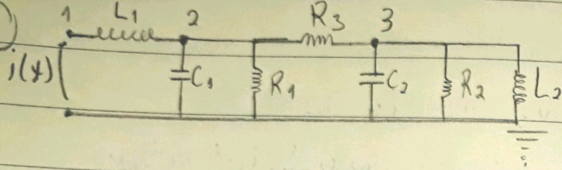
Exercícios da Aula 03/09/2020

Aluno: Luiz Ricardo de Souza Cruz

N°USP: 10334961

1)



$$1: V_1\left(\frac{1}{LD_1}\right) - V_2\left(\frac{1}{LD_1}\right) = i(t)$$

$$2: V_2\left(CD_1 + \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{LD_1}\right) - V_1\left(\frac{1}{LD_1}\right) - V_3\left(\frac{1}{R_3}\right) = 0$$

$$3: V_3\left(CD_2 + \frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{LD_2}\right) - V_2\left(\frac{1}{R_2}\right) = 0$$

• Assim, chegamos a:

$$\boxed{\begin{aligned} & K_1\theta_1 - K_1\theta_2 = T \\ & J_{g1}\ddot{\theta}_2 + B_1\dot{\theta}_2 + B_3\dot{\theta}_2 + K_1\theta_2 - K_1\theta_1 - B_3\dot{\theta}_3 = 0 \\ & J_{g2}\ddot{\theta}_3 + B_3\dot{\theta}_3 + B_2\dot{\theta}_3 + K_2\theta_3 - B_3\dot{\theta}_2 = 0 \end{aligned}}$$

2) a)

$$\begin{cases} T = \dfrac{m_1 v_1^2}{2} + \dfrac{m_2 v_2^2}{2} + \dfrac{m_3 v_3^2}{2} = \dfrac{m_1 \dot{x}_1^2}{2} + \dfrac{m_2 \dot{x}_2^2}{2} + \dfrac{m_3 \dot{x}_3^2}{2} \\ V = \dfrac{k_1 x_1^2}{2} + \dfrac{k_2 (x_2 - x_1)^2}{2} + \dfrac{k_3 (x_3 - x_2)^2}{2} + \dfrac{k_4 (x_3 - x_1)^2}{2} \end{cases}$$

$$L = T - V$$

• $q_{g1} = x_1$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \frac{d}{dt}\left(m_1 \dot{x}_1\right) = m_1 \ddot{x}_1$

$$\frac{\partial L}{\partial x_1} = k_1 x_1 + k_2(x_1 - x_2) + k_4(x_1 - x_3)$$

(1) $m_1\ddot{x}_1 - (k_1 + k_2 + k_4)x_1 + k_2 x_2 + k_4 x_3 = -(b_1 + b_2 + b_4)\dot{x}_1 + b_2\dot{x}_2 + b_4\dot{x}_3$

• $q_{g2} = x_2$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = -b_2(\dot{x}_2 - \dot{x}_1) + b_3(\dot{x}_3 - \dot{x}_2) + F_2(t)$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = \frac{d}{dt}\left(m_2 \dot{x}_2\right) = m_2 \ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = k_2(x_2 - x_1) + k_3(x_2 - x_3)$$

(2) $m_2\ddot{x}_2 + k_2 x_1 - (k_2 + k_3)x_2 + k_3 x_3 = b_2\dot{x}_1 - (b_2 + b_3)\dot{x}_2 + b_3\dot{x}_3 + F_2(t)$

• $q_{g3} = x_3$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) - \frac{\partial L}{\partial x_3} = g_3(t) - b_3(\dot{x}_3 - \dot{x}_2) - b_4(\dot{x}_3 - \dot{x}_1)$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = \frac{d}{dt}\left(m_3 \dot{x}_3\right) = m_3 \ddot{x}_3$$

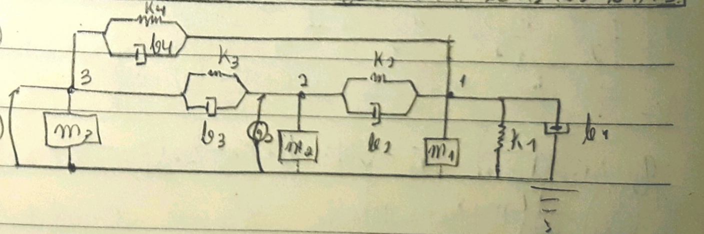
$$\frac{\partial L}{\partial x_3} = k_3(x_3 - x_2) + k_4(x_3 - x_1)$$

