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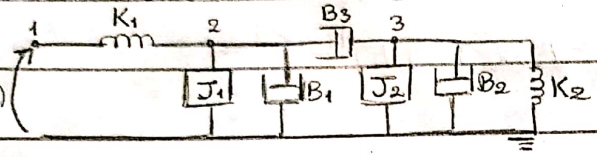
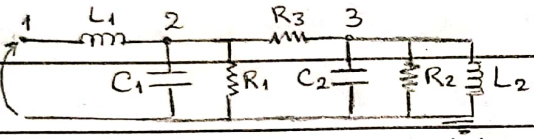
PME3380 - Exercícios da Aula 03/09

1) Sistema rotativo - Circuito Mecânico:



Circuito Elétrico:

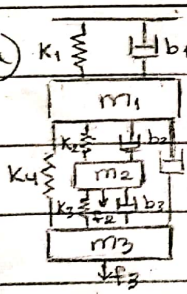


Nó 1: $i(t) = V_1\left(\frac{1}{L_1 D}\right) - V_2\left(\frac{1}{L_2 D}\right) \longrightarrow T(t) = K_1\theta_1 - K_2\theta_2$

Nó 2: $0 = V_2\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{R_1} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{L_1 D}\right) - V_3\left(\frac{1}{R_3}\right) \longrightarrow 0 = J_1\ddot{\theta}_2 + B_1\dot{\theta}_2 + B_3\theta_2 + K_1\theta_1 - K_1\theta_2 - B_3\dot{\theta}_3$

Nó 3: $0 = V_3\left(C_2 D + \frac{1}{L_2 D} + \frac{1}{R_2} + \frac{1}{R_3}\right) - V_2\left(\frac{1}{R_3}\right) \longrightarrow 0 = J_2\ddot{\theta}_3 + B_3\theta_3 + B_2\dot{\theta}_2 + K_2\theta_3 - B_3\dot{\theta}_2$

2) a)



J_{X_1,V_1}

J_{X_2,V_2}

J_{X_3,V_3}

$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_3\dot{x}_3^2$

$V = \frac{1}{2}K_1x_1^2 + \frac{1}{2}K_2(x_2 - x_1)^2 + \frac{1}{2}K_3(x_3 - x_2)^2 + \frac{1}{2}K_4(x_3 - x_1)^2$

$R = \frac{1}{2}b_1\dot{x}_1^2 + \frac{1}{2}b_2(\dot{x}_2 - \dot{x}_1)^2 + \frac{1}{2}b_3(\dot{x}_3 - \dot{x}_2)^2 + \frac{1}{2}b_4(\dot{x}_3 - \dot{x}_1)^2$

Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q$

$q_1 = x_1$: $\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1$

$\frac{\partial L}{\partial x_1} = -K_1x_1 + K_2(x_2 - x_1) + K_3(x_3 - x_2) + K_4(x_3 - x_1)$

$\frac{\partial R}{\partial \dot{x}_1} = b_1\dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) - b_4(\dot{x}_3 - \dot{x}_1)$

$$\boxed{m_1\ddot{x}_1 + K_1x_1 - K_2(x_2 - x_1) - K_3(x_3 - x_2) - K_4(x_3 - x_1) + b_1\dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) - b_4(\dot{x}_3 - \dot{x}_1) = 0}$$

$q_2 = x_2$: $\frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$

$\frac{\partial L}{\partial x_2} = -K_2(x_2 - x_1) + K_3(x_3 - x_2)$

$\frac{\partial R}{\partial \dot{x}_2} = b_2(\dot{x}_2 - \dot{x}_1) - b_3(\dot{x}_3 - \dot{x}_2)^2$

$$\boxed{m_2\ddot{x}_2 + K_2(x_2 - x_1) - K_3(x_3 - x_2) + b_2(\dot{x}_2 - \dot{x}_1) - b_3(\dot{x}_3 - \dot{x}_2) = f_2(t)}$$

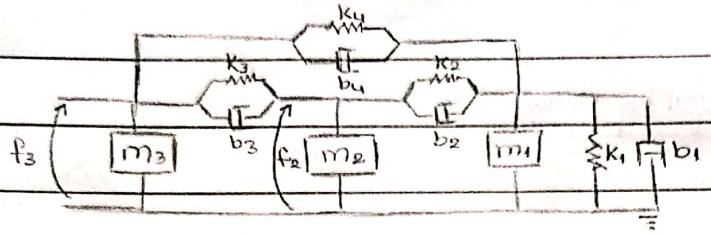
$q_3 = x_3$: $\frac{\partial L}{\partial \dot{x}_3} = m_3 \dot{x}_3 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3 \ddot{x}_3$

