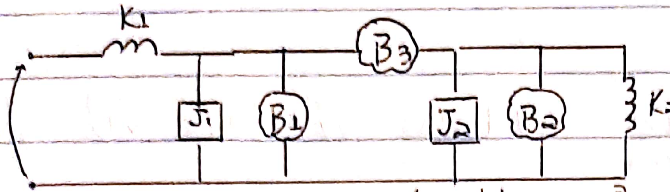
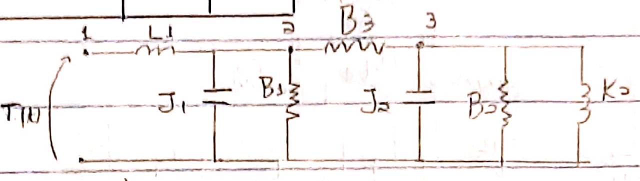
$L = T - V$ / $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot q} = F$

Gabriel Barbosa Pagamini - 10772539 - Modelagem 03/09

ex 1



• Analogia elétrica :

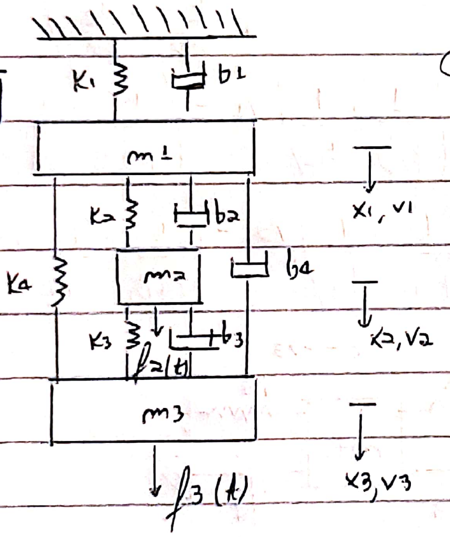


• Nó 1 : $T(t) = \frac{1}{L_1 . D}\left(\dot\theta_1 - \dot\theta_2\right)$

• Nó 2 : $\frac{1}{L_1 D}\left(\dot\theta_2 - \dot\theta_1\right) + \frac{1}{B_3}\left(\dot\theta_2 - \dot\theta_3\right) + \left(J_1 D + \frac{1}{B_1}\right)\dot\theta_2 = 0$

• Nó 3 : $\frac{1}{B_3}\left(\dot\theta_3 - \dot\theta_2\right) + \left(J_2 D + \frac{1}{B_2} + \frac{1}{K_2 D}\right)\dot\theta_3 = 0$

ex 2



ⓐ $T = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{m_3 v_3^2}{2}$

$V = \frac{K_1 x_1^2}{2} + \frac{K_2 (x_2 - x_1)^2}{2} + \frac{K_3 (x_3 - x_2)^2}{2} + \frac{K_4 (x_3 - x_1)^2}{2}$

$R = \frac{b_1 v_1^2}{2} + \frac{b_2 (v_2 - v_1)^2}{2} + \frac{b_3 (v_3 - v_2)^2}{2} + \frac{b_4 (v_3 - v_1)^2}{2}$

(X1) $\frac{\partial L}{\partial v_1} = m_1 v_1 \longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial v_1}\right) = m_1 \dot v_1$

$\frac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (x_2 - x_1) + K_4 (x_3 - x_1)$; $\frac{\partial R}{\partial v_1} = b_1 v_1 - b_2 (v_2 - v_1) - b_4 (v_3 - v_1)$

$\therefore \boxed{m_1 \ddot x_1 + K_1 x_1 - K_2 (x_2 - x_1) - K_4 (x_3 - x_1) + b_1 \dot x_1 - b_2 (\dot x_2 - \dot x_1) - b_4 (\dot x_3 - \dot x_1) = 0}$

(X2) $\frac{\partial L}{\partial v_2} = m_2 v_2 \longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial v_2}\right) = m_2 \dot v_2$; $\frac{\partial L}{\partial x_2} = -K_2 (x_2 - x_1) + K_3 (x_3 - x_2)$