(3) $m_3\ddot{x}_3 + k_4 x_1 + k_3 x_2 - (k_3 + k_4)x_3 = g_3(t) + b_4\dot{x}_1 + b_3\dot{x}_2 - (b_3 + b_4)\dot{x}_3$

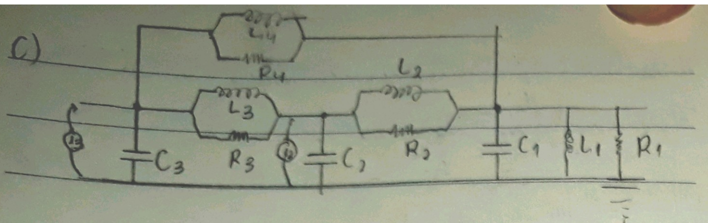
• Assim, o conjunto de equações:

$$\boxed{\begin{aligned} & m_1\ddot{x}_1 - (k_1 + k_2 + k_4)x_1 + k_2 x_2 + k_4 x_3 = -(b_1 + b_2 + b_4)\dot{x}_1 + b_2\dot{x}_2 + b_4\dot{x}_3 \\ & m_2\ddot{x}_2 + k_2 x_1 - (k_2 + k_3)x_2 + k_3 x_3 = b_2\dot{x}_1 - (b_2 + b_3)\dot{x}_2 + b_3\dot{x}_3 + F_2(t) \\ & m_3\ddot{x}_3 + k_4 x_1 + k_3 x_2 - (k_3 + k_4)x_3 = g_3(t) + b_4\dot{x}_1 + b_3\dot{x}_2 - (b_3 + b_4)\dot{x}_3 \end{aligned}}$$

b)



c)



d) Nó 1: $V_1\left(CD_1 + \frac{1}{LD_1} + \frac{1}{R_1} + \frac{1}{LD_2} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{LD_4}\right)$
$- V_2\left(\frac{1}{LD_2} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{LD_4} + \frac{1}{R_4}\right) = 0$

Nó 2: $V_2\left(CD_2 + \frac{1}{LD_2} + \frac{1}{R_2} + \frac{1}{LD_3} + \frac{1}{R_3}\right) - V_3\left(\frac{1}{LD_3} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{LD_2} + \frac{1}{R_2}\right) = i_2$

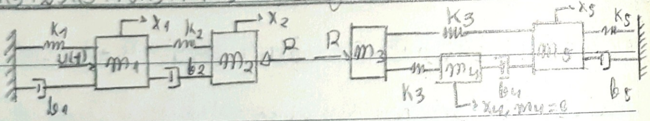
Nó 3: $V_3\left(CD_3 + \frac{1}{LD_3} + \frac{1}{R_3} + \frac{1}{LD_4} + \frac{1}{R_4}\right) - V_2\left(\frac{1}{LD_3} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{LD_4} + \frac{1}{R_4}\right) = i_3$

e) $m_1\ddot{x}_1 + b_1\dot{x}_1 + k_1x_1 + k_2x_1 + b_2\dot{x}_1 + k_4x_1 + b_4\dot{x}_1 = k_2x_2 + b_2\dot{x}_2$
$+ k_4x_3 + b_4\dot{x}_3$

$m_2\ddot{x}_2 + b_2\dot{x}_2 + k_2x_2 + k_3x_2 + b_3\dot{x}_2 = i_2 + k_3x_3 + b_3\dot{x}_3 + k_2x_1 + b_2\dot{x}_1$

$m_3\ddot{x}_3 + b_3\dot{x}_3 + k_3x_3 + k_4x_3 + b_4\dot{x}_3 = i_3 + k_3x_2 + b_3\dot{x}_2 + k_4x_1 + b_4\dot{x}_1$

3) a)



$$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2} + \frac{m_3\dot{x}_3^2}{2} + \frac{m_4\dot{x}_4^2}{2} + \frac{m_5\dot{x}_5^2}{2}$$

$$V = \frac{k_1x_1^2}{2} + \frac{k_2(x_2-x_1)^2}{2} + \frac{k_3(x_5-x_3)^2}{2} + \frac{k_3(x_4-x_3)^2}{2} + \frac{k_5x_5^2}{2}$$

- $q_1 = x_1$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = U(t) - b_1\dot{x}_1 + b_2(\dot{x}_2 - \dot{x}_1)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \frac{d}{dt}(m_1\dot{x}_1) = m_1\ddot{x}_1$

