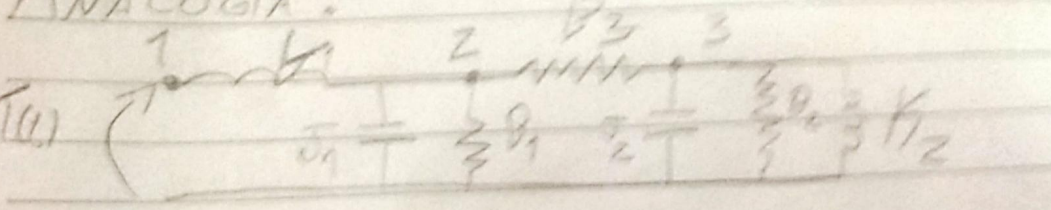
Ex 1:

Analogia:



Nó 1: $\frac{1}{L_1 D} \cdot (V_2 - V_1) = T_{(t)}$

Tal que: $K_1 \cdot \theta_1 - K_1 \cdot \dot{\theta}_2 = T_{(t)}$

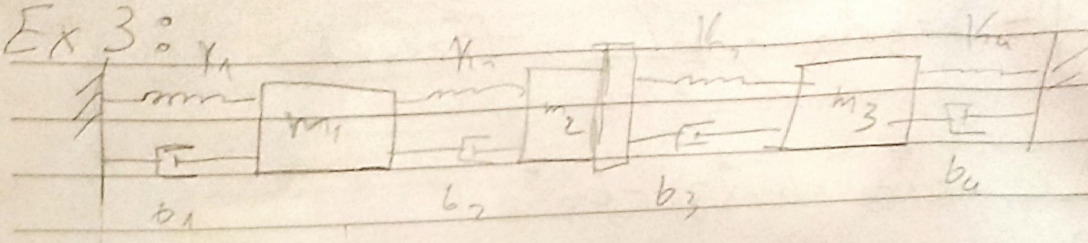
Nó 2: $\frac{1}{L_1 D}(V_2 - V_1) + \frac{1}{B_3}(V_2 - V_3) + \left(J_1 D + \frac{1}{B_1}\right) V_2 = 0$

Tal que: $T_{(t)} + B_3 \dot{\theta}_2 - B_3 \dot{\theta}_3 + B_1 \dot{\theta}_2 + \ddot{\theta}_2 M_1 = 0$

Nó 3: $\frac{1}{B_3}(V_3 - V_2) + \left(J_2 D + \frac{1}{B_2} + \frac{1}{K_2 D}\right) V_3 = 0$

$B_3 \dot{\theta}_3 - B_3 \dot{\theta}_2 + B_2 \dot{\theta}_3 + M_2 \ddot{\theta}_3 + K_2 \theta_3 = 0$

Ex 3:



$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{m_3 \dot{x}_3^2}{2}$$

$$V = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_2 - x_1)^2}{2} + \frac{k_3 (x_3 - x_2)^2}{2} + \frac{k_4 x_3^2}{2}$$

$$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b_3 (\dot{x}_3 - \dot{x}_2)^2}{2} + k_4 \dot{x}_3^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \quad ; \quad \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2 x_2 - k_2 x_1$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_2 \dot{x}_2 + b_2 \dot{x}_1$$

Eq: $m_1 \ddot{x}_1 + b_1 \dot{x}_1 + b_2 \dot{x}_1 - b_2 \dot{x}_2 + k_1 x_1 + k_2 x_1 - k_2 x_2 = u$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \quad ; \quad \frac{\partial L}{\partial x_2} = -k_1 x_2 + k_2 x_1 - k_2 x_2 + k_3 x_2$$

$$\frac{\partial R}{\partial \dot{x}_2} = b_2 \dot{x}_2 - b_2 \dot{x}_1 - b_3 \dot{x}_2 + b_3 \dot{x}_2$$