$\frac{\partial R}{\partial v_2} = b_2 (v_2 - v_1) - b_3 (v_3 - v_2)$

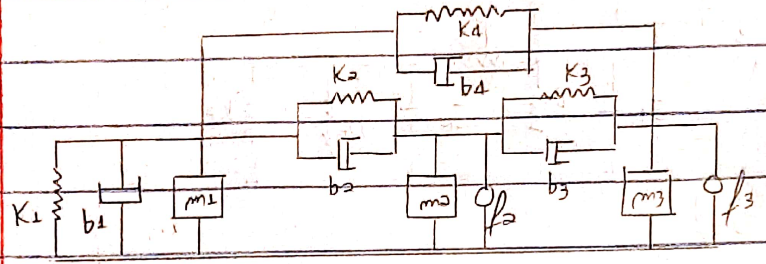
$\therefore m_2 \ddot x_2 + K_2 (x_2 - x_1) - K_3 (x_3 - x_2) + b_2 (\dot x_2 - \dot x_1) - b_3 (\dot x_3 - \dot x_2) = f_2(t)$

(x_3) $\frac{\partial L}{\partial v_3} = m_3 v_3 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial v_3}\right) = m_3 \dot{v}_3$; $\frac{\partial L}{\partial x_2} = -k_3(x_3 - x_2) - k_4(x_3 - x_1)$

$\frac{\partial R}{\partial v_3} = b_3(v_3 - v_2) + b_4(v_3 - v_1)$

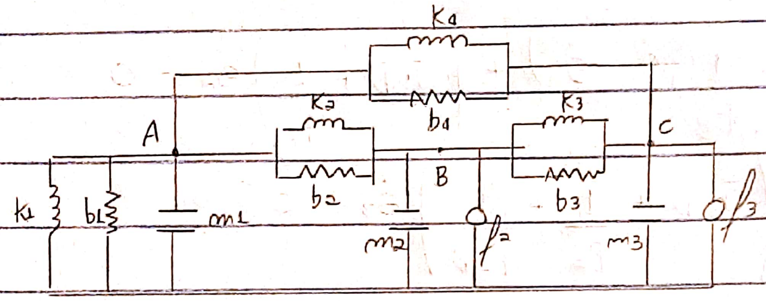
$\therefore m_3 \ddot{x}_3 + k_3(x_3 - x_2) + k_4(x_3 - x_1) + b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1) = f_3(t)$

b)



. circuito mecânico

c)

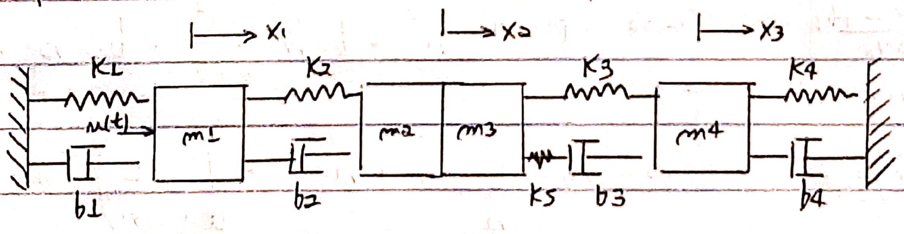


. Nó A: $-\left(\frac{1}{K_4 D} + \frac{1}{b_4}\right) V_3 - \left(\frac{1}{K_2 D} + \frac{1}{b_2}\right) V_2 + \left(\frac{1}{K_1 D} + \frac{1}{b_1} + m_1 D + \frac{1}{K_2 D} + \frac{1}{b_2} + \frac{1}{K_4 D} + \frac{1}{b_4}\right) V_1 = 0$

. Nó B: $\left(\frac{1}{K_2 D} + \frac{1}{b_2} + m_2 D + \frac{1}{K_3 D} + \frac{1}{b_3}\right) V_2 - \left(\frac{1}{K_2 D} + \frac{1}{b_2}\right) V_1 - \left(\frac{1}{K_3 D} + \frac{1}{b_3}\right) V_3 = f_2$

. Nó C: $\left(\frac{1}{K_3 D} + \frac{1}{b_3} + \frac{1}{K_4 D} + \frac{1}{b_4} + m_3 D\right) V_1 - \left(\frac{1}{K_3 D} + \frac{1}{b_3}\right) V_2 - \left(\frac{1}{K_4 D} + \frac{1}{b_4}\right) V_3 = f_3$

$\boxed{ex3}$



$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{(m_2 + m_3)\dot{x}_2^2}{2} + \frac{m_4 \dot{x}_3^2}{2}$

$V = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_2 - x_1)^2}{2} + \frac{k_3 (x_3 - x_2)^2}{2} + \frac{k_4 x_3^2}{2} + \frac{k_5 (x_3 - x_2)^2}{2}$