$\frac{\partial L}{\partial x_3} = -K_3(x_3 - x_2) - K_4(x_3 - x_1)$

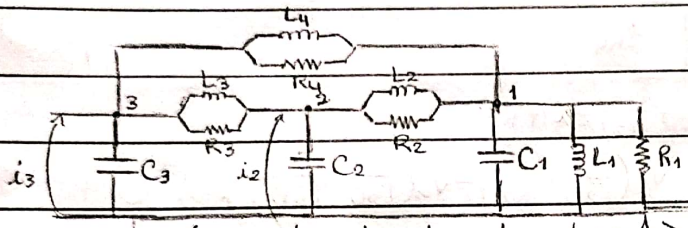
$\frac{\partial R}{\partial \dot{x}_3} = b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1)$

$$m_3\ddot{x}_3 + K_3(x_3 - x_2) + K_4(x_3 - x_1) + b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1) = f_3(t)$$

b)



c)



d) Nó 1: $V_1\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D} + \frac{1}{L_4 D} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - V_2\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{L_4 D} + \frac{1}{R_4}\right) = 0$

Nó 2: $V_2\left(C_2 D + \frac{1}{L_2 D} + \frac{1}{L_3 D} + \frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{L_3 D} + \frac{1}{R_3}\right) = i_2$

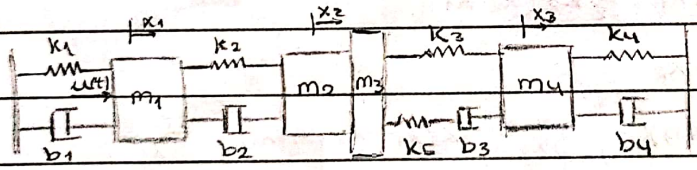
Nó 3: $V_3\left(C_3 D + \frac{1}{L_3 D} + \frac{1}{L_4 D} + \frac{1}{R_3} + \frac{1}{R_4}\right) - V_1\left(\frac{1}{L_4 D} + \frac{1}{R_4}\right) - V_2\left(\frac{1}{L_3 D} + \frac{1}{R_3}\right) = i_3$

e) $m_1\ddot{x}_1 + K_1 x_1 - K_2(x_2 - x_1) - K_3(x_3 - x_2) - K_4(x_3 - x_1) + b_1\dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) - b_4(\dot{x}_3 - \dot{x}_1) = 0$

$m_2\ddot{x}_2 + K_2(x_2 - x_1) - K_3(x_3 - x_2) + b_2(\dot{x}_2 - \dot{x}_1) - b_3(\dot{x}_3 - \dot{x}_2) = f_2$

$m_3\ddot{x}_3 + K_3(x_3 - x_2) + K_4(x_3 - x_1) + b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1) = f_3$

3) e)



$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}(m_2 + m_3)\dot{x}_2^2 + \frac{1}{2}m_4\dot{x}_3^2$

$V = \frac{1}{2}K_1 x_1^2 + \frac{1}{2}K_2(x_2 - x_1)^2 + \frac{1}{2}K_3(x_3 - x_2)^2 + \frac{1}{2}K_4 x_3^2 + \frac{1}{2}K_5(x_3 - x_2)^2$

$R = \frac{1}{2}b_1\dot{x}_1^2 + \frac{1}{2}b_2(\dot{x}_2 - \dot{x}_1)^2 + \frac{1}{2}b_3(\dot{x}_3 - \dot{x}_2)^2 + \frac{1}{2}b_4\dot{x}_3^2$

$q_1 = x_1$: $\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1$

$\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2(x_2 - x_1)$

$\frac{\partial R}{\partial \dot{x}_1} = b_1\dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1)$

$$m_1\ddot{x}_1 + K_1 x_1 - K_2(x_2 - x_1) + b_1\dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) = u(t)$$

$q_2 = x_2$: $\frac{\partial L}{\partial \dot{x}_2} = (m_2 + m_3)\dot{x}_2^2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = (m_2 + m_3)\ddot{x}_2^2$

$\frac{\partial L}{\partial x_2} = -K_2(x_2 - x_1) + K_3(x_3 - x_2) + K_5(x_3 - x_2)$