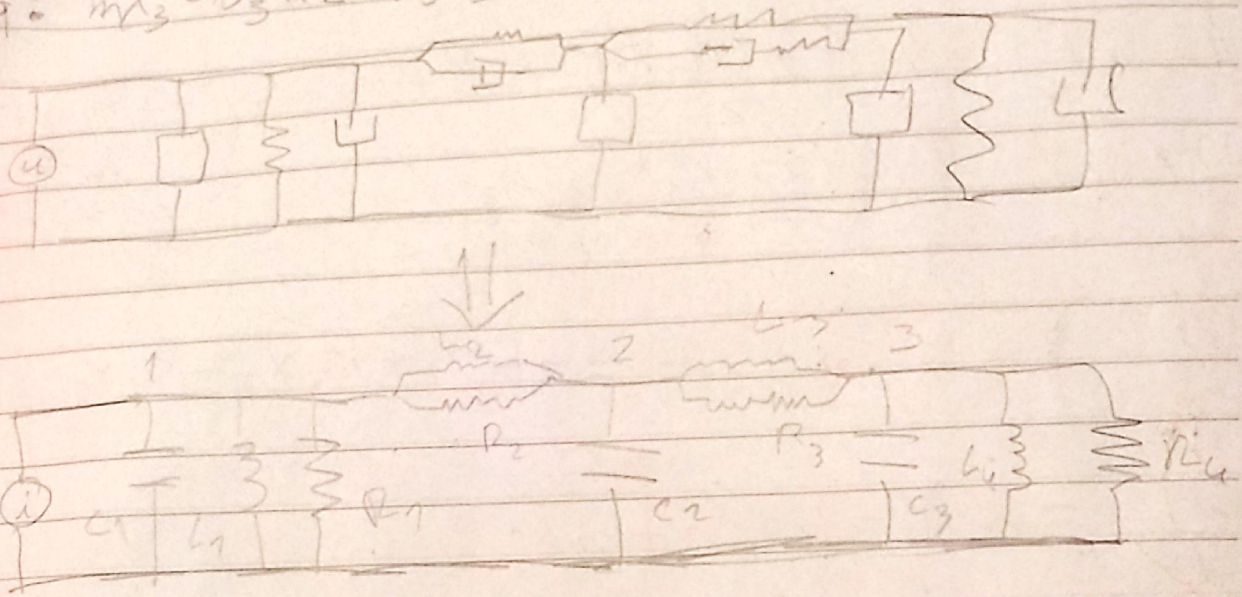
Eq: $m_2 \ddot{x}_2 - k_2 x_1 + k_2 x_2 + k_2 x_2 - k_3 x_3 - b_2 \dot{x}_2 + b_2 \dot{x}_1 + b_3 \dot{x}_2 -$

$- b_3 \dot{x}_2 = 0$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3\ddot{x}_3 \quad \frac{\partial L}{\partial x_3} = -k_3x_3 + k_2x_2 - k_4x_3$$

$$\frac{\partial \Omega}{\partial \dot{x}_3} = -b_3\dot{x}_2 + b_4\dot{x}_3 - b_3\dot{x}_2$$

Eq: $m\ddot{x}_3 - b_3\dot{x}_2 + b_3\dot{x}_3 + b_4\dot{x}_3 - k_3x_2 + k_3x_3 + k_4x_3 = 0$



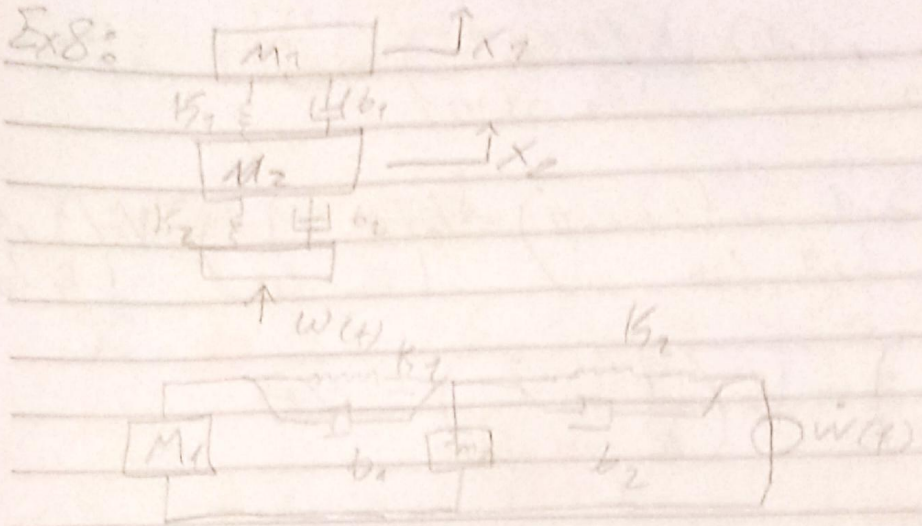
$$V_1\left(\frac{1}{L_1D} + \frac{1}{L_3D} + \frac{1}{R_1} + \frac{1}{R_2} + C_1D\right) - V_2\left(\frac{1}{R_2} + \frac{1}{L_1D}\right) = i(t)$$