$R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_2 - \dot{x}_1)^2}{2} + \frac{b_3 (\dot{x}_3 - \dot{x}_2)^2}{2} + \frac{b_4 \dot{x}_3^2}{2}$

(x_1) $\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$; $\frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2(x_2 - x_1)$

$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1)$

$$\therefore\ m_1\ddot{x}_1 + k_1x_1 - k_2(x_2-x_1) + b_1\dot{x}_1 - b_2(\dot{x}_2-\dot{x}_1) = u(t)$$

(x_2) $\dfrac{\partial L}{\partial \dot{x}_2} = (m_2+m_3)\dot{x}_2 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) = (m_2+m_3)\ddot{x}_2$

$\dfrac{\partial L}{\partial x_2} = -k_2(x_2-x_1) + (k_3+k_5)(x_3-x_2)$ $\qquad \dfrac{\partial R}{\partial \dot{x}_2} = b_2(\dot{x}_2-\dot{x}_1) - b_3(\dot{x}_3-\dot{x}_2)$

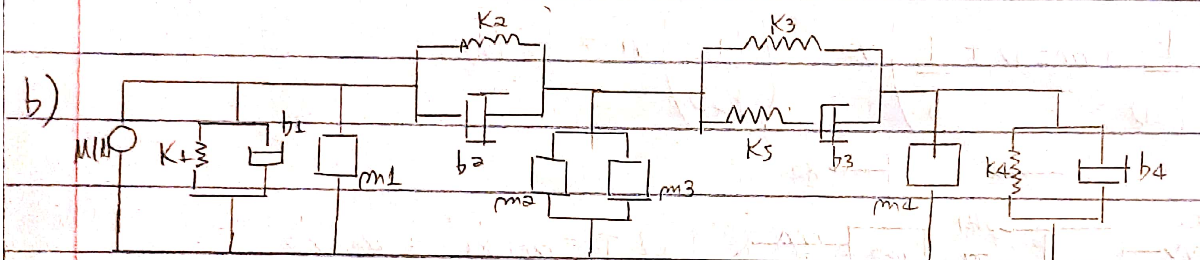
$$\therefore\ (m_2+m_3)\ddot{x}_2 + k_2(x_2-x_1) - (k_3+k_5)(x_3-x_2) + b_2(\dot{x}_2-\dot{x}_1) - b_3(\dot{x}_3-\dot{x}_2) = 0$$

(x_3) $\dfrac{\partial L}{\partial \dot{x}_3} = m_4\dot{x}_3 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_3}\right) = m_4\ddot{x}_3$ $\ /\ \dfrac{\partial R}{\partial \dot{x}_3} = b_3(\dot{x}_3-\dot{x}_2) + b_4\dot{x}_3$

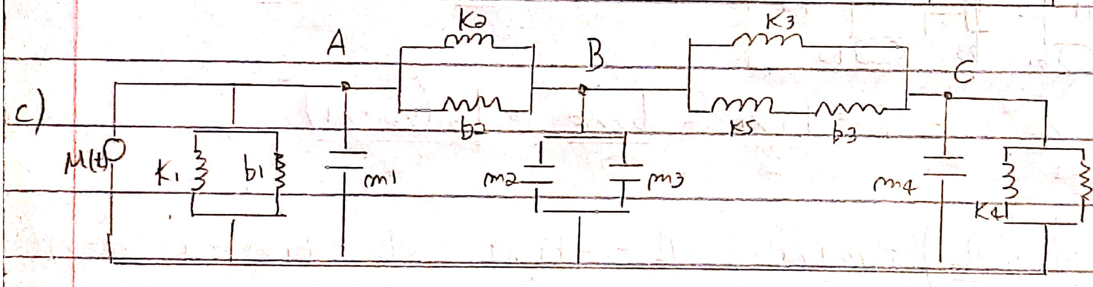
$\dfrac{\partial L}{\partial x_3} = -k_3(x_3-x_2) - k_4x_3 - k_5(x_3-x_2)$

$$\therefore\ m_4\ddot{x}_3 + k_3(x_3-x_2) - k_4x_3 - k_5(x_3-x_2) + b_3(\dot{x}_3-\dot{x}_2) + b_4\dot{x}_3 = 0$$

b)



c)