$\frac{\partial R}{\partial \dot{x}_2} = b_2(\dot{x}_2 - \dot{x}_1) - b_3(\dot{x}_3 - \dot{x}_2)$

$$(m_2 + m_3)\ddot{x}_2 + K_2(x_2 - x_1) - K_3(x_3 - x_2) - K_5(x_3 - x_2) + b_2(\dot{x}_2 - \dot{x}_1) - b_3(\dot{x}_3 - \dot{x}_2) = 0$$

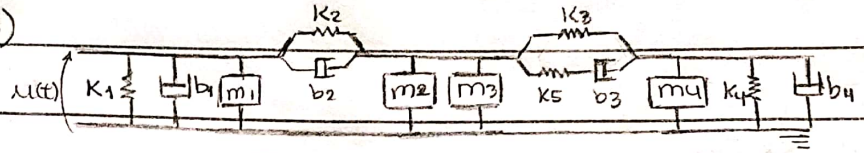
· $q_3 = x_3$: $\frac{\partial L}{\partial \dot{x}_3} = m_4 \dot{x}_3 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_4 \ddot{x}_3$

$\frac{\partial L}{\partial x_3} = -K_3(x_3 - x_2) - K_4 x_3 - K_5(x_3 - x_2)$

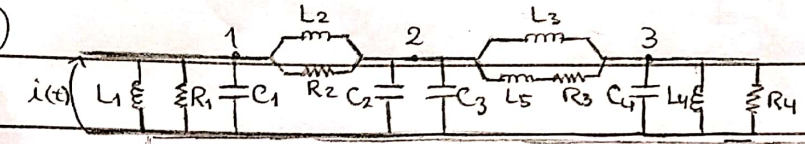
$\frac{\partial R}{\partial \dot{x}_3} = b_3(\dot{x}_3 - \dot{x}_2) + b_4 \dot{x}_3$

$$m_4 \ddot{x}_3 + K_3(x_3 - x_2) + K_4 x_3 + K_5(x_3 - x_2) + b_3(\dot{x}_3 - \dot{x}_2) + b_4 \dot{x}_3 = 0$$

b)



c)



d) Nó 1: $V_1\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D} + \frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) = i(t)$

Nó 2: $V_2\left(C_2 D + C_3 D + \frac{1}{L_2 D} + \frac{1}{L_3 D} + \frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{L_2 D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{L_3 D} + \frac{1}{L_5 D} + \frac{1}{R_3}\right) = 0$

Nó 3: $V_3\left(C_3 D + \frac{1}{L_3 D} + \frac{1}{L_4 D} + \frac{1}{R_3} + \frac{1}{R_4}\right) - V_2\left(\frac{1}{L_3 D} + \frac{1}{L_5 D} + \frac{1}{R_3}\right) = 0$

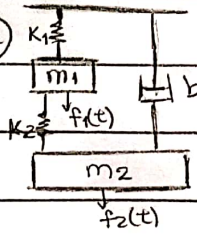
e)

$$m_1 \ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) + b_1 \dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) = u(t)$$

$$(m_2 + m_3)\ddot{x}_2 + K_2(x_2 - x_1) - K_3(x_3 - x_2) - K_5(x_2 - x_2) + b_2(\dot{x}_2 - \dot{x}_1) - b_3(\dot{x}_3 - \dot{x}_2) = 0$$

$$m_4 \ddot{x}_3 + K_3(x_3 - x_2) + K_4 x_3 + K_5(x_3 - x_2) + b_3(\dot{x}_3 - \dot{x}_2) + b_4 \dot{x}_3 = 0$$

4) a)



$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$

$V = \frac{1}{2} K_1 x_1^2 + \frac{1}{2} K_2 (x_2 - x_1)^2$

$R = \frac{1}{2} b \dot{x}_2^2$

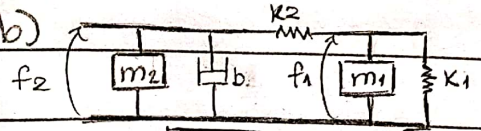
· $q_1 = x_1$: $\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$; $\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2(x_2 - x_1)$; $\frac{\partial R}{\partial \dot{x}_1} = 0$