$\frac{\partial L}{\partial x_1} = -k_1x_1 + k_2(x_2 - x_1)$

$m_1\ddot{x}_1 + k_1x_1 + b_1\dot{x}_1 = U(t) + k_2(x_2-x_1) + b_2(\dot{x}_2 - \dot{x}_1)$ (1)

- $q_2 = x_2$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = -b_2(\dot{x}_2 - \dot{x}_1) - R$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$, $\frac{\partial L}{\partial x_2} = -k_2(x_2 - x_1)$

$m_2\ddot{x}_2 + k_2(x_2 - x_1) + b_2(\dot{x}_2 - \dot{x}_1) = -R$ (2)

- $q_3 = x_3$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) - \frac{\partial L}{\partial x_3} = R$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m_3\ddot{x}_3$; $\frac{\partial L}{\partial x_3} = k_3(x_5 - x_3) + k_3(x_4 - x_3)$

$m_3\ddot{x}_3 + k_3(2x_3 - x_4 - x_5) = R$ (3)

- $q_4 = x_4$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_4}\right) - \frac{\partial L}{\partial x_4} = b_3(\dot{x}_5 - \dot{x}_4)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_4}\right) = m_4\ddot{x}_4$; $\frac{\partial L}{\partial x_4} = -k_3(x_4 - x_3)$

$m_4\ddot{x}_4 + k_3(x_4 - x_3) + b_3(\dot{x}_4 - \dot{x}_5) = 0$ (4)

- $q_5 = x_5$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_5}\right) - \frac{\partial L}{\partial x_5} = -b_4(\dot{x}_5 - \dot{x}_4) + b_5\dot{x}_5$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_5}\right) = m\ddot{x}_5$; $\frac{\partial L}{\partial x_5} = -k_3(x_5 - x_3) - k_5x_5$

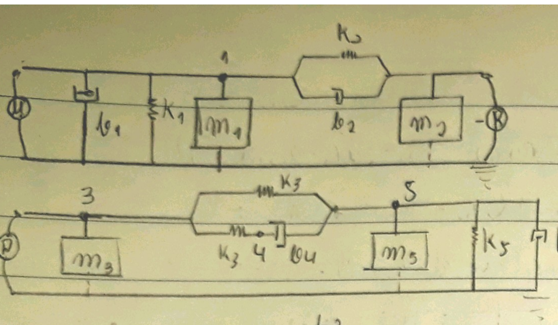
$m_5\ddot{x}_5 + k_3(x_5 - x_3) + k_5x_5 + b_4(\dot{x}_5 - \dot{x}_4) - b_5\dot{x}_5 = 0$ (5)

- Assim, as equações são:

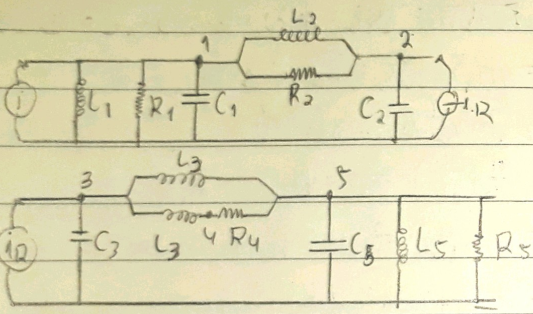
$$\boxed{\begin{aligned} &\ddot{x}_1 + 2\dot{x}_1 + 2x_1 = U(t) + x_2 \\ &\ddot{x}_2 + \dot{x}_2 - \dot{x}_1 + x_2 - x_1 = -R \\ &\ddot{x}_3 + 2x_3 - x_4 - x_5 = R \\ &\ddot{x}_4 + x_4 - x_3 - \dot{x}_5 = 0 \\ &\ddot{x}_5 + 2\dot{x}_5 + 2x_5 - x_3 - \dot{x}_4 = 0 \end{aligned}}$$

, com $x_2 = x_3$

b)



c)



d) Nó 1: $V_1\left(CD_1+\frac{1}{R_1}+\frac{1}{LD_1}+\frac{1}{LD_2}+\frac{1}{R_2}\right)-V_2\left(\frac{1}{LD_2}+\frac{1}{R_2}\right)=i$

Nó 2: $V_2\left(CD_2+\frac{1}{R_2}+\frac{1}{LD_2}\right)-V_1\left(\frac{1}{R_2}+\frac{1}{LD_2}\right)=-i_R$