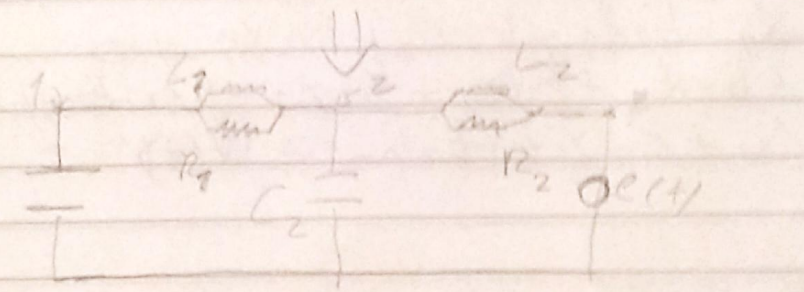
$$V_2\left(\frac{1}{L_1D} + \frac{1}{L_3D} + \frac{1}{R_2} + \frac{1}{R_3} + C_2D\right) - V_1\left(\frac{1}{L_2D} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{R_3} + \frac{1}{L_3D}\right) = 0$$

$$V_3\left(\frac{1}{L_3D} + \frac{1}{L_4D} + \frac{1}{R_3} + \frac{1}{R_4} + C_3D\right) - V_2\left(\frac{1}{L_3D} + \frac{1}{R_3}\right) = 0$$

$$m_1\ddot{x}_1 + b_1\dot{x}_1 + b_2\dot{x}_1 - b_2\dot{x}_2 + k_1x_1 + k_2x_1 - k_2x_2 = 0$$
$$m_2\ddot{x}_2 - b_2\dot{x}_1 + b_2\dot{x}_2 + b_3\dot{x}_2 - b_3\dot{x}_3 - k_2x_1 + k_2x_2 + k_3x_2 - k_3x_3 = 0$$
$$m_3\ddot{x}_3 - b_3\dot{x}_2 + b_3\dot{x}_3 + b_4\dot{x}_3 - k_3x_2 + k_3x_3 + k_4x_3 = 0$$

Ex 8:



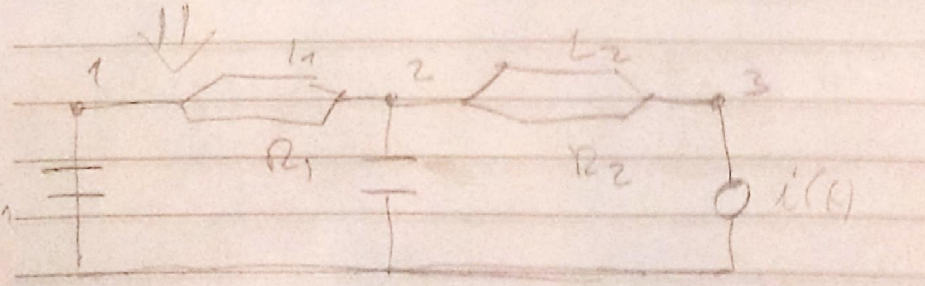


$$V_1\left(\frac{1}{R_1}+\frac{1}{L_1 D}+C_2 D\right)-V_2\left(\frac{1}{R_1}+\frac{1}{L_1 D}\right)=0$$

$$V_2\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{L_1 D}+\frac{1}{L_2 D}+C_2 D\right)-V_1\left(\frac{1}{R_1}+\frac{1}{L_1 D}\right)-e(t)\left(\frac{1}{R_2}+\frac{1}{L_2 D}\right)=0$$

$$m\ddot{x}_1+b_1\dot{x}_1+k_1x_1-b_1\dot{x}_2-k_1x_2=0$$

$$m_2\ddot{x}_2+b_1\dot{x}_2+b_2\dot{x}_2+k_1x_2+k_2x_2-b_1\dot{x}_1-k_1x_1-\dots$$
$$\dots-\dot{w}(t)\,b_2-w(t)=0$$



$$V_1 = \left(\frac{1}{R_1} + \frac{1}{L_1 D} + C_1 D\right) - V_2\left(\frac{1}{R_1} + \frac{1}{L_1 D}\right) = 0$$

$$V_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{L_1 D} + \frac{1}{L_2 D} + C_2 D\right) - V_1\left(\frac{1}{R_1} + \frac{1}{L_1 D}\right) - V_3\left(\frac{1}{R_2} + \frac{1}{L_2 D}\right) =$$

