_2 | $F_2(t) = f_{x2}(t)$

$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \Rightarrow \frac{d}{dt} = m_2 \ddot{x}_2$

$\frac{\partial L}{\partial x_2} = -(k_2 (x_2 - x_1)) - k_3 (x_3 - x_2))$

$\frac{\partial R}{\partial \dot{x}_2} = b_2 (\dot{x}_2 - \dot{x}_1) - b_3 (\dot{x}_3 - \dot{x}_2)$

$m_2 \ddot{x}_2 + b_2 (\dot{x}_2 - \dot{x}_1) - b_3 (\dot{x}_3 - \dot{x}_2) + k_2 (x_2 - x_1) - k_3 (x_3 - x_2) = f_2(t)$

x_3 $F_3(t) = f_3(t)$

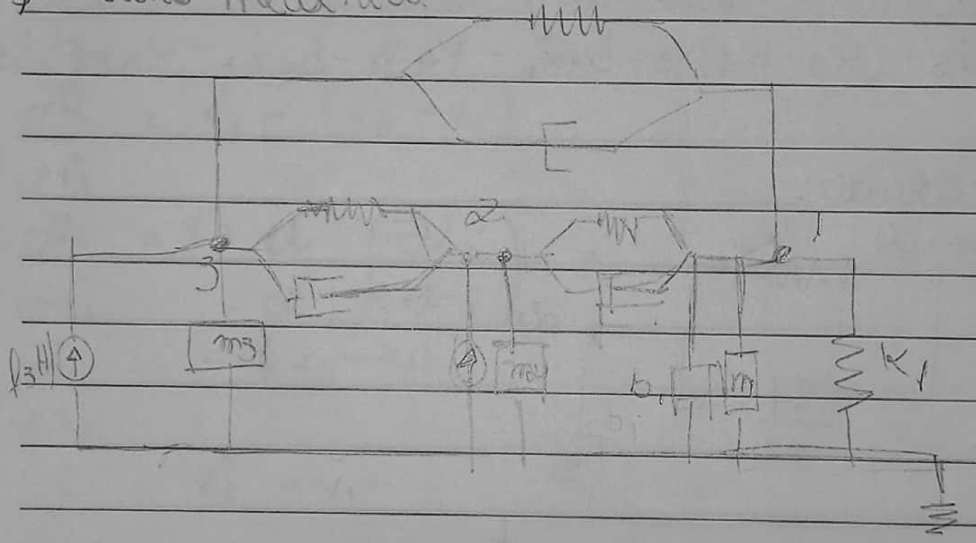
$$\frac{\partial L}{\partial \dot{x}_3} = m_3 \dot{x}_3 \quad \frac{d}{dt} : m_3 \ddot{x}_3$$

$$\frac{\partial L}{\partial x_3} = -\left(K_3(x_3 - x_2) + K_4(x_3 - x_1)\right)$$

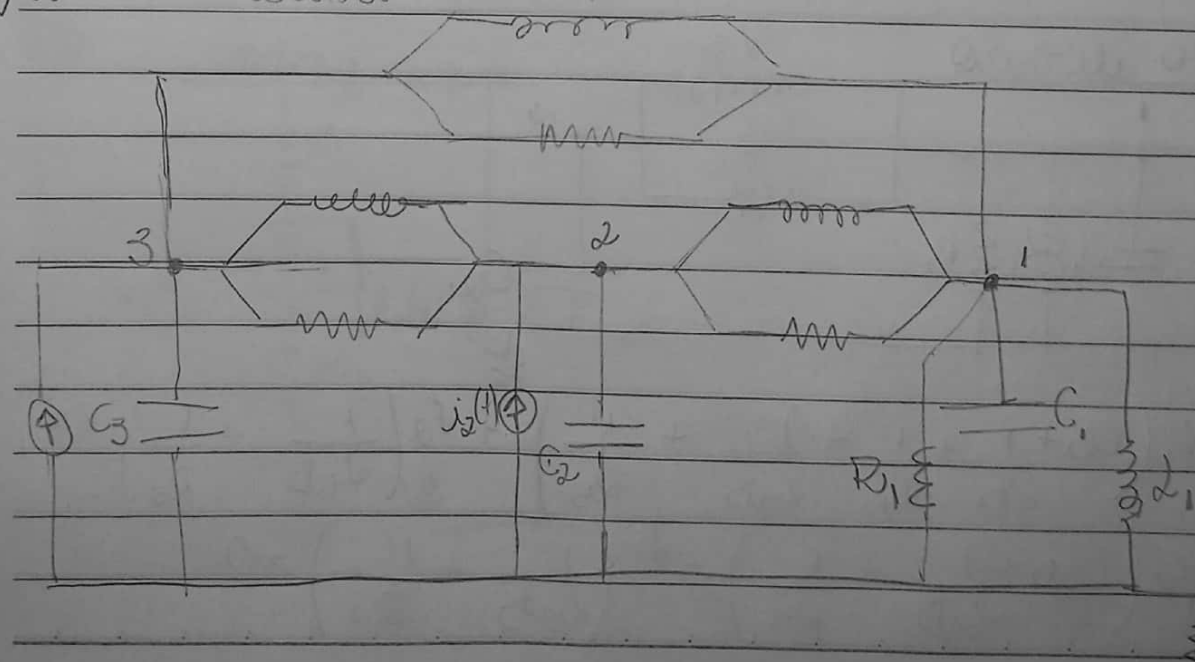
$$\frac{\partial \mathcal{L}}{\partial \dot{x}_3} = b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1)$$