Nó 3: $V_3\left(CD_3+\frac{2}{LD_3}\right)-V_4\left(\frac{1}{LD_3}\right)-V_5\left(\frac{1}{LD_3}\right)=i_R$

Nó 4: $V_4\left(\frac{1}{LD_3}+\frac{1}{R_4}\right)-V_3\left(\frac{1}{LD_3}\right)-V_5\left(\frac{1}{R_4}\right)=0$

Nó 5: $V_5\left(CD_5+\frac{1}{LD_5}+\frac{1}{R_5}+\frac{1}{R_4}+\frac{1}{LD_3}\right)-V_4\left(\frac{1}{R_4}\right)-V_3\left(\frac{1}{LD_3}\right)=0$

com, $V_2=V_3$

e)

$m_1\ddot{x}_1+b_1\dot{x}_1+b_2\dot{x}_1+k_1x_1+k_2x_1-k_2x_2-b_2\dot{x}_2=U$

$m_2\ddot{x}_2+b_2\dot{x}_2+k_2x_2-b_2\dot{x}_1-k_2x_1=-R$

$m_3\ddot{x}_3+2k_3x_3-k_3x_4-k_5x_5=R$

$b_4\ddot{x}_4+k_3x_4-k_3x_3-b_4\dot{x}_5=0$

$m_5\ddot{x}_5+b_5\dot{x}_5+k_5x_5+b_4x_5+k_3x_5-b_4\dot{x}_4-k_3x_3=0$

com, $\dot{x}_2=\dot{x}_3$

4) $T=\frac{m_1\dot{x}_1^2}{2}+\frac{m_2\dot{x}_2^2}{2}$, $V=\frac{k_1x_1^2}{2}+\frac{k_2(x_2-x_1)^2}{2}$

- $q_1=x_1$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)-\frac{\partial L}{\partial x_1}=F_1(t)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)=m_1\ddot{x}_1$; $\frac{\partial L}{\partial x_1}=k_2(x_2-x_1)-k_1x_1$

$m_1\ddot{x}_1+k_1x_1=F_1(t)+k_2(x_2-x_1)$ (1)

- $q_2=x_2$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)=F_2(t)-b\dot{x}_2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)=m_2\ddot{x}_2$; $\frac{\partial L}{\partial x_2}=-k_2(x_2-x_1)$

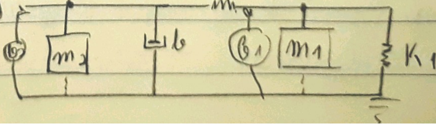
$m_2\ddot{x}_2+k_2(x_2-x_1)+b\dot{x}_2=F_2(t)$ (2)

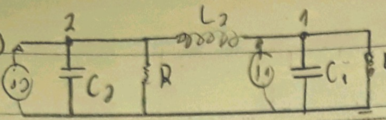
- Assim, as equações são:

$m_1\ddot{x}_1+k_1x_1=F_1(t)+k_2(x_2-x_1)$

$m_2\ddot{x}_2+b\dot{x}_2+k_2(x_2-x_1)=F_2(t)$

b)



c) 

d) $\boxed{\begin{aligned} &\text{nó 1: } V_1\left(CD_1+\frac{1}{LD_1}+\frac{1}{LD_2}\right)-V_2\left(\frac{1}{LD_2}\right)=i_1 \\ &\text{nó 2: } V_2\left(CD_2+\frac{1}{R}+\frac{1}{LD_2}\right)-V_1\left(\frac{1}{LD_2}\right)=i_2 \end{aligned}}$

e) $\boxed{\begin{aligned} &m_1\ddot{x}_1+K_1x_1+K_2x_1-K_2x_2=b_1(t) \\ &m_2\ddot{x}_2+b\dot{x}_2+Kx_2-K_2x_1=b_2(t) \end{aligned}}$

5) a) $T=\frac{m_1\dot{x}_1^2}{2}+\frac{m_2x_2^2}{2}$, $V=\frac{K_1x_1^2}{2}+\frac{K_2x_2^2}{2}$

$R=\frac{b_1\dot{x}_1^2}{2}+\frac{b_2\dot{x}_2^2}{2}+\frac{b_3(\dot{x}_2-\dot{x}_1)^2}{2}$

• $q_1=x_1: \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)-\frac{\partial L}{\partial x_1}+\frac{\partial R}{\partial \dot{x}_1}=b(t)$