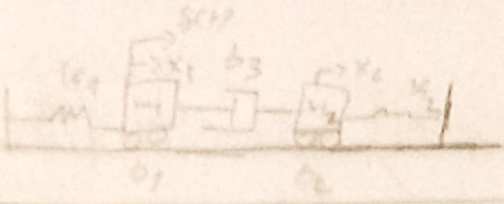
$$V_3\left(\frac{1}{R_2} + \frac{1}{L_2 D}\right) - V_2\left(\frac{1}{R_2} + \frac{1}{L_2 D}\right) = i(t)$$

$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 - b_1 \dot{x}_2 - k_1 x_2 + k_1 x_1 = 0$$

$$m_2 \ddot{x}_2 + b_1 \dot{x}_1 + b_2 \dot{x}_2 + k_1 x_1 + k_2 x_1 - b \dot{x}_1 - k x_1 - b \dot{x}_2 - k x_2 = 0$$

$$b_2 \dot{x}_3 + k_2 x_3 + k_2 x_2 - b_2 \dot{x}_2 = W(t)$$

Ex 5:



$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} \quad ; \quad V = \frac{k_1 x_1^2}{2} + \frac{k_2 x_2^2}{2}$$

$$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 \dot{x}_2^2}{2} + \frac{b_3 (\dot{x}_2 - \dot{x}_1)^2}{2}$$

$$\frac{\partial L}{\partial x_1} = m_1 \dot{x}_1 \; ; \; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \; ; \; \frac{\partial L}{\partial x_1} = -k_1 x_1$$

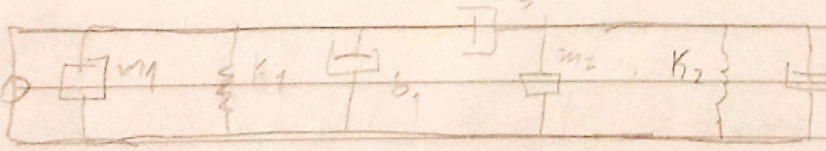
$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 + b_3 (\dot{x}_2 - \dot{x}_1)$$

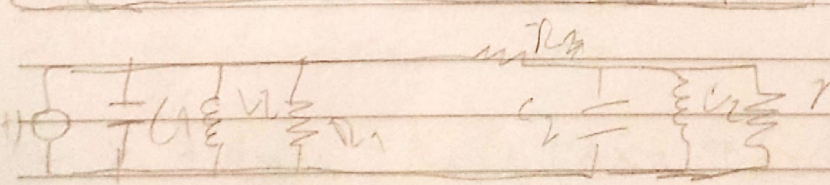
$$m_1 \ddot{x}_1 + k_1 x_1 + b_1 \dot{x}_1 + b_3 \dot{x}_1 - b_3 \dot{x}_2 = S(t)$$

$$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \; ; \; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \; ; \; \frac{\partial L}{\partial x_2} = -k_2 x_2$$

$$\frac{\partial R}{\partial \dot{x}_2} = b_2 \dot{x}_2 + b_3 (\dot{x}_2 - \dot{x}_1)$$

$$m_2 \ddot{x}_2 + k_2 x_2 + b_2 \dot{x}_2 + b_3 \dot{x}_2 - b_3 \dot{x}_1 = 0$$

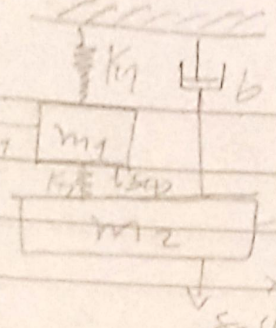




$$V_1\left(\frac{1}{L_1 D} + \frac{1}{R_3} + \frac{1}{R_1} + C_1 D\right) - \frac{V_2}{R_3} = x(t)$$

$$V_2\left(\frac{1}{L_2 D} + \frac{1}{R_3} + \frac{1}{R_2} + C_2 D\right) - \frac{V_1}{R_3} = 0$$