$$m_3\ddot{x}_3 + b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1) + K_3(x_3 - x_2) + K_4(x_3 - x_1) = F_3(t)$$

b) Circuito mecânico



c) Circuito elétrico:



d) Nó 1: $V_1\left(\frac{1}{R_1} + C_1D + \frac{1}{L_1D} + \frac{1}{R_2} + \frac{1}{L_2D} + \frac{1}{R_4} + \frac{1}{L_4D}\right) - V_2\left(\frac{1}{L_2D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{L_4D} + \frac{1}{R_4}\right) = 0$

Nó 2: $V_2\left(\frac{1}{L_2D} + \frac{1}{R_2} + \frac{1}{L_3D} + \frac{1}{R_3} + C_2D\right) - V_1\left(\frac{1}{L_2D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{L_3D} + \frac{1}{R_3}\right) = 0$ (1)

Nó 3: $V_3\left(\frac{1}{L_3D} + \frac{1}{R_3} + C_3D + \frac{1}{L_4D} + \frac{1}{R_4}\right) - V_1\left(\frac{1}{L_4D} + \frac{1}{R_4}\right) - V_2\left(\frac{1}{L_3D} + \frac{1}{R_3}\right) = 0$

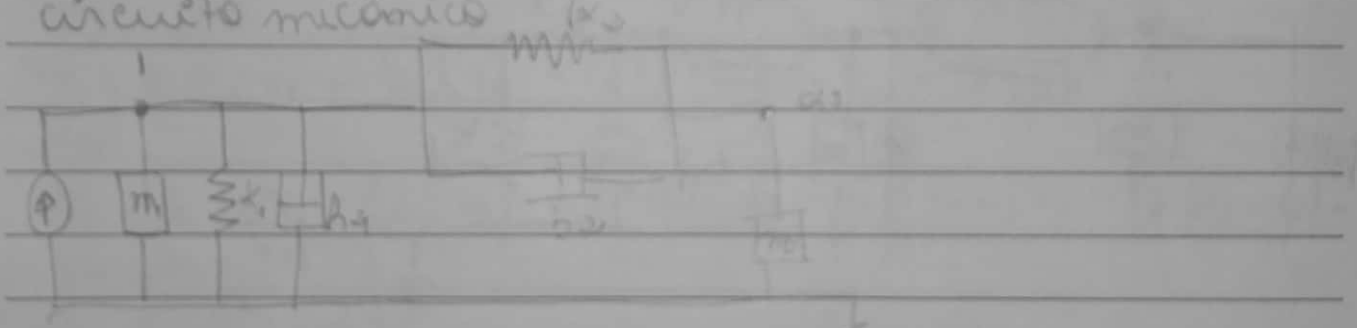
e) $m_1\ddot{x}_1 + (b_1 + b_2 + b_4)\dot{x}_1 + (K_1 + K_2 + K_4)x_1 - b_2\dot{x}_2 - K_2x_2 - b_4\dot{x}_3 - K_4x_3 = 0$

$m_2\ddot{x}_2 + (b_2 + b_3)\dot{x}_2 + (K_2 + K_3)x_2 - b_2\dot{x}_1 - K_2x_1 - b_3\dot{x}_3 - K_3x_3 = f_2(t)$