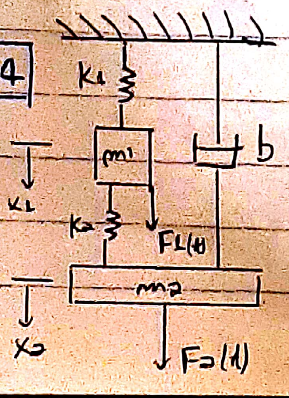


d) **Nó A:** $\left(\dfrac{1}{k_1D} + \dfrac{1}{b_1} + m_1D + \dfrac{1}{k_2D} + \dfrac{1}{b_2}\right)v_1 - \left(\dfrac{1}{k_2D} + \dfrac{1}{b_2}\right)v_2 = u(t)$

Nó B: $\left(\dfrac{1}{k_2D} + \dfrac{1}{b_2} + m_2D + m_3D + \dfrac{1}{k_3D} + \dfrac{1}{k_5D} + \dfrac{1}{b_3}\right)v_2 - \left(\dfrac{1}{k_2D} + \dfrac{1}{b_2}\right)v_1 - \left(\dfrac{1}{k_3D} + \dfrac{1}{k_5D} + \dfrac{1}{b_3}\right)v_3 = 0$

Nó C: $\left(\dfrac{1}{k_3D} + \dfrac{1}{k_5D} + \dfrac{1}{b_3} + m_4D + \dfrac{1}{k_4D} + \dfrac{1}{b_4}\right)v_3 - \left(\dfrac{1}{k_3D} + \dfrac{1}{k_5D} + \dfrac{1}{b_3}\right)v_2 = 0$

ex.4



a) $T = \dfrac{m_1\dot{x}_1^2}{2} + \dfrac{m_2\dot{x}_2^2}{2}$ $\ /\ R = \dfrac{b\dot{x}_2^2}{2}$

$V = \dfrac{k_1x_1^2}{2} + \dfrac{k_2(x_2-x_1)^2}{2}$

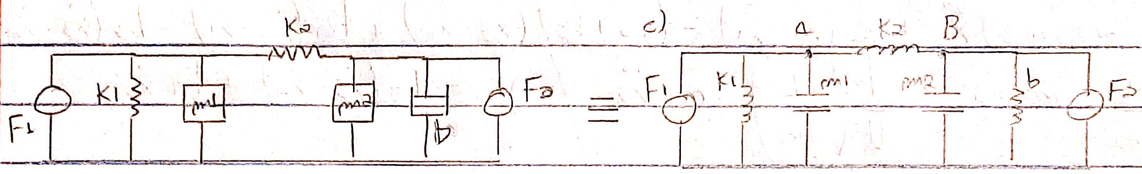
(x_1) $\dfrac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$ / $\dfrac{\partial L}{\partial x_1} = -k_1 x_1 + k_2(x_2 - x_1)$ / $\dfrac{\partial R}{\partial \dot{x}_1} = 0$

$\therefore\ m_1 \ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) = F_1(t)$

(x_2) $\dfrac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$ / $\dfrac{\partial L}{\partial x_2} = -k_2(x_2 - x_1)$ / $\dfrac{\partial R}{\partial \dot{x}_2} = b\dot{x}_2$

$\therefore\ m_2 \ddot{x}_2 + k_2(x_2 - x_1) + b\dot{x}_2 = F_2(t)$

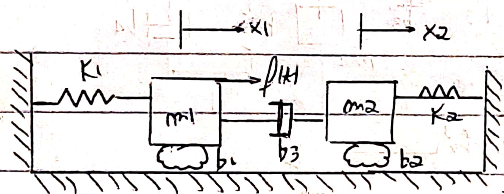
b)



Nó A: $\left(\dfrac{1}{k_1 D} + m_1 D + \dfrac{1}{k_2 D}\right) v_1 - \left(\dfrac{1}{k_2 D}\right) v_2 = F_1(t)$

Nó B: $\left(\dfrac{1}{k_2 D} + m_2 D + \dfrac{1}{b}\right) v_2 - \left(\dfrac{1}{k_2 D}\right) v_1 = F_2(t)$

ex 5



a) $T = \dfrac{m_1 \dot{x}_1^2}{2} + \dfrac{m_2 \dot{x}_2^2}{2}$;