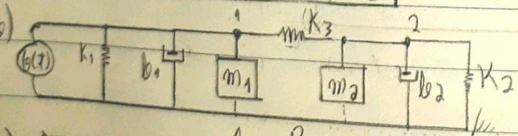
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)=m_1\ddot{x}_1;\ \frac{\partial L}{\partial x_1}=-K_1x_1;\ \frac{\partial R}{\partial \dot{x}_1}=b_1\dot{x}_1-b_3(\dot{x}_2-\dot{x}_1)$

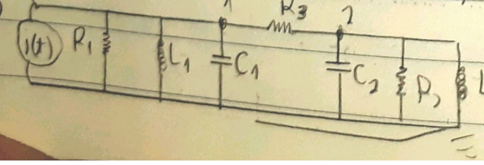
$\boxed{m_1\ddot{x}_1+K_1x_1+b_1\dot{x}_1-b_3(\dot{x}_2-\dot{x}_1)=b(t)}$

• $q_2=x_2: \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)-\frac{\partial L}{\partial x_2}+\frac{\partial R}{\partial \dot{x}_2}=0$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)=m_2\ddot{x}_2;\ \frac{\partial L}{\partial x_2}=-K_2x_2;\ \frac{\partial R}{\partial \dot{x}_2}=b_2\dot{x}_2+b_3(\dot{x}_2-\dot{x}_1)$

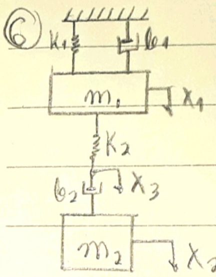
$\boxed{m_2\ddot{x}_2+K_2x_2+b_2\dot{x}_2+b_3(\dot{x}_2-\dot{x}_1)=0}$

b) 

c) 

d) $\boxed{\begin{aligned} &\text{nó 1: } V_1\left(CD_1+\frac{1}{LD_1}+\frac{1}{R_1}+\frac{1}{R_3}\right)-V_2\left(\frac{1}{R_3}\right)=i(t) \\ &\text{nó 2: } V_2\left(CD_2+\frac{1}{R_2}+\frac{1}{LD_2}+\frac{1}{R_3}\right)-V_1\left(\frac{1}{R_3}\right)=0 \end{aligned}}$

e) $\boxed{\begin{aligned} &m_1\ddot{x}_1+k_1x_1+b_1\dot{x}_1+b_3\dot{x}_1-b_3\dot{x}_2=b(t) \\ &m_2\ddot{x}_2+b_2\dot{x}_2+k_2x_2+b_3\dot{x}_2-b_3\dot{x}_1=0 \end{aligned}}$

6) 

a) $T=\frac{m_1\dot{x}_1^2}{2}+\frac{m_2\dot{x}_2^2}{2}$

$V=\frac{K_1x_1^2}{2}+\frac{K_2(x_3-x_1)^2}{2}$

$R=\frac{b_1\dot{x}_1^2}{2}+\frac{b_2(\dot{x}_2-\dot{x}_3)^2}{2}$

• $q_1=x_1: \frac{d}{dt}\left(\frac{\partial L}{\dot{x}_1}\right)=m_1\ddot{x}_1;\ \frac{\partial R}{\partial \dot{x}_1}=b_1x_1$

$\frac{\partial L}{\partial x_1}=-K_1x_1+K_2(x_3-x_1)$

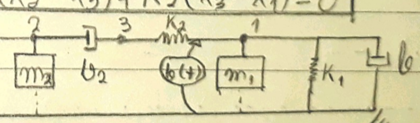
$\boxed{m_1\ddot{x}_1+K_1x_1-K_2(x_3-x_1)+b_1\dot{x}_1=b(t)}$

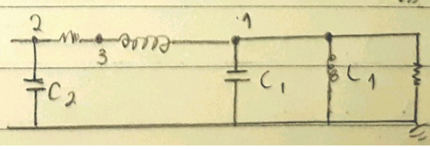
• $q_2=x_2: \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)=m_2\ddot{x}_2;\ \frac{\partial L}{\partial x_2}=0;\ \frac{\partial R}{\partial \dot{x}_2}=b_2(\dot{x}_2-\dot{x}_3)$

$\boxed{m_2\ddot{x}_2+b_2(\dot{x}_2-\dot{x}_3)=0}$

• $q_3=x_3: \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right)=0;\ \frac{\partial L}{\partial x_3}=-K_2(x_3-x_1);\ \frac{\partial R}{\partial \dot{x}_3}=-b(\dot{x}_2-\dot{x}_3)$