$$m_1 \ddot{x}_1 + k_1 x_1 - K_2(x_2 - x_1) = f_1(t)$$

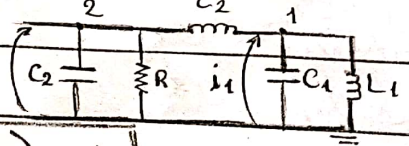
· $q_2 = x_2$: $\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$; $\frac{\partial L}{\partial x_2} = -K_2(x_2 - x_1)$; $\frac{\partial R}{\partial \dot{x}_2} = b \dot{x}_2$

$$m_2 \ddot{x}_2 + K_2(x_2 - x_1) + b \dot{x}_2 = f_2(t)$$

b)



c)



d) Nó 1: $V_1\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D}\right) - V_2\left(\frac{1}{L_2 D}\right) = i_1$

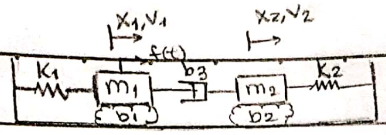
Nó 2: $V_2\left(C_2 D + \frac{1}{L_2 D} + \frac{1}{R}\right) - V_1\left(\frac{1}{L_2 D}\right) = i_2$

e)

$$m_1 \ddot{x}_1 + k_1 x_1 - K_2(x_2 - x_1) = f_1$$

$$m_2 \ddot{x}_2 + K_2(x_2 - x_1) + b \dot{x}_2 = f_2$$

5) a)



$T=\frac{1}{2}m_1\dot{x}_1^2+\frac{1}{2}m_2\dot{x}_2^2$; $V=\frac{1}{2}k_1x_1^2+\frac{1}{2}k_2x_2^2$, $R=\frac{1}{2}b_1\dot{x}_1^2+\frac{1}{2}b_2\dot{x}_2^2+\frac{1}{2}b_3(\dot{x}_2-\dot{x}_1)^2$

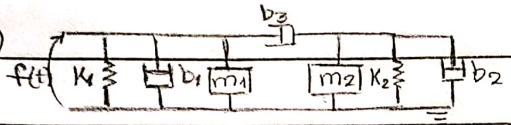
$q_1=x_1$: $\frac{\partial L}{\partial \dot{x}_1}=m_1\dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)=m_1\ddot{x}_1$; $\frac{\partial L}{\partial x_1}=-k_1x_1$; $\frac{\partial R}{\partial \dot{x}_1}=b_1\dot{x}_1-b_3(\dot{x}_2-\dot{x}_1)$

$$m_1\ddot{x}_1+k_1x_1+b_1\dot{x}_1-b_3(\dot{x}_2-\dot{x}_1)=f(t)$$

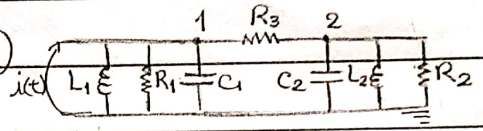
$q_2=x_2$: $\frac{\partial L}{\partial \dot{x}_2}=m_2\dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)=m_2\ddot{x}_2$; $\frac{\partial L}{\partial x_2}=+k_2x_2$; $\frac{\partial R}{\partial \dot{x}_2}=b_2\dot{x}_2+b_3(\dot{x}_2-\dot{x}_1)$

$$m_2\ddot{x}_2+k_2x_2+b_2\dot{x}_2+b_3(\dot{x}_2-\dot{x}_1)=0$$

b)



c)



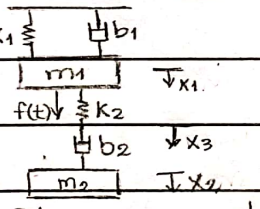
d) Nó 1: $V_1\left(C_1D+\frac{1}{L_1D}+\frac{1}{R_1}+\frac{1}{R_3}\right)-V_2\frac{1}{R_3}=i(t)$

Nó 2: $V_2\left(C_2D+\frac{1}{L_2D}+\frac{1}{R_2}+\frac{1}{R_3}\right)-V_1\frac{1}{R_3}=0$

e) $m_1\ddot{x}_1+k_1x_1+b_1\dot{x}_1-b_3(\dot{x}_2-\dot{x}_1)=f(t)$

$m_2\ddot{x}_2+k_2x_2+b_2\dot{x}_2+b_3(\dot{x}_2-\dot{x}_1)=0$

6) a)