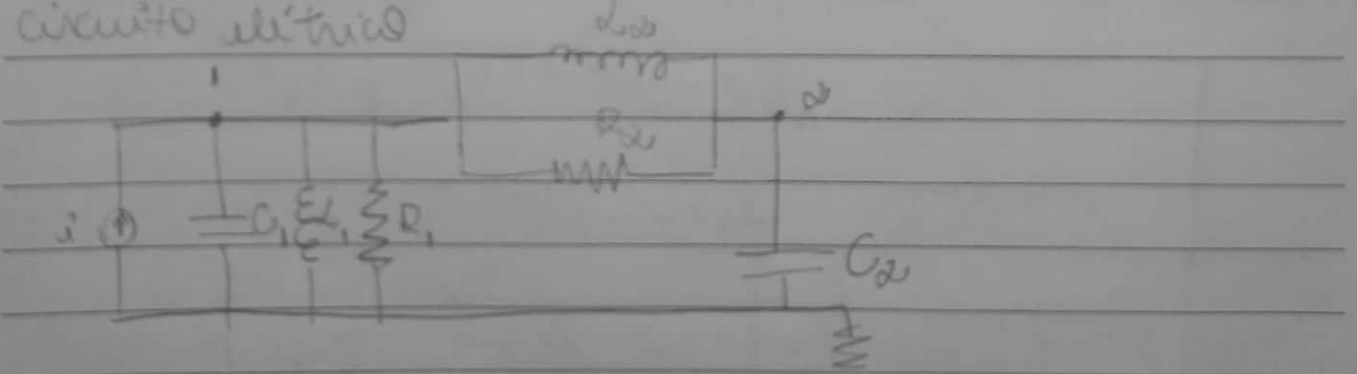
$m_3\ddot{x}_3 + (b_3 + b_4)\dot{x}_3 + (K_3 + K_4)x_3 - b_4\dot{x}_1 - K_4x_1 - b_3\dot{x}_2 - K_3x_2 = f_3(t)$

③ Contato desacoplado:

circuito mecânico



circuito elétrico



Nó 1: $V_1\left(C_1D + \frac{1}{L_1D} + \frac{1}{R_1} + \frac{1}{L_2D} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{L_2D} + \frac{1}{R_2}\right) = i$

Nó 2: $V_2\left(C_2D + \frac{1}{L_2D} + \frac{1}{R_2}\right) - V_1\left(\frac{1}{L_2D} + \frac{1}{R_2}\right) = 0$

$\ddot{x}_1 + 2x_1 - \ddot{x}_2 + 2x_1 - x_2 = 0$

$\ddot{x}_2 - x_1 + \ddot{x}_2 - x_1 + x_2 = 0$

$L = \frac{\dot{x}_1^2}{2} + \frac{\dot{x}_2^2}{2} - \frac{x_1^2}{2} - \frac{(x_2 - x_1)^2}{2}$

$R = \frac{\dot{x}_1^2}{2} + \frac{(\dot{x}_2 - \dot{x}_1)^2}{2}$

$\frac{\partial L}{\partial \dot{x}_1} = \dot{x}_1 \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \ddot{x}_1$

$\frac{\partial L}{\partial x_1} = -x_1 + (x_2 - x_1)$

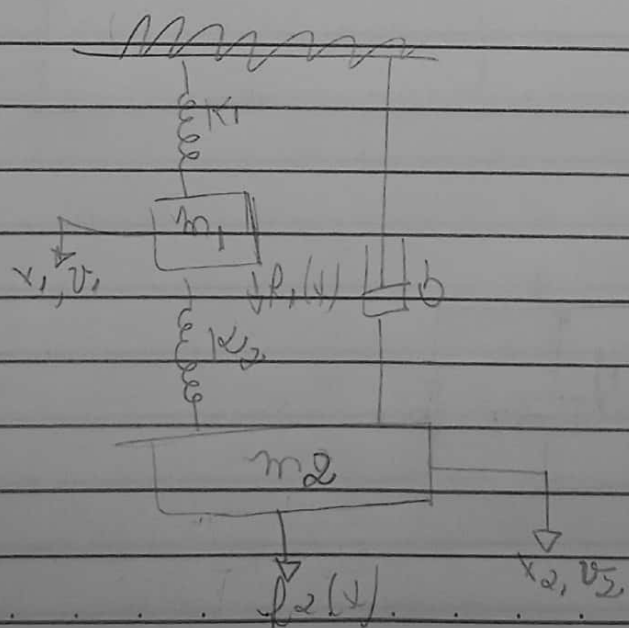
$\frac{\partial R}{\partial \dot{x}_1} = \dot{x}_1 - (\dot{x}_2 - \dot{x}_1)$