Ex 9:



$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$$

$$V = \frac{K_1 x_1^2}{2} + \frac{K_2 (x_2 - x_1)^2}{2}$$

$$R = \frac{b \dot{x}_2^2}{2}$$

$$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \; ; \; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \; ; \; \frac{\partial R}{\partial \dot{x}_1} = 0$$

$$\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (x_2 - x_1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} + \frac{\partial R}{\partial \dot{x}_1} = f(t)$$

$$\downarrow$$

$$m_1 \ddot{x}_1 + K_1 x_1 - K_2 x_2 + K_2 x_1 = f(t)$$

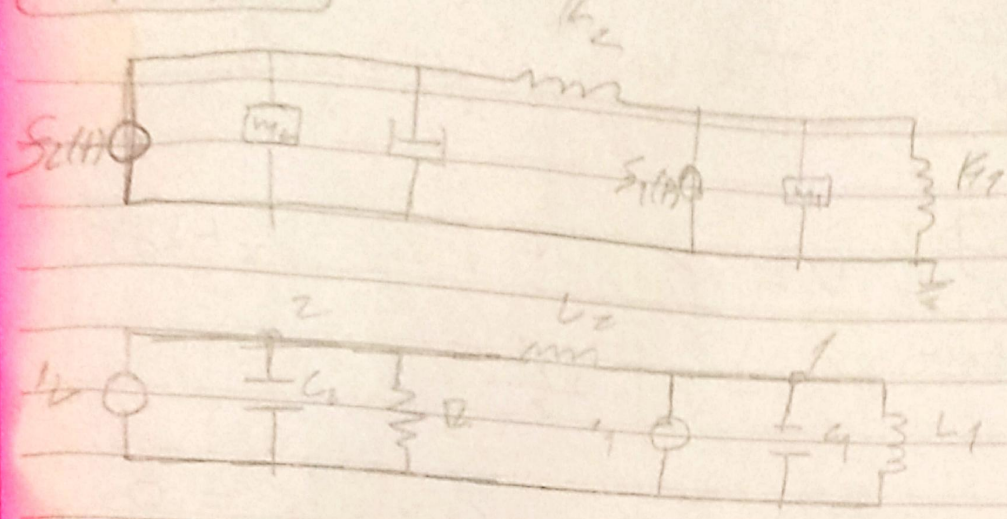
$$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \; ; \; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \; ; \; \frac{\partial R}{\partial \dot{x}_2} = b \dot{x}_2$$

$$\frac{\partial L}{\partial x_2} = -K_2 x_2 + K_2 x_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} + \frac{\partial R}{\partial \dot{x}_2} = f_2(t)$$

$$\downarrow$$

$$m_2 \ddot{x}_2 + K_2 x_2 - K_2 x_1 + b \dot{x}_2 = f_2(t)$$



$$i_1 = V_1\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D}\right) - V_2 \frac{1}{L_2 D}$$

$$i_2 = V_2\left(C_2 D + \frac{1}{R_2} + \frac{1}{L_2 D}\right) - V_1 \frac{1}{L_2 D}$$

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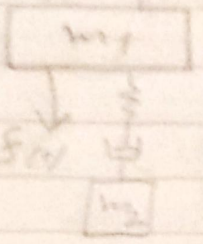
Ex 6 :



$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$

$V = \frac{1}{2}\left(k_1 x_1^2 + k_2 (x_2 - x_1)^2\right)$

$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_2 - \dot{x}_1)^2}{2}$

$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \; ; \; \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2 x_2 - k_2 x_1$

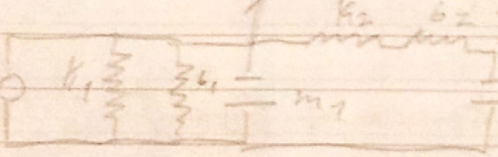
$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_2 \dot{x}_2 + b_2 \dot{x}_1$

tal que : $m_1 \ddot{x}_1 + b_1 \dot{x}_1 + b_2 \dot{x}_1 + k_1 x_1 + k_2 x_1 - k_2 x_2 = f(t)$