$T=\frac{1}{2}m_1\dot{x}_1^2+\frac{1}{2}m_2\dot{x}_2^2$

$V=\frac{1}{2}k_1x_1^2+\frac{1}{2}k_2(x_3-x_1)^2$

$R=\frac{1}{2}b_1\dot{x}_1^2+\frac{1}{2}b_2(\dot{x}_2-\dot{x}_3)^2$

$q_1=x_1$: $\frac{\partial L}{\partial \dot{x}_1}=m_1\dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)=m_1\ddot{x}_1$; $\frac{\partial L}{\partial x_1}=-k_1x_1+k_2(x_3-x_1)$; $\frac{\partial R}{\partial \dot{x}_1}=b_1\dot{x}_1-b_2(\dot{x}_2-\dot{x}_3)$

$$m_1\ddot{x}_1+k_1x_1-k_2(x_3-x_1)+b_1\dot{x}_1-b_2(\dot{x}_2-\dot{x}_3)=f(t)$$

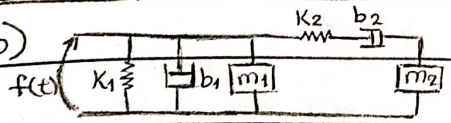
$q_2=x_2$: $\frac{\partial L}{\partial \dot{x}_2}=m_2\dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)=m_2\ddot{x}_2$; $\frac{\partial L}{\partial x_2}=0$; $\frac{\partial R}{\partial \dot{x}_2}=b_2(\dot{x}_2-\dot{x}_3)$

$$m_2\ddot{x}_2+b_2(\dot{x}_2-\dot{x}_3)=0$$

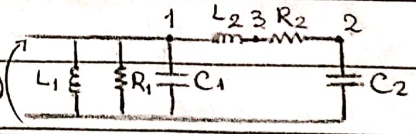
$q_3=x_3$: $\frac{\partial L}{\partial \dot{x}_3}=0$; $\frac{\partial L}{\partial x_3}=-k_2(x_3-x_1)$; $\frac{\partial R}{\partial \dot{x}_3}=-b_2(\dot{x}_2-\dot{x}_3)$

$$k_2(x_3-x_1)-b_2(\dot{x}_2-\dot{x}_3)=0$$

b)



c)



d) Nó 1: $V_1\left(C_1D+\frac{1}{L_1D}+\frac{1}{L_2D}+\frac{1}{R_1}\right)-V_3\left(\frac{1}{L_2D}\right)=i(t)$

Nó 2: $V_2\left(C_2D+\frac{1}{R_2}\right)-V_3\left(\frac{1}{R_2}\right)=0$

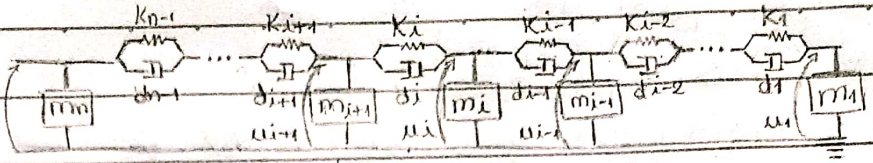
Nó 3: $V_3\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)-V_1\frac{1}{L_2D}-V_2\frac{1}{R_2}=0$

e) $m_1\ddot{x}_1+k_1x_1-k_2(x_3-x_1)+b_1\dot{x}_1-b_2(\dot{x}_2-\dot{x}_3)=f(t)$

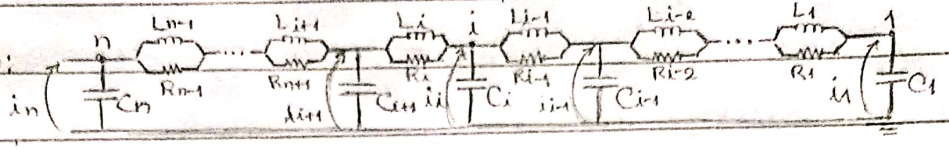
$m_2\ddot{x}_2+b_2(\dot{x}_2-\dot{x}_3)=0$

$k_2(x_3-x_1)-b_2(\dot{x}_2-\dot{x}_3)=0$

⑦ Circuito mecânico:



Circuito elétrico:



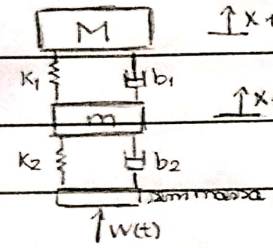
Nó n: $V_n\left(C_nD+\frac{1}{L_{n-1}D}+\frac{1}{R_{n-1}}\right)-V_{n-1}\left(\frac{1}{L_{n-1}D}+\frac{1}{R_{n-1}}\right)=i_n$