$\frac{\partial L}{\partial \dot{x}_1} = \dot{x}_2 \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) \dot{x}_2$

$\frac{\partial L}{\partial x_2} = -(x_2 - x_1)$

$\frac{\partial R}{\partial \dot{x}_2} = \dot{x}_2 - \dot{x}_1$

(4)



x_1

$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \qquad \frac{d}{dt} = m_1 \ddot{x}_1$

$\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (x_2 - x_1)$

$\frac{\partial D}{\partial \dot{x}_1} = 0$

$m\ddot{x}_1 + (K_1 + K_2) x_1 = K_2 x_2 + F_1(t)$

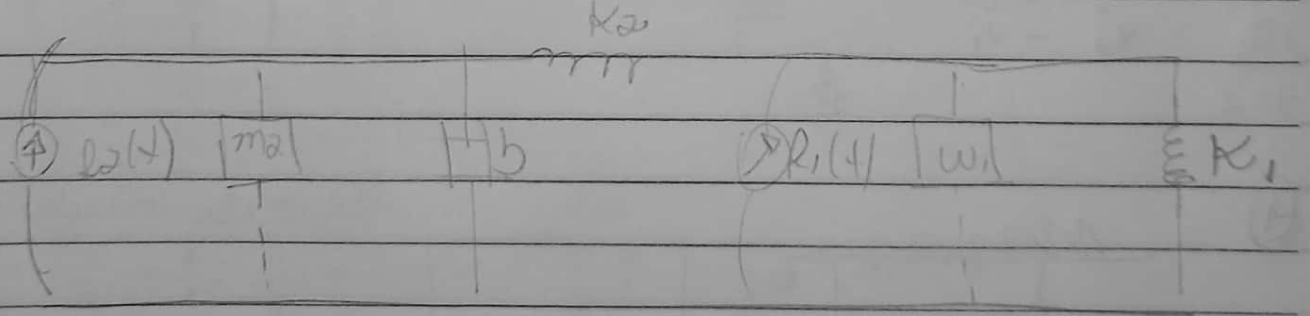
x_2

$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \qquad \frac{d}{dt} = m_2 \ddot{x}_2$

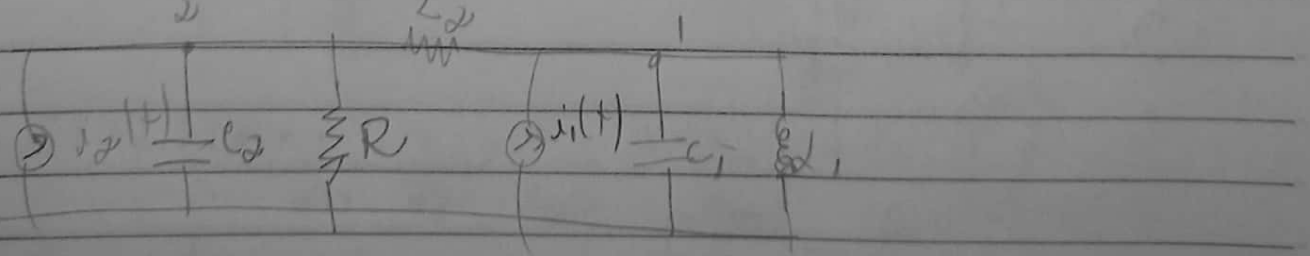
$\frac{\partial L}{\partial x_2} = -K_2 (x_2 - x_1) \qquad \frac{\partial D}{\partial \dot{x}_2} = b\dot{x}_2$

$m_2 \ddot{x}_2 + K_2 x_2 + b\dot{x}_2 = K_2 x_1 + F_2(t)$

Circuito mecânico



Circuito elétrico



Nó 1: $V_1\left(C_1D + \frac{1}{\alpha_1 D} + \frac{1}{L_2 D}\right) - \frac{V_2}{L_2 D} = \lambda_1(t)$

Nó 2: $V_2\left(C_2D + \frac{1}{R} + \frac{1}{L_2 D}\right) - \frac{V_1}{L_2 D} = \lambda_2(t)$

$m_1 \ddot{x}_1 + (K_1 + K_{12})x_1 = f_1(t) + K_2 x_2$

$m_2 \ddot{x}_2 + b\dot{x}_2 + K_2 x_2 = f_2(t) + K_{12} x_1$