$V = \dfrac{k_1 x_1^2}{2} + \dfrac{k_2 x_2^2}{2}$; $R = \dfrac{b_1 \dot{x}_1^2}{2} + \dfrac{b_2 \dot{x}_2^2}{2} + \dfrac{b_3(\dot{x}_2 - \dot{x}_1)^2}{2}$

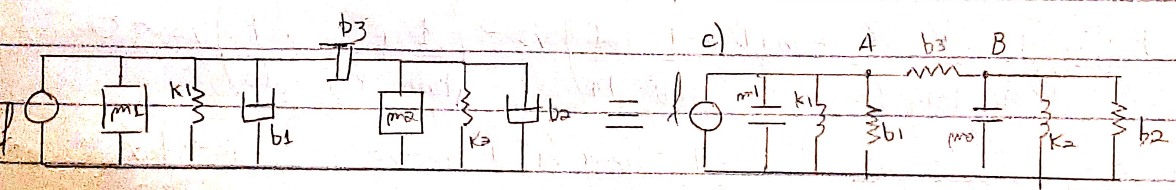
(x_1) $\dfrac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$ / $\dfrac{\partial L}{\partial x_1} = -k_1 x_1$ / $\dfrac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_3(\dot{x}_2 - \dot{x}_1)$

$\therefore\ m_1 \ddot{x}_1 + k_1 x_1 + b_1 \dot{x}_1 - b_3(\dot{x}_2 - \dot{x}_1) = f(t)$

(x_2) $\dfrac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$ / $\dfrac{\partial L}{\partial x_2} = -k_2 x_2$ / $\dfrac{\partial R}{\partial \dot{x}_2} = b_2 \dot{x}_2 + b_3(\dot{x}_2 - \dot{x}_1)$

$\therefore\ m_2 \ddot{x}_2 + k_2 x_2 + b_2 \dot{x}_2 + b_3(\dot{x}_2 - \dot{x}_1) = 0$

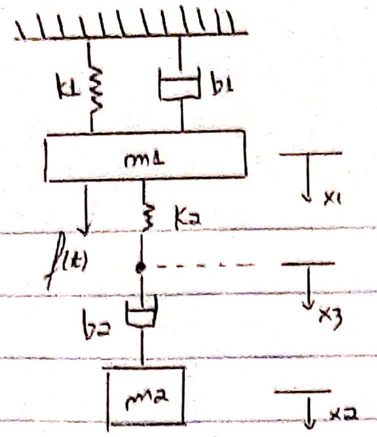
b)



Nó A: $\left(m_1 D + \dfrac{1}{k_1 D} + \dfrac{1}{b_1} + \dfrac{1}{b_3}\right) v_1 - \left(\dfrac{1}{b_3}\right) v_2 = f(t)$

Nó B: $\left(m_2 D + \dfrac{1}{b_3} + \dfrac{1}{k_2 D} + \dfrac{1}{b_2}\right) v_2 - \left(\dfrac{1}{b_3}\right) v_1 = 0$

ex6



a) $T = \dfrac{m_1 \dot{x}_1^2}{2} + \dfrac{m_2 \dot{x}_2^2}{2}$

$V = \dfrac{K_1 x_1^2}{2} + \dfrac{K_2 (x_3 - x_1)^2}{2}$

$R = \dfrac{b_1 \dot{x}_1^2}{2} + \dfrac{b_2 (\dot{x}_2 - \dot{x}_3)^2}{2}$

(x_1) $\dfrac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$

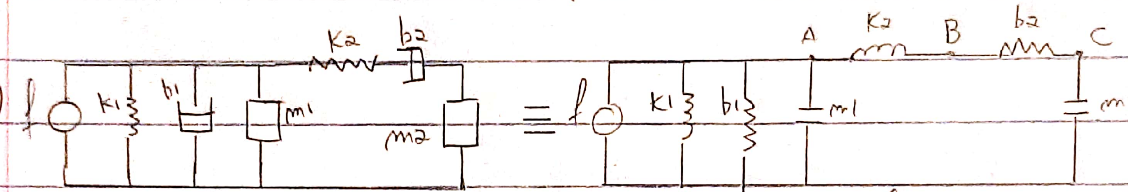
$\dfrac{\partial L}{\partial x_1} = -K_1 x_1 + K_2 (x_3 - x_1)$ / $\dfrac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1$ $\Big\}$ $\therefore m_1 \ddot{x}_1 + K_1 x_1 - K_2 (x_3 - x_1) + b_1 \dot{x}_1 = F(t)$