$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \; ; \; \frac{\partial L}{\partial x_2} = -k_2 x_2 + k_2 x_1$

$\frac{\partial R}{\partial \dot{x}_2} = b_2 \dot{x}_2 - b_2 \dot{x}_1$

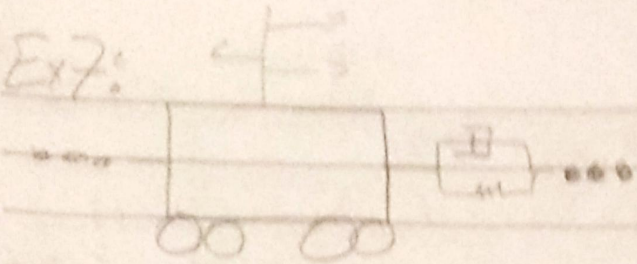
tal que : $m_2 \ddot{x}_2 - b_2 \dot{x}_1 + b_2 \dot{x}_2 + k_2 x_2 - k_2 x_1 = 0$

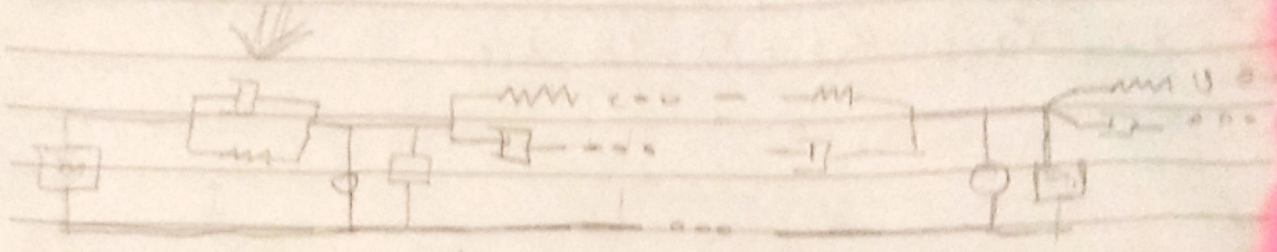


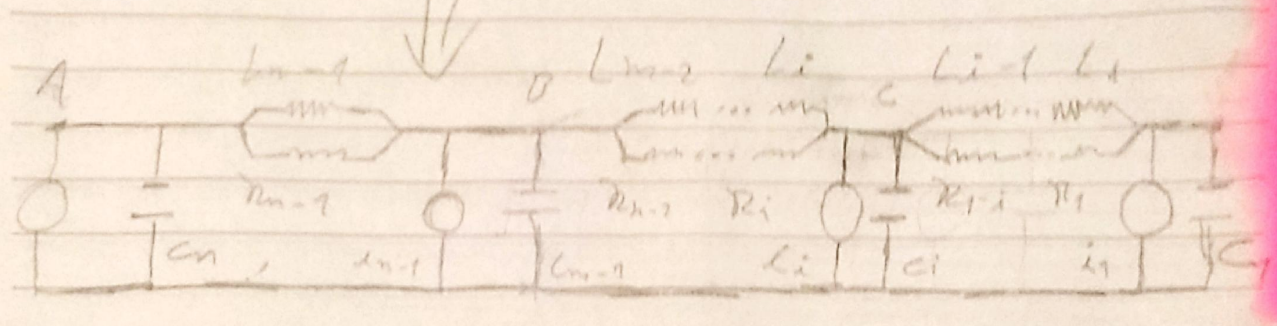
1 : $f(t) = \left(\frac{1}{k_1 D} + \frac{1}{b_1} + m_1 D + \frac{1}{k_2 D}\right) v_1 - \frac{1}{k_2 D} v_2$

2 : $\frac{1}{k_2 D} v_2 - \left(\frac{1}{b_2} + m_2 D\right) v_2 - \frac{1}{k_2 D} v_2 = 0$

Ex7:







A:

$$V_A\left(C_n D + \frac{1}{L_{n-1}D} + \frac{1}{R_{n-1}}\right) - V_{A-1}\left(\frac{1}{L_{n-1}D} + \frac{1}{R_{n-1}}\right) = i_n(t)$$