$\boxed{-b_2(\dot{x}_2-\dot{x}_3)+K_2(x_3-x_1)=0}$

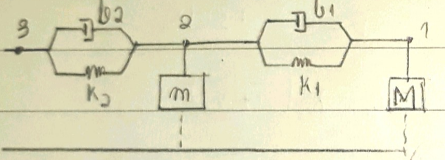
b) 

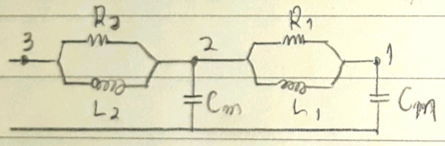
c) 

d) $\boxed{\begin{aligned} &\text{Nó 1: } V_1\left(CD_1+\frac{1}{LD_1}+\frac{1}{R_1}+\frac{1}{LD_2}\right)-V_3\left(\frac{1}{LD_2}\right)=i(t)\\ &\text{Nó 2: } V_2\left(CD_2+\frac{1}{R_2}\right)-V_3\left(\frac{1}{R_2}\right)=0\\ &\text{Nó 3: } V_3\left(\frac{1}{R_2}+\frac{1}{LD_2}\right)-V_2\left(\frac{1}{R_2}\right)-V_1\left(\frac{1}{LD_2}\right)=0 \end{aligned}}$

e) $\boxed{\begin{aligned} &m_1\ddot{x}_1+K_1x_1+b_1\dot{x}_1+K_2x_1-K_2x_3=b(t)\\ &m_2\ddot{x}_2+b_2\dot{x}_2-b_2\dot{x}_3=0\\ &K_2x_3+b_2\dot{x}_3-b_2\dot{x}_2-K_2x_1=0 \end{aligned}}$

8 a)





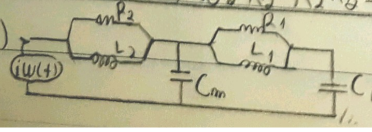
Nó 1: $V_1\left(CD_M+\frac{1}{LD_1}+\frac{1}{R_1}\right)-V_2\left(\frac{1}{LD_1}+\frac{1}{R_1}\right)=0$

Nó 2: $V_2\left(CD_m+\frac{1}{LD_1}+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{LD_2}\right)-V_1\left(\frac{1}{LD_1}+\frac{1}{R_1}\right)-V_3\left(\frac{1}{LD_2}+\frac{1}{R_2}\right)=0$

Nó 3: $V_3\left(\frac{1}{LD_2}+\frac{1}{R_2}\right)-\left(\frac{1}{LD_2}+\frac{1}{R_2}\right)V_2=0$; $V_3=?\,\dot{w}(t)$

$\boxed{\begin{aligned} &M\ddot{x}_1+b_1\dot{x}_1+K_1x_1-b_1\dot{x}_2-K_1x_2=0\\ &m\ddot{x}_2+b_1\dot{x}_2+b_2\dot{x}_2+K_1x_2+K_2x_2-b_1\dot{x}_1-K_1x_1-K_2w(t)-b_2\dot{w}(t)=0\\ &b_2\dot{w}(t)+K_2w(t)-b_2\dot{x}_2-K_2x_2=0 \end{aligned}}$

b)



Nó 1: $V_1\left(CD_M+\frac{1}{LD_1}+\frac{1}{R_1}\right)-V_2\left(\frac{1}{LD_1}+\frac{1}{R_1}\right)=0$

Nó 2: $V_2\left(CD_m+\frac{1}{LD_1}+\frac{1}{R_1}+\frac{1}{LD_2}+\frac{1}{R_2}\right)-V_1\left(\frac{1}{LD_1}+\frac{1}{R_1}\right)-V_3\left(\frac{1}{LD_2}+\frac{1}{R_2}\right)=0$

Nó 3: $V_3\left(\frac{1}{LD_2}+\frac{1}{R_2}\right)-V_2\left(\frac{1}{LD_2}+\frac{1}{R_2}\right)=iw(t)$

$\boxed{\begin{aligned} &M\ddot{x}_1+b_1\dot{x}_1+K_1x_1-b_1\dot{x}_2-K_1x_2=0\\ &m\ddot{x}_2+b_1\dot{x}_2+b_2\dot{x}_2+K_1x_1+K_2x_2-b_1\dot{x}_1-K_1x_1-K_2x_3-b_2\dot{x}_3=0\\ &b_2\dot{x}_3+K_2x_3-b_2\dot{x}_2-K_2x_2=0 \end{aligned}}$