(x_2) $\dfrac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \longrightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$ / $\dfrac{\partial L}{\partial x_2} = 0$ / $\dfrac{\partial R}{\partial \dot{x}_2} = b_2 (\dot{x}_2 - \dot{x}_3)$

$\therefore m_2 \ddot{x}_2 + b_2 (\dot{x}_2 - \dot{x}_3) = 0$

(x_3) $\dfrac{\partial L}{\partial \dot{x}_3} = 0$ / $\dfrac{\partial L}{\partial x_3} = -K_2 (x_3 - x_1)$ / $\dfrac{\partial R}{\partial \dot{x}_3} = -b_2 (\dot{x}_2 - \dot{x}_3)$ $\Big\}$ $K_2 (x_3 - x_1) - b_2 (\dot{x}_2 - \dot{x}_3) = 0$

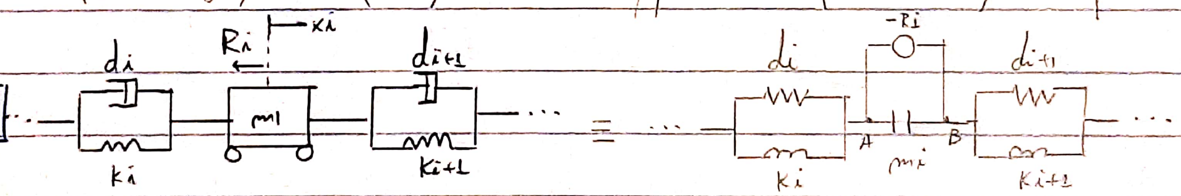
b)



. Nó A: $\left(\dfrac{1}{K_1 D} + \dfrac{1}{b_1} + m_1 D + \dfrac{1}{K_2 D}\right) V_1 - \left(\dfrac{1}{K_2 D}\right) V_2 = f(t)$

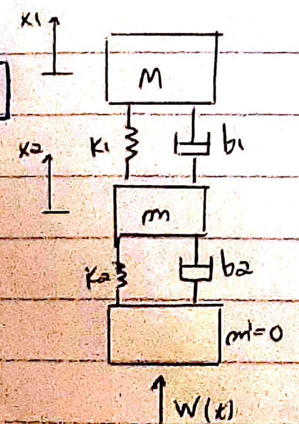
. Nó B: $\left(\dfrac{1}{K_2 D} + \dfrac{1}{b_2}\right) V_2 - \left(\dfrac{1}{b_2}\right) V_3 - \dfrac{1}{K_2 D} V_1 = 0$ / . Nó C: $\left(\dfrac{1}{b_2} + m_2 D\right) V_3 - \left(\dfrac{1}{b_2}\right) V_2 = 0$

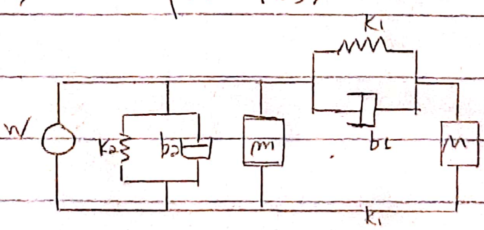
ex7



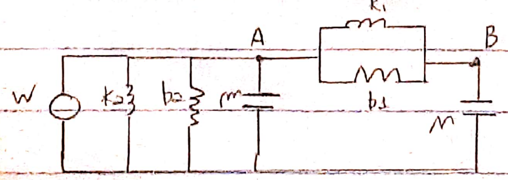
. Nó A: $\left(\dfrac{1}{d_i} + \dfrac{1}{K_i D} + m_i D\right) V_i - \left(\dfrac{1}{d_i} + \dfrac{1}{K_i D}\right) V_{i-1} - m_i D V_{i+1} = -R_i$

ex8





. circuito mecânico



. Analogia elétrica

. Nó A: $\left(\dfrac{1}{K_2 D} + \dfrac{1}{b_2} + mD + \dfrac{1}{K_1 D} + \dfrac{1}{b_1}\right) V_1 - \left(\dfrac{1}{K_1 D} + \dfrac{1}{b_1}\right) V_2 = W(t)$

. Nó B: $\left(\dfrac{1}{K_1 D} + \dfrac{1}{b_1} + MD\right) V_2 - \left(\dfrac{1}{K_1 D} + \dfrac{1}{b_1}\right) V_1 = 0$