B:

$$V_B\left(C_i D + \frac{1}{L_i D} + \frac{1}{L_{i-1}D} + \frac{1}{R_i} + \frac{1}{R_{i-1}}\right) - V_{B-1}\left(\frac{1}{L_i D} + \frac{1}{R_i}\right) \cdots$$

$$-V_{B-1}\left(\frac{1}{L_{i-1}D} + \frac{1}{R_{i-1}}\right) = i_i$$

C: $V_C\left(C_1 D + \frac{1}{L_1 D} + \frac{1}{R_1}\right) - V_{C-1}\left(\frac{1}{L_1 D} + \frac{1}{R_1}\right) = i_1(t)$

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Tal que:

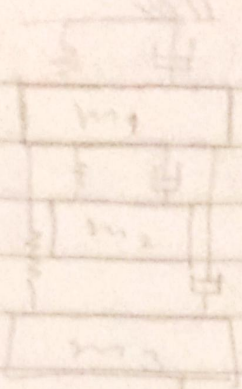
$$m_1\ddot{x}_1 + d_1\dot{x}_1 + k_1x_1 - d_1\dot{x}_2 - k_1x_2 = u(t)$$

$$m_2\ddot{x}_n + d_{n-1}\dot{x}_n + k_{n-1}x_n - k_{n-1}x_{n-1} - d_{n-1}\dot{x}_{n-1} = u(t)$$

$$m_3\ddot{x}_i + d_1\dot{x}_i + d_{i-1}\dot{x}_1 + k_ix_i + k_{i-1} - d_ix_{i+1} - k_ix_{i+1}$$

$$- d_{i-1}\dot{x}_{i-1} - k_{i-1}x_{i-1} = u(t)$$

Ex 2:



$$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2} + \frac{m_3\dot{x}_3^2}{2}$$

$$V = \frac{k_1x_1^2}{2} + \frac{k_{12}(x_2-x_1)^2}{2} + \frac{k_3(x_3-x_2)^2}{2} + \frac{k_4(x_3-x_1)^2}{2}$$

$$R = \frac{b_1\dot{x}_1^2}{2} + \frac{b_2(\dot{x}_2-\dot{x}_1)^2}{2} + \frac{b_3(\dot{x}_3-\dot{x}_2)^2}{2} + \frac{b_4(\dot{x}_3-\dot{x}_1)^2}{2}$$

$$\frac{\partial L}{\partial x_1} = -k_1x_1 + k_2(x_2-x_1) + k_4(x_3-x_1)$$

$$\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \;;\; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1$$

$$\frac{\partial L}{\partial x_2} = -k_2(x_2-x_1) + k_3(x_3-x_2)$$

$$\frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 \;;\; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$$

$$\frac{\partial L}{\partial x_3} = -k_3(x_3-x_2) - k_4(x_3-x_1)$$

$$\frac{\partial L}{\partial \dot{x}_3} = m_3\dot{x}_3 \;;\; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3\ddot{x}_3$$

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$$\frac{\partial R}{\partial \dot{x}_1} = b_4\dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) - b_1(\dot{x}_2 - \dot{x}_1)$$

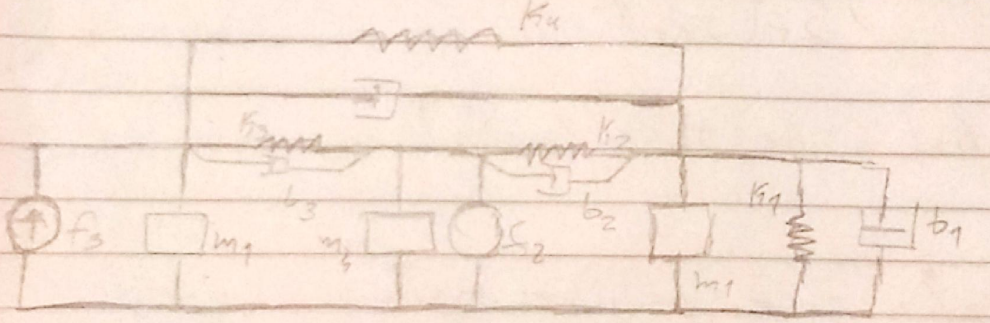
$$\frac{\partial R}{\partial \dot{x}_2} = b_2\dot{x}_2 - b_2\dot{x}_1 - b_1\dot{x}_2 + b_2\dot{x}_2$$