Nó i: $V_i\left(C_iD+\frac{1}{L_iD}+\frac{1}{L_{i-1}D}+\frac{1}{R_i}+\frac{1}{R_{i-1}}\right)-V_{i+1}\left(\frac{1}{L_iD}+\frac{1}{R_i}\right)-V_{i-1}\left(\frac{1}{L_{i-1}D}+\frac{1}{R_{i-1}D}\right)=i_i$

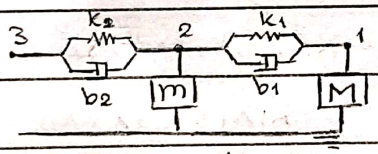
Nó 1: $V_1\left(C_1D+\frac{1}{L_1D}+\frac{1}{R_1}\right)-V_2\left(\frac{1}{L_1D}+\frac{1}{R_1}\right)=i_1$

$$\begin{cases} m_n\ddot{x}_n+d_{n-1}\dot{x}_n+k_{n-1}x_n-d_{n-1}\dot{x}_{n-1}-k_{n-1}x_{n-1}=u_n(t) \\ m_i\ddot{x}_i+(d_i+d_{i-1})\dot{x}_i+(k_i+k_{i-1})x_i-d_i\dot{x}_{n+i}-k_i x_{i+1}-d_{i-1}\dot{x}_{i-1}-k_{i-1}x_{i-1}=u_i(t) \\ m_1\ddot{x}_1+d_1\dot{x}_1+k_1x_1-d_1\dot{x}_2-k_1x_2=u_1(t) \end{cases}$$

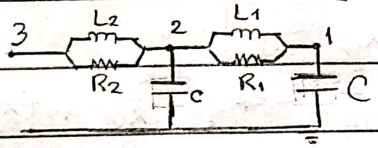
⑧



a) Circuito mecânico:



circuito elétrico:



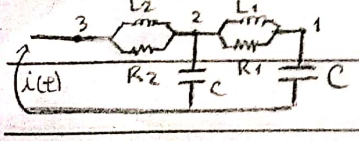
Nó 1: $V_1\left(CD+\frac{1}{L_1D}+\frac{1}{R_1}\right)-V_2\left(\frac{1}{L_1D}+\frac{1}{R_1}\right)=0$

Nó 2: $V_2\left(cD+\frac{1}{L_1D}+\frac{1}{L_2D}+\frac{1}{R_1}+\frac{1}{R_2}\right)-V_1\left(\frac{1}{L_1D}+\frac{1}{R_1}\right)-V_3\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)=0$

Nó 3: $V_3\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)-V_2\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)=0$

$$\begin{cases} M\ddot{x}_1+b_1\dot{x}_1+k_1x_1-b_1\dot{x}_2-k_1x_2=0 \\ m\ddot{x}_2+b_1\dot{x}_2+b_2\dot{x}_2+k_1x_2+k_2x_2-b_1\dot{x}_1-k_1x_1-b_2\dot{w}(t)-k_2w(t)=0 \\ b_2\dot{w}(t)+k_2w(t)-b_2\dot{x}_2-k_2x_2=0 \end{cases}$$

b) circuito elétrico:



Nó 1: $V_1\left(CD+\frac{1}{L_1D}+\frac{1}{R_1}\right)-V_2\left(\frac{1}{L_1D}+\frac{1}{R_1}\right)=0$

Nó 2: $V_2\left(cD+\frac{1}{L_1D}+\frac{1}{L_2D}+\frac{1}{R_1}+\frac{1}{R_2}\right)-V_1\left(\frac{1}{L_1D}+\frac{1}{R_1}\right)-V_3\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)=0$

Nó 3: $V_3\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)-V_2\left(\frac{1}{L_2D}+\frac{1}{R_2}\right)=i(t)$

$$\begin{cases} M\ddot{x}_1+b_1\dot{x}_1+k_1x_1-b_1\dot{x}_2-k_1x_2=0 \\ m\ddot{x}_2+b_1\dot{x}_2+b_2\dot{x}_2+k_1x_2+k_2x_2-b_1\dot{x}_1-k_1x_1-b_2\dot{x}_3-k_2x_3=0 \\ b_2\dot{x}_3+k_2x_3-b_2\dot{x}_2-k_2x_2=w(t) \end{cases}$$