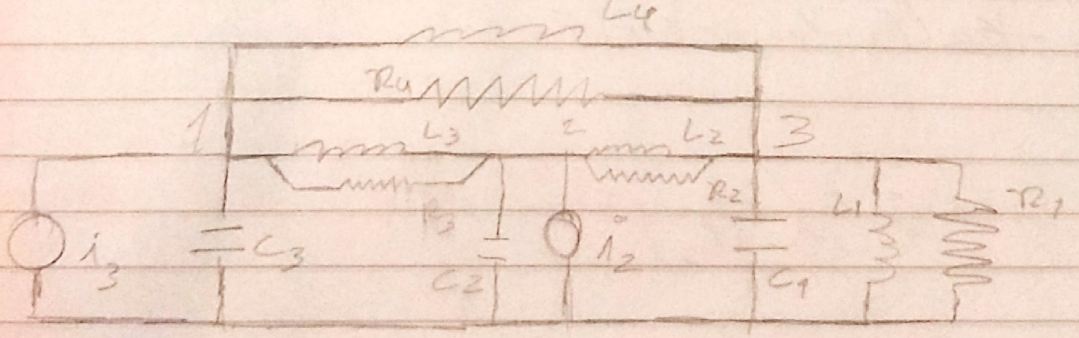
$$\frac{\partial R}{\partial \dot{x}_3} = b_3\dot{x}_3 - b_3\dot{x}_2 + b_4\dot{x}_3 - b_4\dot{x}_1$$

$$m_1\ddot{x}_1 + k_4x_1 + k_3x_1 - k_2x_2 + k_4x_1 - k_4x_3 + b\dot{x}_1 + b_4\dot{x}_1 - b_2\dot{x}_2 + b_4\dot{x}_1 - b_4\dot{x}_3 = 0$$

$$m_1\ddot{x} + k_2x_1 + k_2x_2 + k_3x_2 - k_3x_3 + b_2\dot{x}_2 - b_2\dot{x}_1 + b_3\dot{x}_2 + b_3\dot{x}_3 = f_2$$

$$m_2\ddot{x}_3 + k_3x_3 - k_3x_2 + k_4x_3 - k_4x_1 + b_3\dot{x}_3 - b_3\dot{x}_2 + b_4\dot{x}_3 - b_4\dot{x}_1 = f_1$$





$1^{\circ}_{0}\ +V_1\left(\frac{1}{L_{10}}+\frac{1}{L_{11}}\right)-V_2\left(\frac{1}{L_{10}}+\frac{1}{R_2}\right)+V_3\left(\frac{1}{C_3 D}+\frac{1}{R_3}+\frac{1}{L_{10}}+\frac{1}{L_{10}}\right)=i_3$

$M_3\ddot{x}_3-b_4\dot{x}_1-b_3\dot{x}_2+(b_3+b_4)\dot{x}_3-k_4x_1-k_3x_2+(k_3+k_4)x_3=f_3$

$2^{\circ}_{0}\ -V_1\left(\frac{1}{L_{20}}+\frac{1}{R_2}\right)+V_2\left(\frac{1}{L_{10}}+\frac{1}{R_2}+\frac{1}{L_{30}}+\frac{1}{R_3}+C_2D\right)-V_3\left(\frac{1}{L_{10}}+\frac{1}{R_3}\right)=i_2$

$m_2\ddot{x}_2-b_2\dot{x}_1+b_2\dot{x}_2+b_3\dot{x}_2-b_3\dot{x}_3-k_2x_1+k_2x_2+k_3x_2-k_3x_3=f_2$

$3^{\circ}_{0}\ V_2-V_1\left(\frac{1}{L_{20}}+\frac{1}{R_2}\right)+(V_2-V_1)\left(\frac{1}{L_{10}}+\frac{1}{R_1}\right)-V_1\left(CD+\frac{1}{L_{10}}+\frac{1}{R_1}\right)=0$

$m_1\ddot{x}_1+b_1\dot{x}_1+b_2\dot{x}_1+b_4\dot{x}_1-b_2\dot{x}_2-b_4\dot{x}_3+k_1x_1+k_2x_1+k_4x_1-k_2x_2-k_4x_3=0$